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PASSIVE VIBRATION SUPPRESSION OF STRUCTURES IN THE VICINITY OF NATURAL FREQUENCIES USING PIEZOEFFECT

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Abstract: The proposed method of passive vibration suppression of structures is based on the use of the piezoelectric effect, which consists in the ability of the piezoelectric material to convert electrical energy into mechanical energy, and conversely. As it is known if the frequency of forced vibrations tends to the resonant frequency, all the desired quantities (forcers, moments, displacements and deformations) grow indefinitely. A new idea is that as we approach the resonant frequency, we change the electrical conditions on the electrodes of piezoelectric layers, thereby obtaining a different boundary value problem and a different spectrum of natural frequencies. Thus, we manage to get away from the resonant vibrations of the structure. Using the example of a laminated beam with elastic and piezoelectric layers the possibility of damping vibrations caused by mechanical load is studied. In this paper, a mathematically based model is used to solve the problem in question. The calculations are performed and the results are presented in the form of graphs. It is shown that forcers, moments, displacements and deformations of beam in the vicinity of the natural frequency can be significantly reduced by as a result of changes in the electrical conditions on the electrodes of the piezoelectric layers.

Keywords: passive vibration suppression, natural frequencies, piezoeffect, laminated electroelastic beam

ПАССИВНОЕ ГАШЕНИЕ ВИБРАЦИЙ КОНСТРУКЦИИ В ОКРЕСТНОСТИ РЕЗОНАНСНЫХ ЧАСТОТ С ИСПОЛЬЗОВАНИЕМ ПЬЕЗОЭФФЕКТА

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Аннотация: Предлагаемый метод пассивного гашения вибраций конструкции основывается на использовании пьезоэффекта, который заключается в способности пьезоэлектрического материала преобразовывать электрическую энергию в механическую и наоборот. Известно, что если частота вынужденных колебаний стремится к резонансной частоте, то все искомые величины (усилия, моменты, перемещения и деформации) неограниченно растут. Идея гашения заключается в том, что при приближении к резонансной частоте мы изменяем электрические условия на электродах пьезоэлектрических слоев в результате чего получаем другую краевую задачу с другими резонансными частотами. Таким образом, удается избежать резонанса конструкции. Возможность гашения вибрации, вызванной механической нагрузкой, исследуется на примере слоистой балки с упругими и пьезоэлектрическими слоями. Для решения проблемы используется математически обоснованная модель. Выполнены расчеты, результаты которых представлены в виде графиков. Показано, что усилия, моменты, перемещения и деформации балки могут существенно уменьшаться в результате изменения электрических условий на электродах пьезоэлектрических слоев.

Ключевые слова: пассивное гашение вибраций, собственные частоты, пьезоэффект, слоистая электроупругая балка

1. INTRODUCTION

Operation of structures and equipment in dynamic conditions led to the problems of vibration isolation and vibration suppression. For vibration isolation and vibration suppression passive, active systems and their combinations are used. Active vibration isolation and vibration damping systems use external energy sources. These are pneumatic, hydropneumatic and hydromechanical devices and so on. Recently, electro-elastic and magneto-elastic systems [1] - [4] began to be used for vibration isolation and active vibration suppression. As a rule, the analysis of the work of such systems consists in the development of an experimental layout and a schematic diagram. Passive vibration isolation usually consists in the fact that the protected object relies on extremely dimensional springs and vibration isolators. Vibration isolation systems containing only passive elastic and damping elements are called passive. Passive vibration isolation and vibration damping systems do not use external energy sources. So, for effective passive vibration damping and vibration isolation, complex and massive equipment is required. Here, another quench route is proposed using the piezoelectric effect. The method is based on the fact that with changing electrical conditions on the surfaces of the piezoelectric elements, the natural frequencies of the structure are changed. We encounter a similar situation in the dynamics of elastic structures: it is known that if we change the mechanical boundary conditions, for example, if we replace the free from fixation edge on the rigidly fixed edge, then the natural frequencies of the structure will change. Of course, in a working structure, nobody changes the mechanical boundary conditions. Another thing is the electrical boundary conditions: it is easy to short or break the electrodes. Structures with different electrical conditions are described by different boundary value problems, which correspond to different spectrum of natural frequencies. Our goal is to find the optimal way to change the natural vibration frequencies and, by changing the boundary value problem, escape from resonance. On this way, many questions arise: what piezoelements to choose from which material (properties, characteristics, electromechanical coupling coefficient, and so on), what form, what direction of pre-polarization, where and how to place them on the structure, how many electrodes to use, where to place them on the piezoelectric elements, and so on. The paper is the first step of our research of the problem under discussion.

2. BASIC EQUATIONS

A three-layer beam with one elastic layer and two piezoelectric layers located symmetrically with respect to the elastic layer is considered. The middle layer is elastic, the outer layers are made of a piezoelectric material. The number of the elastic layer is 1, the numbers of the upper and lower layers are ± 2, respectively. The thickness of the elastic layer is equal $2h_1$, the thickness of each piezoelectric layer is equal h_2, the length of the rod is l, the width of the beam is g (Fig. 1).

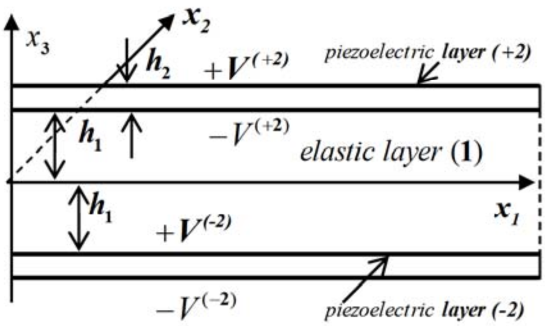

Figure 1. Schematic representation of the structure of the layered beam.

The axis x_1 is directed along the length of the beam, the axis x_2 is directed along the width of the beam, the axis x_3 is orthogonal to them. It is assumed that the piezoelectric layers are pre-polarized in the direction x_3 [5] - [8].

In [8], we constructed the theory of the multilayer electroelastic beams. Here we briefly present these results for a particular case - a three-layer beam.

In the case of thin-walled beams in the equations of state, the stresses σ_{22} and σ_{33} can be neglected compared to the stress σ_{11}. In addition, it is assumed that the electroelastic state does not depend on the coordinate x_2.

Taking into account the assumptions made, the equations for the elastic and electroelastic layers will be written as

Equilibrium equations

$$\frac{\partial \sigma_{ii}^{(k)}}{\partial x_i} + \frac{\partial \sigma_{ij}^{(k)}}{\partial x_j} = \rho \frac{\partial^2 u_i^{(k)}}{\partial t^2}, \quad i \neq j = 1,3 \qquad (1)$$

Strain - displacement formulas

$$e_1^{(k)} = \frac{\partial u_1^{(k)}}{\partial x_1}, \; k = -2,1,2 \qquad (2)$$

Equation of state (Hooke's law) for the elastic layer

$$\sigma_{11}^{(1)} = E_1 e_1^{(1)} \qquad (3)$$

Equations of state for piezoelectric layers

$$\sigma_{11}^{(\pm 2)} = \frac{1}{s_{11}^E} e_1^{(\pm 2)} - \frac{d_{31}}{s_{11}^E} E_3^{(\pm 2)} \qquad (4)$$

$$D_3^{(\pm 2)} = \varepsilon_{33}^T E_3^{(\pm 2)} + d_{31} \sigma_{11}^{(\pm 2)} \qquad (5)$$

where $$E_3^{(\pm 2)} = -\frac{\partial \varphi^{(\pm 2)}}{\partial x_3} \qquad (6)$$

In formulas (1)-(6) u_1 and e_1 are the displacement and deformation in the direction x_1, respectively, E_3 and D_3 are the components of the electric field vector and electric induction vector in the direction x_3, φ is the electric potential, s_{11}^E is the elastic compliance at zero electric field, d_{31} is the piezoelectric constant, ε_{33}^T is the dielectric constant at zero voltages. The notation used is the same as that used in [8].

For our purposes, we will consider piezoelectric layers, in which the faces $x_3 = \text{const}$ are completely covered with electrodes. Here we will consider only two kinds of conditions on the electrodes:

- the electrodes are short-circuited (the electric potential is zero on the electrodes) and
- the electrodes are open.

On short-circuited electrodes, the electric potential is zero

$$\varphi^{(\pm 2)}\Big|_{x_3 = \pm h} = \varphi^{(\pm 2)}\Big|_{x_3 = \pm h_1} = 0 \qquad (7)$$

On open electrodes the electric potential is not zero. It is equal to

$$\varphi^{(\pm 2)}\Big|_{x_3 = \pm h} = \pm V^{(\pm 2)}, \; \varphi^{(\pm 2)}\Big|_{x_3 = \pm h_1} = \mp V^{(\pm 2)} \qquad (8)$$

where the values $V^{(\pm 2)}$ are determined from following integral condition:

$$I = \int_{\Omega} \frac{\partial D_3}{\partial t} d\Omega = 0 \qquad (9)$$

Here the integral is evaluated over the surface Ω of one of the electrodes and t denotes the time.

On the surfaces of the beam, the mechanical surface load is usually specified as

$$\sigma_{13}^{(\pm 2)}\Big|_{x_3 = \pm h} = \pm q_1^{\pm}, \quad \sigma_{33}^{(\pm 2)}\Big|_{x_3 = \pm h} = \pm q_3^{\pm} \qquad (10)$$

The superscript in parentheses indicates the layer number. Hereinafter, each formula with double signs $\pm, \mp$ contains two formulas. To get one formula, one should take only the upper signs, to get the second formula one need to leave only the lower signs.

2. DERIVATION OF EQUATIONS FOR ELECTROELASTIC BEAM THEORY

In order to construct the theory of electroelastic beams, one should accept some assumptions regarding electrical quantities. As in the construction of the theory of piezoelectric plates and shells [7], the content of accepted hypotheses depends on the electrical conditions on the surfaces of the piezoelectric layers. For piezoelectric layers, we accept assumptions that were substantiated by the asymptotic method electro-elastic plates and shells [7].

The mechanical quantities of any layer for which the Kirchhoff hypotheses are valid can be written as the following linear functions of the coordinate x_3

$$u_1 = u_{1,0} + x_3 u_{1,1}, \quad e_1 = \varepsilon + x_3 \kappa \\ \sigma_{11}^{(k)} = \sigma_{11,0}^{(k)} + x_3 \sigma_{11,1}^{(k)}, \quad k = \text{-}2,1,2 \tag{11}$$

where ε and κ are the components of the tangential and bending deformations of the midline of the beam, respectively, k is the number of the layer

$$\varepsilon = \frac{\partial u}{\partial x_1}, \ \kappa = \frac{\partial^2 w}{\partial x_1{}^2}, \ u = u_1\big|_{x_3=0}, \ w = -u_3\big|_{x_3=0} \tag{12}$$

It was shown in the work [7] that the electric potential is a quadratic function of the thickness coordinate x_3

$$\varphi^{(\pm 2)} = \varphi_{,0}^{(\pm 2)} + x_3 \varphi_{,1}^{(\pm 2)} + x_3^2 \varphi_{,2}^{(\pm 2)} \tag{13}$$

We write out the basic formulas for the beam with open electrodes. On open electrodes, the electric potential is non-zero. It is constant at each electrode. This value is determined from the condition (9).

If the electric potential is set on the electrodes (8), then formula (13) can be converted to

$$\varphi^{(\pm 2)} = \mp V^{(\pm 2)} + (x_3 \mp \boldsymbol{h}_1)\left(\frac{\boldsymbol{2}V^{(\pm 2)}}{h_2} \mp h_2 \varphi_{,2}^{(\pm)}\right) \\ + (x_3 \mp h_1)^2 \varphi_{,2}^{(\pm 2)} \tag{14}$$

Taking into account formulas (14) and (6), we get

$$E_{3,0}^{(\pm 2)} = -\frac{\boldsymbol{2}V^{(\pm 2)}}{h_2} \pm (\boldsymbol{h} + h_1)\varphi_{,2}^{(\pm 2)} \\ E_{3,1}^{(\pm 2)} = -\boldsymbol{2}\varphi_{,2}^{(\pm 2)} \tag{15}$$

where

$$E_3^{(\pm 2)} = E_{3,0}^{(\pm 2)} + x_3 E_{3,1}^{(\pm 2)} \tag{16}$$

As a result of the simple transformations the equations of state for the piezoelectric layers with open electrodes can be rewritten as

$$\sigma_{11,0}^{(\pm 2)} = \frac{\mathbf{1}}{s_{11}^E}\varepsilon + \frac{\mathbf{2}d_{31}}{h_2 s_{11}^E} V^{(\pm 2)} \mp \frac{(h+h_1)k_{31}{}^2}{\mathbf{2}s_{11}^E(\mathbf{1}-k_{31}{}^2)}\kappa \\ \sigma_{11,1}^{(\pm 2)} = \frac{\mathbf{1}}{s_{11}^E(\mathbf{1}-k_{31}{}^2)}\kappa \\ \sigma_{11}^{(\pm 2)} = \sigma_{11,0}^{(\pm 2)} + x_3 \sigma_{11,1}^{(\pm 2)} \\ \varphi_{,2}^{(\pm)} = \frac{d_{31}}{2\varepsilon_{33}^T}\sigma_{11,1}^{(\pm 2)} = \frac{k_{31}{}^2}{2d_{31}(1-k_{31}{}^2)}\kappa \\ E_{3,0}^{(\pm 2)} = -\frac{\mathbf{2}V^{(\pm 2)}}{\boldsymbol{h}_2} \pm \frac{(\boldsymbol{h}+h_1)k_{31}{}^2}{\mathbf{2}d_{31}(\mathbf{1}-k_{31}{}^2)}\kappa \\ E_{3,1}^{(\pm 2)} = -\frac{k_{31}{}^2}{d_{31}(\mathbf{1}-k_{31}{}^2)}\kappa \\ D_{3,0}^{(\pm 2)} = \varepsilon_{33}^T(\mathbf{1}-k_{31}{}^2)E_{3,0}^{(\pm 2)} + \frac{d_{31}}{s_{11}^E}\varepsilon \tag{17}$$

$$\int_\Omega \frac{\partial D_{3,0}}{\partial t} d\Omega \\ = \frac{\partial}{\partial t}\int_\Omega \left(\varepsilon_{33}^T(\mathbf{1}-k_{31}{}^2)\boldsymbol{E}_{3,0}^{(\pm 2)} + \frac{d_{31}}{s_{11}^E}\varepsilon\right)d\Omega = \mathbf{0} \tag{18}$$

We write out the basic formulas for the beam with short-circuited electrodes taking into account formulas (7) ($V^{(\pm 2)} = \mathbf{0}$) in equations (17)

$$\sigma_{11,0}^{(\pm 2)} = \frac{\mathbf{1}}{s_{11}^E}\varepsilon \mp \frac{(\boldsymbol{h}+h_1)k_{31}{}^2}{\mathbf{2}s_{11}^E(\mathbf{1}-k_{31}{}^2)}\kappa \\ \sigma_{11,1}^{(\pm 2)} = \frac{\mathbf{1}}{s_{11}^E(\mathbf{1}-k_{31}{}^2)}\kappa \\ E_{3,0}^{(\pm 2)} = \pm(\boldsymbol{h}+h_1)\varphi_{,2}^{(\pm 2)} \\ D_{3,0}^{(\pm 2)} = \pm\frac{\varepsilon_{33}^T}{\mathbf{2}d_{31}}((\boldsymbol{h}+h_1)k_{31}{}^2\kappa \pm \mathbf{2}k_{31}{}^2\varepsilon) \tag{19}$$

We apply the notations of beam theory to our equations. Integrating the stresses in thickness of the beam, we find the resulting tangential force T and bending moment G

$$T = \int_{-h}^{+h} \sigma_{11} dx_3, \quad G = -\int_{-h}^{+h} \sigma_{11} x_3 dx_3 \qquad (20)$$

Having integrated the equations of motion and the equations of state for each layer, we obtain one-dimensional equations for a three-layer electroelastic beam. As an example, we consider the damping of harmonic vibrations of a three-layer beam (all values vary according to the variable t by the law $e^{-i\omega t}$, where the variable t is the time, ω is the circular frequency of vibrations). In the future, we will write down all the equations and boundary conditions with respect to the amplitude values of the unknown quantities.

The equations of the theory of layered electroelastic beams of a symmetric structure have exactly the same form as in the case of elastic beams. The problem in question, as in the theory of elasticity, is divided into two problems - a plane problem and a bending problem.

Note that the electric potential is odd function in plane problem

$$V^{(2)} = V^{(-2)} = V_p$$

Plane problem for beam with shot-circuited electrodes ($V_p = 0$,)

$$\frac{dT}{dx_1} + X + 2h\rho\omega^2 u = 0, \; T = A\varepsilon, \; \varepsilon = \frac{du}{dx_1} \qquad (21)$$

$$\sigma_{11,0}^{(\pm 2)} = \frac{1}{s_{11}^E}\varepsilon, \; E_{3,0}^{(\pm 2)} = 0, \; D_{3,0}^{(\pm 2)} = \frac{d_{31}}{s_{11}^E}\varepsilon \qquad (22)$$

$$X = q_1^+ + q_1^-, \; A = 2h_1 E + \frac{2h_2}{s_{11}^E}, \; \rho = \frac{1}{h}(\rho_1 h_1 + \rho_2 h_2)$$

Plane problem for beam with open-electrodes

$$\frac{dT}{dx_1} + X + 2h\rho\omega^2 u = 0$$

$$T = A\varepsilon + P^{(d)}, \; \varepsilon = \frac{du}{dx_1} \qquad (23)$$

$$P^{(d)} = \frac{2}{s_{11}^E}\frac{h_2}{l}\frac{k_{31}^2}{1-k_{31}^2}\left(u\big|_{x_1=l} - u\big|_{x_1=0}\right)$$

$$\sigma_{11,0}^{(\pm 2)} = \frac{1}{s_{11}^E}\varepsilon + \frac{2d_{31}}{h_2 s_{11}^E}V_p, \; E_{3,0} = -\frac{2V_p}{h_2}$$

$$D_{3,0}^{(\pm 2)} = \varepsilon_{33}^T(1-k_{31}^2)E_{3,0}^{(\pm 2)} + \frac{d_{31}}{s_{11}^E}\varepsilon \qquad (24)$$

$$V_p = \frac{h_2}{2ld_{31}}\frac{k_{31}^2}{1-k_{31}^2}\left(u\big|_{x_1=l} - u\big|_{x_1=0}\right)$$

We determine the value V_p by equation (18).

The electric potential is an even function of the variable x_3 in bending problem

$$V^{(2)} = -V^{(-2)} = V_b$$

Bending problem for beam with shot-circuited electrodes ($V_b = 0$)

$$\frac{dN}{dx_1} + Z + 2h\rho\omega^2 w = 0, \; N = \frac{dG}{dx_1}$$

$$G = M\kappa, \; \kappa = \frac{d^2 w}{dx_1^2} \qquad (25)$$

$$Z = -(q_3^+ + q_3^-)$$

$$M = -\frac{2h_1^3 E}{3} - \frac{2(h^3 - h_1^3)\kappa}{3s_{11}^E(1-k_{31}^2)}\left(1 - \frac{3h_2(h^2 - h_1^2)k_{31}^2}{4(h^3 - h_1^3)}\right)$$

$$\sigma_{11,0}^{(\pm 2)} = \mp\frac{(h+h_1)k_{31}^2}{2s_{11}^E(1-k_{31}^2)}\kappa$$

$$\sigma_{11,1}^{(\pm 2)} = \frac{1}{s_{11}^E(1-k_{31}^2)}\kappa$$

$$E_{3,0}^{(\pm 2)} = \pm\frac{(h+h_1)k_{31}^2}{2d_{31}(1-k_{31}^2)}\kappa \qquad (26)$$

$$E_{3,1}^{(\pm 2)} = \frac{-k_{31}^2}{d_{31}(1-k_{31}^2)}\kappa$$

$$D_{3,0}^{(\pm 2)} = \varepsilon_{33}^T(1-k_{31}^2)E_{3,0}^{(\pm 2)}$$

Here N is the shear force

$$N = -\int_{-h}^{+h} \sigma_{13} dx_3$$

Bending problem for beam with open-electrodes

$$\frac{dN}{dx_1} + Z + 2h\rho\omega^2 w = 0,\ N = \frac{dG}{dx_1},$$
$$G = M\kappa + Q,\quad \kappa = \frac{d^2 w}{dx_1^2}$$
$$Q = -\frac{h_2(h+h_1)^2}{2l}\frac{k_{31}{}^2}{s_{11}^E(1-k_{31}{}^2)} \cdot \left(\left.\frac{dw}{dx}\right|_{x=l} - \left.\frac{dw}{dx}\right|_{x=0}\right) \tag{27}$$
$$Q = -\frac{d_{31}}{s_{11}^E}\frac{h^2 - h_1^2}{h_2}2V_b$$

$$\sigma_{11,0}^{(\pm 2)} = \pm\frac{2d_{31}}{h_2 s_{11}^E}V_b \mp \frac{(h+h_1)k_{31}{}^2}{2s_{11}^E(1-k_{31}{}^2)}\kappa \tag{28}$$
$$E_{3,0}^{(\pm 2)} = \mp\frac{2V_b}{h_2} \pm \frac{(h+h_1)k_{31}{}^2}{2d_{31}(1-k_{31}{}^2)}\kappa$$

The quantities $\boldsymbol{Z}, \boldsymbol{M}$, $\sigma_{11,1}^{(\pm 2)}, D_{3,0}^{(\pm 2)}, E_{3,1}^{(\pm 2)}$ are determined by formulas (26).

3. DYNAMIC PLANE PROBLEM

We will consider the forced harmonic vibrations of the beam under the action of a constant distributed load with the following boundary conditions:

$$u\big|_{x_1=0} = 0,\ T\big|_{x_1=l} = 0 \tag{29}$$

For numerical examples, we introduce dimensionless coordinates and dimensionless sought quantities

$$\xi = \frac{x}{l},\ u_* = \frac{u}{l},\ \varepsilon = \varepsilon_*,\ T_* = \frac{T}{A}$$
$$X_* = \frac{Xl}{A}, E_{3*} = d_{31}E_3,\ D_{3*} = \frac{s_{11}^E}{d_{31}}D_3 \tag{30}$$

Let the beam electrodes be short-circuited. Submitting the formulas (30) in the equations (21), we obtain the following system of equations

$$\frac{dT_*}{d\xi} + X_* + \lambda_1^2 u_* = 0,\ T_* = \varepsilon_*,\quad \varepsilon_* = \frac{du_*}{d\xi}$$
$$\lambda_1^2 = \frac{2h\rho\rho^2 l^2}{A} \tag{31}$$

where λ_1^2 is the dimensionless frequency parameter.

The resolving equation is

$$\frac{d^2 u_*}{d\xi^2} + \lambda_1^2 u_* + X_* = 0 \tag{32}$$

Its solution is

$$u_* = c_1 \sin\lambda_1\xi + c_2\cos\lambda_1\xi - \frac{1}{\lambda_1^2}X_*$$
$$T_* = \lambda_1(c_1\cos\lambda_1\xi - c_2\sin\lambda_1\xi) \tag{33}$$

Arbitrary integration constants c_1 and c_2 are determined from the conditions at the ends of the beam

$$u_*\big|_{\xi=0} = 0,\ T_*\big|_{\xi=1} = 0 \tag{34}$$

Satisfying conditions (34), we get

$$c_1 = \frac{\sin\lambda_1}{\lambda_1{}^2\cos\lambda_1}X_*,\quad c_2 = \frac{1}{\lambda_1{}^2}X_* \tag{35}$$

The natural frequencies are determined from the equation $\cos\lambda_1 = 0$ and they are equal to

$$n\pi + \pi/2,\ n = 0,1,2,3,... \tag{36}$$

Consider the plane problem for beam with open-electrodes. The system of equations according to the sought quantities has the form

$$\frac{dT_*}{d\xi} + X_* + \lambda_1^2 u_* = 0,\ T_* = \varepsilon_* + P_*,\ \varepsilon_* = \frac{du_*}{d\xi}$$
$$P_* = ru_*\big|_{\xi=1},\ r = \frac{2h_2}{As_{11}^E}\frac{k_{31}^2}{1-k_{31}^2},\ k_{31}^2 = \frac{d_{31}^2}{s_{11}^E\varepsilon_{33}^T} \tag{37}$$

After solving the problem (37), (34), the following desired quantities can be calculated by the formulas:

$$\sigma_{11,0}^{(\pm 2)} = \frac{1}{s_{11}^E}\varepsilon + \frac{2d_{31}}{h_2 s_{11}^E}V_p, \quad P = 4\frac{d_{31}}{s_{11}^E}V_p$$

$$V_p = \frac{h_2}{2ld_{31}}\frac{k_{31}^2}{1-k_{31}^2}u\Big|_{x_1=l}, \quad E_{3,0} = -\frac{2V_p}{h_2} \qquad (38)$$

$$D_{3,0}^{(\pm 2)} = \varepsilon_{33}^T(1-k_{31}{}^2)E_{3,0}^{(\pm 2)} + \frac{d_{31}}{s_{11}^E}\varepsilon$$

The solution resolving equation has the form (33). The force T_* is determined by formula

$$T_* = \lambda_1(c_1\cos\lambda_1\xi - c_2\sin\lambda_1\xi) + P_* \qquad (39)$$

Arbitrary integration constants c_1 and c_2 are determined from the conditions at the ends of the beam (34)

$$c_1 = \frac{X_*}{\lambda_1{}^2}\frac{\lambda_1\sin\lambda_1 - r(\cos\lambda_1 - 1)}{\lambda_1\cos\lambda_1 + r\sin\lambda_1}, \quad c_2 = \frac{X_*}{\lambda_1{}^2} \qquad (40)$$

The natural frequencies are determined from the equation

$$\lambda_1\cos\lambda_1 + r\sin\lambda_1 = 0 \qquad (41)$$

The first natural frequency for different values r is 1.602 (r=0.05), 1.630 (r=0.1), and 1.689 (r=0.2).

Let a beam with short-circuited electrodes vibrates with a frequency close to the first resonant frequency $\lambda_1 = 1.56$. In order to reduce the amplitudes of vibrations of the desired quantities, we will open the electrodes of the piezoelectric layers. Perform a numerical calculation using formulas (33), (35), (39), (40). Figs. 2, 3 show the displacement u_* and the force T_* as function of the longitudinal coordinate ξ. The dashed line represents the same values for a beam with short-circuited electrodes. Thick (thin) line represents the same values for a beam with open electrodes at r = 0.05 (r = 0.1).

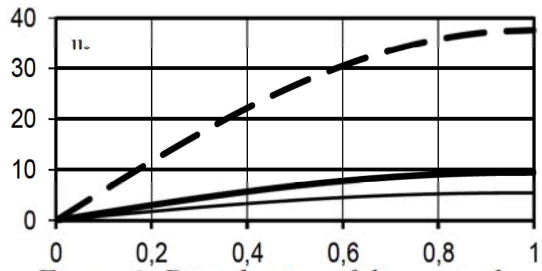

Figure 2. Distribution of dimensionless displacement u_* along the length of a beam.

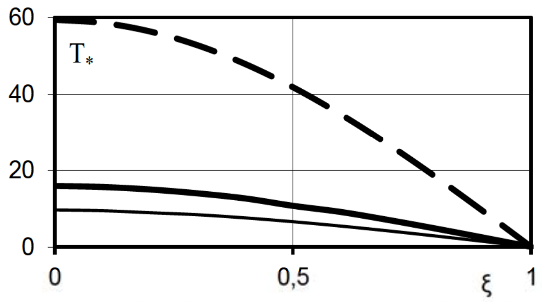

Figure 3. Distribution of dimensionless force T_* along the length of a beam.

From the graphs it can be seen that the vibration amplitudes for a beam with open electrodes are substantially less than for a beam with short-circuited electrodes.

4. DYNAMIC BENDING PROBLEM

Consider the case when only constant distributed load Z_* acts on the beam. We assume that the edge $x_1 = 0$ rigidly fixed, and the edge $x_1 = l$ is free from fixings

$$w\big|_{x_1=0} = 0, \quad \frac{dw}{dx_1}\bigg|_{x_1=0} = 0, \quad G\big|_{x_1=l} = 0, \quad N\big|_{x_1=l} = 0$$

We introduce the dimensionless sought values

$$\xi = \frac{x}{l},\ w_* = \frac{w}{l},\ \kappa_* = l\kappa = \frac{d^2 w_*}{d\xi^2}$$

$$N_* = \frac{l^2}{M}N,\ G_* = \frac{l}{M}G, \tag{42}$$

$$Z_* = \frac{l^3}{M}Z, Q_* = Q\frac{l}{M}$$

Let the beam electrodes be short-circuited. Taking into account the formulas (42), the system of equations (25) will be rewritten as ($V_b = 0,\ Q = 0$)

$$\frac{dN_*}{d\xi} + Z_* - \lambda_2^4 w_* = 0,\ N_* = \frac{dG_*}{d\xi}$$

$$G_* = \kappa_*,\ \kappa_* = \frac{d^2 w_*}{d\xi^2} \tag{43}$$

The resolving equation for the bending problem is written as

$$\frac{d^4 w_*}{d\xi^4} - \lambda_2^4 w_* + Z_* = 0,\ \lambda_2^4 = -\frac{2h\rho\omega^2 l^4}{M} \tag{44}$$

The solution of equation (44) is

$$w_* = c_1 ch\lambda_2\xi + c_2 sh\lambda_2\xi + c_3 cos\lambda_2\xi + c_4 \sin\lambda_2\xi + \frac{1}{\lambda_2^4}Z_* \tag{45}$$

Arbitrary integration constants c_1 and c_2 are determined from the conditions at the ends of the beam

$$w_*\big|_{\xi=0} = 0,\ \frac{dw_*}{d\xi}\bigg|_{\xi=0} = 0,\ G_*\big|_{\xi=1} = 0,\ N_*\big|_{\xi=1} = 0$$

Find arbitrary integration constants c_1, c_2, c_3, c_4

$$c_1 = -c_3 - \frac{1}{\lambda_2^4}Z_* = -\frac{Z_*}{2\lambda_2^4\delta}(cc + ss + 1)$$
$$c_2 = -c_4 = \frac{Z_*}{2\lambda_2^4\delta}(cs + sc) \tag{46}$$

$$\delta = cc + 1 \tag{47}$$

The new notations are introduced in formulas (46)

$$cc = ch\lambda_2\cos\lambda_2,\ ss = sh\lambda_2\sin\lambda_2$$
$$sc = sh\lambda_2\cos\lambda_2,\ cs = ch\lambda_2\sin\lambda_2 \tag{48}$$

We calculate the natural frequencies of the beam using the equation (47). The first three natural frequencies are 1.875, 4.694, 7.855.
Consider the bending problem for beam with open-electrodes $V^{(2)} = -V^{(-2)} = V_b$. The system of equations according to the dimensionless sought quantities has the form

$$\frac{dN_*}{d\xi} + Z_* - \lambda_2^4 w_* = 0,\ N_* = \frac{dG_*}{d\xi}$$

$$G_* = \kappa_* + Q_*,\ \kappa_* = \frac{d^2 w_*}{d\xi^2}$$

$$Q_* = t\frac{dw_*}{d\xi}\bigg|_{\xi=1},\ t = -\frac{h_2(h+h_1)^2}{2s_{11}^E M}\frac{k_{31}^2}{1-k_{31}^2}$$

Satisfying the boundary conditions, we obtain arbitrary integration constants and the equation for determining the resonant frequencies

$$c_1 = -c_3 - \frac{Z_*}{\lambda_2^4} = -\frac{Z_*}{2\lambda_2^4\delta}\left(\lambda_2(cc + ss + 1) + 2t\cdot cs\right)$$
$$c_2 = -c_4 = -\frac{Z_*}{2\lambda_2^4\delta}\left(\lambda_2(cs + sc + 1) + 2t\cdot ss\right) \tag{49}$$

$$\delta = \lambda_2(cc + 1) + t(cs + sc) \tag{50}$$

In formulas (49), (50), the notation (48) is used.
We calculate the natural frequencies of the beam using the equation (50). The first three natural frequencies are 1.8891; 4.705; 7.8611 for $t = 0.05$ and 1.9022; 4.7563; 7.8673 for $t = 0.1$.
The results of the calculations are presented in the form of graphs. Figures 4-6 show the dependence of the deflection, the shear force and the bending moment on the longitudinal coordi-

nate of the beam in the vicinity of the second resonant frequency of the beam with short-circuited electrodes.

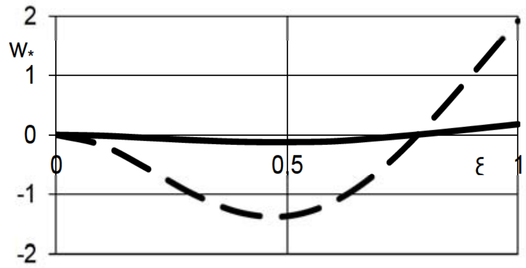

Figure 4. The deflection w_* as a function of the coordinate ξ near the second resonance of the beam at $\lambda_2 = 4.693$.

In all figures, the dashed line represents the quantities of a beam with short-circuited electrodes and the solid line represents the quantities for a beam with disconnected electrodes.

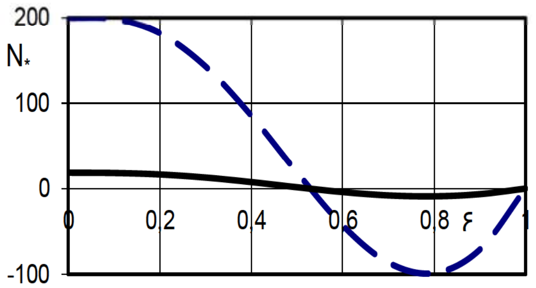

Figure 5. The shear force N_* as a function of the coordinate ξ near the second resonance of the beam at $\lambda_2 = 4.693$.

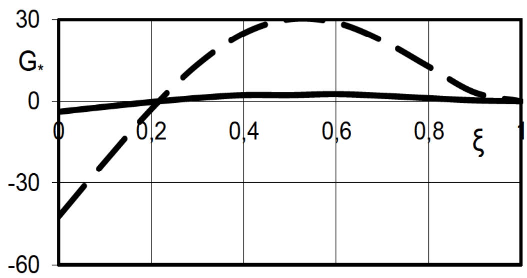

Figure 6. The moment G_* as function of the coordinate ξ near the second resonance at $\lambda_2 = 4.693$.

The performed calculations confirm that a change in the electrical conditions on the electrodes of the piezoelectric layers makes it possible to avoid resonance and, as a consequence, an increase in the amplitude values of the desired values.

CONCLUSION

A study was carried out on the new method of vibration damping of a structure at resonance using the example of a multi-layer beam. Analytical solutions of the problem of beam passive vibration suppression are obtained. Numerical calculations were performed confirming the effectiveness of using the piezoelectric effect for passive vibration control. It is shown that forcers, moments, displacements and deformations of beam in the vicinity of the natural frequency can be significantly reduced by as a result of changes in the electrical conditions on the surfaces of the piezoelectric layers.

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