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ABOUT SEVERAL NUMERICAL AND SEMIANALYTICAL METHODS OF LOCAL STRUCTURAL ANALYSIS

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Abstract: Numerical or semianalytical solution of problems of structural mechanics with immense number of unknowns is time-consuming process. High-accuracy solution at all points of the model is not required normally, it is necessary to find only the most accurate solution in some pre-known domains. The choice of these domains is a priori data with respect to the structure being modelled. Designers usually choose domains with the so-called edge effect (with the risk of significant stresses that could lead to destruction of structures) and regions which are subject to specific operational requirements. Stress-strain state in such domains is important. Wavelets provide effective and popular tool for local structural analysis. Operational and variational formulations of problems of structural mechanics with the use of method of extended domain are presented. After discretization and obtaining of governing equations, problems are transformed to a multilevel space by multilevel wavelet transform. Discrete wavelet basis is used and corresponding direct and inverse algorithms of transformations are performed. Due to special algorithms of averaging, reduction of the problems is provided. Wavelet-based methods allows reducing the size of the problems and obtaining accurate results in selected domains simultaneously. These are rather efficient methods for evaluation of local phenomenon in structures.

Keywords: numerical methods, semianalytical methods, local structural analysis, structural mechanics, wavelet-based methods, reduction, operational formulations, variational formulation, boundary problem

О НЕКОТОРЫХ ЧИСЛЕННЫХ И ПОЛУАНАЛИТИЧЕСКИХ МЕТОДАХ ЛОКАЛЬНОГО РАСЧЕТА СТРОИТЕЛЬНЫХ КОНСТРУКЦИЙ

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Аннотация: Численное и полуаналитическое (численно-аналитическое) решение краевых задач строительной механики, нередко характеризующих огромным количеством неизвестных, сопряжено с большим объемом вычислительной работы и значительными временными затратами. Как правило, отсутствует необходимость в обеспечении высокой точности решения во всех точках соответствующей вычислительной модели, зачастую имеется потребность в нахождении высокоточного решения в некотором наборе областей (зон) конструкции, расположение которых, как правило, заранее известно (это своего рода априорная оценка). Расчетчики в этом отношении традиционно выбирают зоны так называемого краевого эффекта (потенциально опасные с точки зрения уровня возникающих напряжений, способных повлечь разрушение конструкций), а также зоны, внимание к которым обусловлено, например, разного рода технологической спецификой и т.д. Для указанных областей важнейшим вопросом является достоверное определение напряженно-деформированного состояния. Вейвлет-анализ является исключительно эффективным инструментарием для построения локальных решений соответствующих краевых задач строительной механики. Постановка последних в статье приводится в операторном и вариационном видах на основе использования метода расширенной (стандартной) области. После введения соответствующей аппроксимации реализуется переход от указанных континуальных постановок к дискретным и дискретно-континуальным. Далее реализуется прямое вейвлет-преобразование с привлечением дискретного вейвлет-базиса (заметим, что предложены соответствующие эффективные алгоритмы прямого и обратного вейвлет-преобразования). Разработанные вейвлет-версии методов локального расчета строительных конструкций позволяют с одной стороны значительно сократить вычислительную размерность решаемых задач, а с другой стороны обеспечить высокую точность получаемых результатов в выбранных областях (зонах) строительных конструкций.

Ключевые слова: численные методы, полуаналитические методы, локальный расчет строительных конструкций, локальные решения краевых задач, строительная механика, вейвлет-реализации, редукция, операторные постановки задач, вариационные постановки задач, краевая задача

1. BASIC FORMULAS OF FAST DIRECT AND INVERSE DISCRETE HAAR TRANSFORMS AND AVERAGING

1.1. One-dimensional problems

1.1.1. Algorithms of fast direct and inverse discrete Haar transforms. Let us consider the one-dimensional region

$$\omega = \{ x : a \le x \le b \} ,$$

where x is coordinate, a, b are lower and upper limits of interval. Let us divide ω into $(n-1)$ equal parts, where $n = 2^M$, M is the number of levels in the Haar basis [1-5]. Coordinates of mesh nodes are

$$x_i = a + (i-1)h,\ i = 1, 2, ..., n; \quad h = (b-a)/(n-1) . \quad (1)$$

Haar mesh functions are defined by formulas (N_p is the number of Haar functions at level p):

$$\psi_j^p(i) = \alpha_p^{-1} \begin{cases} 1, & 2^{p+1}(j-1) < i \le 2^p(2j-1) \\ -1, & 2^p(2j-1) < i \le 2^{p+1} j \\ 0, & i \le 2^{p+1}(j-1) \ \bigcup \ i > 2^{p+1} j, \end{cases}$$

$$i = 1, 2, ..., n,\ \ 0 \le p < M;$$

$$\psi_1^M(i) = \alpha_M^{-1},\ \ i = 1, 2, ..., n; \quad (2)$$

$$N_p = \begin{cases} n / 2^{p+1},\ 0 \le p < M \\ 1,\ p = M; \end{cases}$$

$$\alpha_p = \begin{cases} \sqrt{2^{p+1}},\ 0 \le p < M \\ \sqrt{2^M} = \sqrt{n},\ p = M. \end{cases} \quad (3)$$

Let $f(i)$ be arbitrary mesh function. Then we have

$$f(i)=\sum_{p=0}^{M}\sum_{j=1}^{N_p} v_j^p \psi_j^p(i),$$
$$v_j^p=\sum_{i=1}^{n} f(i)\psi_j^p(i),\ \ j=1,2,...,N_p, \quad p=0,1,...,M, \tag{4}$$

where $v_j^p,\ j=1,2,...,N_p,\ p=0,1,...,M$ are Haar expansion coefficients.

Algorithm of fast direct discrete Haar transform is described below.

$$u_j^0=f(j),\ j=1,2,...,n;\quad \alpha_0=\sqrt{2}. \tag{5}$$

We have (for all $p=0,1,...,M-1,\ j=1,2,...,N_p$):

$$v_j^p=\alpha_p^{-1}(u_{2j-1}^p-u_{2j}^p);$$
$$u_j^{p+1}=u_{2j-1}^p+u_{2j}^p;\quad \alpha_{p+1}=\sqrt{2}\alpha_p; \tag{6}$$
$$\alpha_M=\sqrt{n};\quad v_1^M=\alpha_M^{-1}u_1^M, \tag{7}$$

where $u_j^p,\ j=1,2,...,N_p,\ p=0,1,...,M$ are auxiliary quantities.

Algorithm of fast inverse Haar transform is described below.

$$\alpha_M=\sqrt{n};\quad \alpha_{M-1}=\sqrt{n};\quad u_1^M=\alpha_M^{-1}v_1^M. \tag{8}$$

We have ($p=M-1,M-2,...,0,\ i=1,2,...,N_p$):

$$j=[(i+1)/2];\quad z=(-1)^{i+1};$$
$$u_j^p=\alpha_p^{-1}zv_j^p+u_j^{p+1};\quad \alpha_{p-1}=\alpha_p/\sqrt{2}. \tag{9}$$

Thus,

$$f(i)=u_i^0,\ \ i=1,2,...,n. \tag{10}$$

1.1.2. Algorithm of averaging. In many cases it is not necessary to obtain global solution in the domain. Local solution for several prescribed subdomains is normally required. If we don’t need to find a complete solution we can reduce the number of unknowns without significant loss of accuracy or with a small error in local solutions. It is reasonable to eliminate unknown expansion coefficients of the basis functions with supports substantially distant from the considering area. Algorithm of averaging in one-dimensional case is described below.

Let us assume that it is necessary to make averaging at some level number q. For all $p=0,1,\dots,q$ and $j=1,2,\dots,N_p$ we suppose

$$(Du^p)_{2j-1}\approx(Du^p)_{2j}\approx(D\tilde{u}^p)_{2j-1},$$
$$v_{2j-1}^p=v_{2j}^p,\quad j=1,2,\dots,N_{p+1}, \tag{11}$$
$$\tilde{u}_{2j-1}^p=(u_{2j-1}^p+u_{2j}^p)/2;$$
$$(D\tilde{u}^p)_{2j-1}=(\tilde{u}_{2j}^p-\tilde{u}_{2j-1}^p)/(2^{p+1}h); \tag{12}$$

Then formulas of averaging have the form

$$v_{2j-1}^p=v_{2j}^p=\beta v_j^{p+1},\quad j=1,2,...,N_{p+1};$$
$$\beta=1/(2\sqrt{2}). \tag{13}$$

1.2. Two-dimensional problems.

1.2.1. Algorithms of fast direct and inverse discrete Haar transforms. Let us consider the two-dimensional rectangular domain

$$\omega=\{(x_1,x_2):\ 0\le x_1\le l_1,\ 0\le x_2\le l_2\},$$

where x_1,x_2 are coordinates; l_1,l_2 are dimensions along x_1,x_2. Let us divide ω into $(n-1)$ equal parts along x_1 and into $(n-1)$ equal parts along x_2, where $n=2^M$, M is the number of levels in the Haar basis. We have the following formulas for coordinates of mesh nodes:

$$x_{1,i}=(i_1-1)h_1,\ i_1=1,2,...,n;$$
$$x_{2,i}=(i_2-1)h_2,\ i_2=1,2,...,n,$$
$$h_1=l_1/(n-1);\quad h_2=l_2/(n-1). \tag{14}$$

Haar mesh functions

$$\psi_{s_1,s_2,j_1,j_2}^p(i_1,i_2),\ \ p=1,2,...,M,$$
$$j_1,\ j_2=1,2,...,N_p,\ \ s_1,s_2=0,1$$

(except $s_1 = s_2 = 0$) can be defined by formulas:

$$\psi^p_{s_1,s_2,j_1,j_2}(i_1,i_2) = \\ = \alpha_p^{-1}\begin{cases} (-1)^{k_1 s_1 + k_2 s_2}, \\ \bigcap_{q=1}^{2}\bigcup_{k_q=0}^{1}\left(\begin{array}{l} 2^{p+1}(j_q - 1 + \frac{k_q}{2}) < i_q \wedge \\ i_q \le 2^{p+1}(j_q - \frac{1}{2} + \frac{k_q}{2}) \end{array}\right), \\ 0, \quad \text{in other cases}; \end{cases}$$

$$\psi^M_{0,0,1,1}(i_1,i_2) = \alpha_M^{-1}; \quad (15)$$

$$N_p = \begin{cases} n/2^{p+1}, 0 \le p < M \\ 1, \ p = M; \end{cases}$$

$$\alpha_p = \begin{cases} 2^{p+1}, \ 0 \le p < M \\ 2^M = n, \ p = M. \end{cases} \quad (16)$$

Let $f(i_1,i_2)$ be an arbitrary mesh function. Consequently we have

$$f(i_1,i_2) = v^M_{0,0,1,1}\psi^M_{0,0,1,1} + \\ + \sum_{p=0}^{M-1}\sum_{j_1=1}^{N_p}\sum_{j_2=1}^{N_p}(v^p_{1,0,j_1,j_2}\psi^p_{1,0,j_1,j_2}(i_1,i_2) + \\ + v^p_{0,1,j_1,j_2}\psi^p_{0,1,j_1,j_2}(i_1,i_2) + v^p_{1,1,j_1,j_2}\psi^p_{1,1,j_1,j_2}(i_1,i_2)), \quad (17)$$

where $v^p_{1,0,j_1,j_2}, v^p_{0,1,j_1,j_2}, v^p_{1,1,j_1,j_2}$, $j_1, j_2 = 1,2,...,N_p$, $p = 1, 2, ..., M$ are Haar expansion coefficients,

$$v^p_{s_1,s_2,j_1,j_2} = \sum_{i_1=1}^{N}\sum_{i_2=1}^{N} f(i_1,i_2)\psi^p_{s_1,s_2,j_1,j_2}(i_1,i_2). \quad (18)$$

Algorithm of fast direct discrete Haar transform is described below.

$$u^0_{j_1,j_2} = f(j_1,j_2), \ j_1 = 1,2,...,n, \ j_2 = 1,2,...,n; \\ \alpha_0 = 2. \quad (19)$$

We have (for all $p = 0,1,...,M-1$, $j_1, j_2 = 0,1,...,N_p$, $s_1, s_2 = 0,1$ (except $s_1 = s_2 = 0$)):

$$z_1 = (-1)^{s_1}, \quad z_2 = (-1)^{s_2}; \quad \alpha_{p+1} = 2\cdot\alpha_p; \quad (20)$$

$$v^p_{s_1,s_2,j_1,j_2} = \alpha_p^{-1}(u^p_{2j_1-1,2j_2-1} + z_1 u^p_{2j_1,2j_2-1} + \\ + z_2 u^p_{2j_1-1,2j_2} + z_1 z_2 u^p_{2j_1,2j_2}); \quad (21)$$

$$u^{p+1}_{j_1,j_2} = u^p_{2j_1-1,2j_2-1} + u^p_{2j_1,2j_2-1} + \\ + u^p_{2j_1-1,2j_2} + u^p_{2j_1,2j_2}; \quad (22)$$

$$\alpha_M = n; \quad v^M_{0,0,1,1} = \alpha_M^{-1}u^M_{1,1}, \quad (23)$$

where $u^p_{j_1,j_2}$, $j_1, j_2 = 1, 2, ..., N_p$, $p = 1, 2, ..., M$ are auxiliary quantities.

Algorithm of fast inverse Haar transform is described below.

$$\alpha_M = n; \quad \alpha_{M-1} = n; \quad u^M_{1,1} = \alpha_M^{-1}v^M_{0,0,1,1}. \quad (24)$$

We have ($p = M-1, M-2, ..., 0, i_1, i_2 = 1, 2, ..., N_p$):

$$j_1 = [(i_1+1)/2]; \quad j_2 = [(i_2+1)/2]; \\ z_1 = (-1)^{i_1+1}; \quad z_2 = (-1)^{i_2+1}; \quad \alpha_{p-1} = \alpha_p/2; \quad (25)$$

$$u^p_{i_1,i_2} = \\ = \alpha_p^{-1}(z_1 v^p_{1,0,j_1,j_2} + z_2 v^p_{0,1,j_1,j_2} + z_1 z_2 v^p_{1,1,j_1,j_2}) + \\ + u^{p+1}_{j_1,j_2}. \quad (26)$$

Thus,

$$f(i_1,i_2) = u^0_{i_1,i_2}, \quad i_1 = 0,1,...,n, \quad i_2 = 0,1,...,n. \quad (27)$$

1.2.2. Algorithm of averaging. Let us assume that it is necessary to make averaging at level q. For all $p = 1, 2, ..., q$, $j_1, j_2 = 1, 2, ..., N_p$, $s_1, s_2 = 0,1$ (except $s_1 = s_2 = 0$) we suppose

$$(D_1 u^p)_{2j_1-1,2j_2-1} = (D_1 u^p)_{2j_1-1,2j_2} = \\ = (D_1 u^p)_{2j_1,2j_2-1} = (D_1 u^p)_{2j_1,2j_2} \approx \\ \approx (D_1 \tilde{u}^p)_{2j_1-1,2j_2-1}; \quad (28)$$

$$(D_2 u^p)_{2j_1-1,2j_2-1} = (D_2 u^p)_{2j_1-1,2j_2} = \\ = (D_2 u^p)_{2j_1,2j_2-1} = (D_2 u^p)_{2j_1,2j_2} \approx \\ \approx (D_2 \tilde{u}^p)_{2j_1-1,2j_2-1}; \quad (29)$$

$$(D_2^+ D_1^+ u^p)_{2j_1-1,2j_2-1} = (D_2^+ D_1^+ u^p)_{2j_1-1,2j_2} = \\ = (D_2^+ D_1^+ u^p)_{2j_1,2j_2-1} = (D_2^+ D_1^+ u^p)_{2j_1,2j_2} \approx \\ \approx (D_2^+ D_1^+ \tilde{u}^p)_{2j_1-1,2j_2-1}; \tag{30}$$

$$v^p_{s_1,s_2,2j_1-1,2j_2-1} = v^p_{s_1,s_2,2j_1,2j_2-1} = \\ = v^p_{s_1,s_2,2j_1-1,2j_2} = v^p_{s_1,s_2,2j_1,2j_2}; \tag{31}$$

$$\tilde{u}^p_{j_1,j_2} = (u^p_{j_1,j_2} + u^p_{j_1+1,j_2} + u^p_{j_1,j_2+1} + u^p_{j_1+1,j_2+1})/4; \tag{32}$$

$$(D_1^+ u^p)_{j_1,j_2} = (u^p_{j_1+1,j_2} - u^p_{j_1,j_2})/(2^p h); \\ (D_2^+ u^p)_{j_1,j_2} = (u^p_{j_1,j_2+1} - u^p_{j_1,j_2})/(2^p h); \tag{33}$$

$$(T_1^+ u^p)_{j_1,j_2} = u^p_{j_1+1,j_2} + u^p_{j_1,j_2}; \\ (T_2^+ u^p)_{j_1,j_2} = u^p_{j_1,j_2+1} + u^p_{j_1,j_2}; \tag{34}$$

$$D_1 = 0.5 \cdot T_2^+ D_1^+; \quad D_2 = 0.5 \cdot T_1^+ D_2^+. \tag{35}$$

Final formulas of averaging have the form

$$v^p_{1,0,2j_1-1,2j_2-1} = v^p_{1,0,2j_1,2j_2-1} = v^p_{1,0,2j_1-1,2j_2} = \\ = v^p_{1,0,2j_1,2j_2} = \beta_{1,0} v^{p+1}_{1,0,j_1,j_2}, \; j_1, j_2 = 1, 2, ..., N_{p+1}; \tag{36}$$

$$v^p_{0,1,2j_1-1,2j_2-1} = v^p_{0,1,2j_1,2j_2-1} = v^p_{0,1,2j_1-1,2j_2} = \\ = v^p_{0,1,2j_1,2j_2} = \beta_{0,1} v^{p+1}_{0,1,j_1,j_2}, \; j_1, j_2 = 1, 2, ..., N_{p+1}; \tag{37}$$

$$v^p_{1,1,2j_1-1,2j_2-1} = v^p_{1,1,2j_1,2j_2-1} = v^p_{1,1,2j_1-1,2j_2} = \\ = v^p_{1,1,2j_1,2j_2} = \beta_{1,1} v^{p+1}_{1,1,j_1,j_2}, \; j_1, j_2 = 1, 2, ..., N_{p+1}; \tag{38}$$

$$\beta_{1,0} = 0.25; \quad \beta_{0,1} = 0.25; \quad \beta_{1,1} = 0.125. \tag{39}$$

1.3. Three-dimensional problems.

This most cumbersome case is described in [6].

2. MULTILEVEL WAVELET-BASED NUMERICAL METHOD OF LOCAL STRUCTURAL ANALYSIS

2.1. Formulation of the problem

Effective qualitative multilevel analysis of local and global stress-strain states of the structure is normally required in various technical problems. As is known, defects and failures are mostly local in nature. However total load-carrying ability of the structure, associated with the condition of limit equilibrium, is determined by the global behavior of the considering project. Therefore corresponding multilevel approach is peculiarly relevant and apparently preferable in all aspects for qualitative and quantitative analysis of calculation data. Wavelet analysis provides effective and popular tool for such researches. After expansion of the solution with the use of local wavelet basis corresponding components are considered at each level of the basis.

In accordance with the method of extended domain [7], the domain Ω, occupied by considering structure, is embordered by extended one ω of arbitrary shape, particularly elementary. Operational formulation of the problem in domain ω normally has the form

$$Lu = F, \tag{40}$$

where L is the operator of boundary problem, which takes into account the boundary conditions; u is the unknown function; F is the given right-side function.

Directly from operational formulation we have variational formulation of the problem:

$$\Phi(u) = 0.5 \cdot (Lu, u) - (F, u), \tag{41}$$

Solution of (41) is the critical point of (40). (f, g) denotes dot product of functions f and g.

Discrete formulation of the problem has the form:

$$A\bar{u} = \bar{f}, \tag{42}$$

where $A = \{a_{i,j}\}_{i,\,j=1,2,...,n_{gl}}$ is the difference approximation of operator L; $\bar{u} = [\,u_1 \;\; u_2 \;\; ... \;\; u_{n_{gl}}\,]^T$ is the unknown mesh function; $\bar{f} = [\,f_1 \;\; f_2 \;\; ... \;\; f_{n_{gl}}\,]^T$ is the given right-side mesh function; n_{gl} is dimension of problem.

Various methods can be used to form the matrix of the discrete operator. We recommend method of basis (local) variations. Its major peculiarities include universality and computer orientation. We can use the following formulas for linear problems:

$$a_{i,j} = \Phi(\bar{e}^{(i)} + \bar{e}^{(j)}) - \Phi(\bar{e}^{(i)}) - \Phi(\bar{e}^{(j)}) + \Phi(\bar{0})];$$
$$f_i = 0.5 \cdot [\Phi(\bar{e}^{(i)}) - \Phi(-\bar{e}^{(i)})], \quad (43)$$

$$\bar{e}^{(i)} = [\, e_1^{(i)} \;\; e_2^{(i)} \;\; \dots \;\; e_{n_{gl}}^{(i)} \,]^T, \;\; i = 1, 2, \dots, n_{gl}; \quad e_j^{(i)} = \delta_{i,j}, \;\; j = 1, 2, \dots n_{gl}; \quad (44)$$

$\bar{e}^{(i)}$, $i = 1, 2, \dots, n_{gl}$ are basis mesh vectors; $\bar{0}$ is the null function; $\delta_{i,j}$ is the Kronecker delta.

2.2. Haar-based formulation of the problem

Let us consider Haar-based formulation of the problem:

$$\begin{aligned}\Phi(\bar{u}) &= 0.5 \cdot (A\bar{u}, \bar{u}) - (f, \bar{u}) = \\ &= 0.5 \cdot (LQ\bar{v}, Q\bar{v}) - (\bar{f}, Q\bar{v}) = \\ &= 0.5 \cdot (Q^* LQ\bar{v}, \bar{v}) - (Q^* \bar{f}, \bar{v}),\end{aligned} \quad (45)$$

where Q is transition matrix consisting from Haar basis vectors, located in rows. Thus,

$$\tilde{\Phi}(\bar{v}) = 0.5 \cdot (Q^* LQ\bar{v}, \bar{v}) - (Q^* \bar{f}, \bar{v}), \quad (46)$$

where $\bar{v}$ is vector of Haar expansion coefficients of the vector $\bar{u}$. Corresponding operational formulation of the problem has the form

$$\tilde{L}\bar{v} = \tilde{\bar{f}}, \quad \tilde{L} = Q^* LQ; \quad \tilde{f} = Q^* \bar{f}. \quad (47)$$

Further reduction of the problem is based on the averaging algorithm specified above.

3. MULTILEVEL WAVELET-BASED SEMIANALYTICAL METHOD OF LOCAL STRUCTURAL ANALYSIS

The objects of the multilevel wavelet-based semianalytical (discrete-continual) method are structures with piecewise constancy of physical and geometrical parameters in one dimension (it is so-called “basic direction”). Special discrete-continual design model is introduced. It presupposes wavelet approximation of extended domain along non-basic directions, while along the basic direction problem remains continual. Analytical solution is apparently preferable in all aspects for qualitative analysis of calculation data. It allows investigator to consider boundary effects when some components of solution are rapidly varying functions. Due to the abrupt decrease inside of mesh elements in many cases their rate of change can’t be adequately considered by conventional numerical methods while analytics enables study. Another feature of the proposing method is the absence of limitations on lengths of structures. Semianalytical formulation are contemporary mathematical models which currently becoming available for computer realization. Resultant multipoint boundary problem after reduction has the form [8-10]

$$\begin{cases} \bar{y}' = A_k \bar{y} + \bar{f}_k, \quad x \in (x_k^b, x_{k+1}^b), \\ \qquad\qquad k = 1, \dots, n_k - 1 \\ B_k^- \bar{y}(x_k^b - 0) + B_k^+ \bar{y}(x_k^b + 0) = \bar{g}_k^- + \bar{g}_k^+, \\ \qquad\qquad k = 2, \dots, n_k - 1 \\ B_1^+ \bar{y}(x_1^b + 0) + B_{n_k}^- \bar{y}(x_{n_k}^b - 0) = \bar{g}_1^+ + \bar{g}_{n_k}^-, \end{cases} \quad (48)$$

where $x_k^b = x_{3,k}^b$, $k = 1, \dots, n_k$ are coordinates of boundary points; A_k, $k = 1, 2, \dots, n_k - 1$ are matrices of constant coefficients of order n; B_k^-, B_k^+, $k = 2, \dots, n_k - 1$ and $B_1^+, B_{n_k}^-$ are matrices of boundary conditions of order n at point x_k^b; $\bar{g}_k^-, \bar{g}_k^+$, $k = 2, \dots, n_k - 1$ and $\bar{g}_1^+, \bar{g}_{n_k}^-$ are right-side vectors of boundary conditions at point x_k^b; $\bar{y} = \bar{y}(x) = [\, y_1(x) \;\; y_2(x) \;\; \dots \;\; y_n(x) \,]^T$ is the unknown vector function;

$$\bar{y}^{(1)} = \bar{y}^{(1)}(x) = d\bar{y}/dx;$$
$$\bar{f}_k = \bar{f}_k(x) = [\, f_{k,1}(x) \;\; f_{k,2}(x) \dots f_{k,n}(x) \,]^T, \quad k = 1, 2, \dots, n_k - 1$$

are right-side vector functions.

Solution of considering multipoint boundary problem of structural analysis is accentuated by numerous factors. They include boundary effects (stiff systems) and considerable number of differential equations (several thousands). Matrices of coefficients of a system normally have eigenvalues of opposite signs and corresponding Jordan matrices are not diagonal. Method of solution of multipoint boundary problems for systems of ordinary differential equations with piecewise constant coefficients in structural analysis has been developed. Not only does it overcome all difficulties, but its peculiarities also include universality, computer-oriented algorithm, computational stability, optimal conditionality of resultant systems and partial Jordan decomposition of matrix of coefficient, eliminating necessity of calculation of root vectors.

CONCLUSION

Currently, high-tech work is underway to integrate the developed numerical and semianalytical methods and corresponding algorithms of local structural analysis into the STADYO software package [19,20].

It should be noted that STADYO is the universal software package, which rovides temperature fields, static, stability and dynamic analysis (including response spectra and accelerations definition) as well as fracture mechanics and strength analysis and optimization of arbitrary combined 2-D and 3-D solid, shell, plate and beam mechanical systems by the finite elements, superelement and other modern numerical methods:

- STADYO-FIELD – stationary field (thermoconduction, filtration, fluid flow, etc) problems;
- STADYO-STAT – linear-elastic static stress-strain analysis;
- STADYO-EIG – solving the eigenvalue problems (natural frequencies and modes, loads and forms of buckling);
- STADYO-SEISM – “normative” spectral analysis of seismic responce under exitations, defined by acceleration spectra;
- STADYO-VIBR – evaluation of system stationary vibration parameters;
- STADYO-SPEC – linear spectral (modern superposition) dynamic analysis;
- STADYO-DYN – direct step-by-step integration of dynamic equations;
- STADYO-NFIELD – solving the non-stationary field problems;
- STADYO-FRAC – solving the linear problems of fracture mechanics, including intensivity ratio coefficients and J-integral definitions;
- STADYO-NLIN – solving the nonlinear static and dynamic problems of motion equations (large displacement, plasticity and viscoplasticity of metals, concrete and ground, opening cracks and joints etc.);
- STADYO-WIND – object-oriented code for 3D static and dynamic analysis of typical wind units;
- STADYO-ASTRA – object-oriented cod for 3D static analysis of typical pipe elements (elbows, tees, weld connections, etc);
- STADYO-INTER – object-oriented code for 3D static and dynamic analysis of combined “soilstructure” systems.

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