

UNIVERSAL SOFTWARE SYSTEM “STADYO” FOR THE NUMERICAL SOLUTION OF LINEAR AND NONLINEAR PROBLEMS OF THE FIELD THEORY, STATICS, STABILITY AND DYNAMICS OF SPATIAL COMBINED SYSTEMS: GENERAL PARAMETERS AND SUPERELEMENTAL FEATURES

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Abstract: The distinctive paper presents the general papers of the science-based universal software system “STADYO” intended for the numerical solution of stationary and nonstationary problems of field theory, analysis of static, temperature and dynamic stress-strain state (SSS), stability and strength of arbitrary combined mechanical systems (massive-shell-lamellar-membrane-rod with “rigid” bodies, fluid cavities and internal bonds, isotropic and orthotropic materials) into a flat axis, axisymmetric and three-dimensional linear and nonlinear formulations. The composition of the “STADYO” software system is briefly described; in terms of its theoretical foundations, the main provisions and matrix relationships of the superelement method (which is known to be one of the most effective ways to increase the universality and increase the computational efficiency of finite element algorithms) are presented, as well as methods for dynamic synthesis of substructures and submodelling. The superelement algorithm is also extended to solving a system of linear equations at each step of the implicit scheme of direct integration of the equations of motion, and at each iteration in the calculation of natural oscillations; however, an alternative and more efficient approach consists in constructing special superelement algorithms based on the direct condensation of the equations of motion and ideologically close to the method of dynamic synthesis of substructures. The submodelling options, in particular, are important for the refined analysis of the three-dimensional SSS of heavily loaded component parts of the objects under consideration. In general, the presentation of the global design model of the system as a set of substructures is also very convenient for its description and creates the prerequisite for the creation (application) of effective pre- and postprocessor software. The paper also provides information on verification and experience in the use of the “STADYO” software system, as well as prospects for development.

Keywords: buildings, dynamic substructure synthesis method, finite element method, mathematical modelling, submodelling, substructure method, superelement method, “STADYO” software system, structures

УНИВЕРСАЛЬНЫЙ ПРОГРАММНЫЙ КОМПЛЕКС СТАДИО ДЛЯ ЧИСЛЕННОГО РЕШЕНИЯ ЛИНЕЙНЫХ И НЕЛИНЕЙНЫХ ЗАДАЧ ТЕОРИИ ПОЛЯ, СТАТИКИ, УСТОЙЧИВОСТИ И ДИНАМИКИ ПРОСТРАНСТВЕННЫХ КОМБИНИРОВАННЫХ СИСТЕМ: ОБЩАЯ ХАРАКТЕРИСТИКА И СУПЕРЭЛЕМЕНТНЫЕ ВОЗМОЖНОСТИ

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Аннотация: В настоящей статье представлена общая характеристика наукоемкого универсального программного комплекса СТАДИО, предназначенного для численного решения стационарных и нестационарных задач теории поля, расчетов статического, температурного и динамического напряженно-деформированного состояния (НДС), устойчивости и прочности произвольных комбинированных механических систем (массивно-оболочечно-пластинчато-мембранно-стержневых с «жесткими» телами, полостями жидкости и внутренними связями, изо- и ортотропными материалами) в плоской, осесимметричной и трехмерной линейной и нелинейной постановках. Кратко описан состав программного комплекса СТАДИО, в части его теоретических основ приведены основные положения и матричные соотношения метода суперэлементов (являющегося, как известно, одним из наиболее действенных способов повышения универсальности и увеличения вычислительной эффективности конечноэлементных алгоритмов), а также методов динамического синтеза подконструкций и субмоделирования. Суперэлементный алгоритм распространяется также на решение системы линейных уравнений на каждом шаге неявной схемы прямого интегрирования уравнений движения и на каждой итерации при расчете собственных колебаний, однако альтернативный и более эффективный подход состоит в построении специальных суперэлементных алгоритмов, основанных на непосредственной конденсации уравнений движения и идеологически близких методу динамического синтеза подконструкций. Опции субмоделирования, в частности, являются важными для уточненного анализа трехмерного НДС тяжело нагруженных узлов-деталей рассматриваемых объектов. В целом, представление глобальной расчетной модели системы в виде совокупности подконструкций является также очень удобным для ее описания и создает предпосылку для создания (применения) эффективных пре- и постпроцессорных программных средств. В статье также приведены сведения о верификации и опыте использования программного комплекса СТАДИО, а также рассмотрены перспективы развития.

Ключевые слова: математическое моделирование, метод конечных элементов, метод суперэлементов, метод подконструкций, метод динамического синтеза подконструкций, субмоделирование, программный комплекс СТАДИО, конструкции, здания, сооружения

1. GENERAL CHARACTERISTICS OF “STADYO” SOFTWARE SYSTEM

Founded in far 1975, the Russian multipurpose software system “STADYO”TM provides a numerical solution of stationary and nonstationary field theory problems, analysis of the static, temperature and dynamic stress-strain state (SSS), stability and strength of arbitrary combined mechanical systems (massively shell-lamellar-membrane-rod with “rigid” bodies, fluid cavities and internal bonds, isotropic and orthotropic materials) in a plane, axisymmetric and three-dimensional linear and nonlinear statements [1, 2, 8].

In the “STADYO” software product, a representative library of straight- and curvilinear rod, flexural-membrane, curved super- (thin- and

medium-walled shells) and isoparametric volumetric finite elements (total more than 120 types), elements with given stiffness, inertia and damping matrices see, for example, Figure 1), modern optimized numerical algorithms (the methods of Cholesky and the preconditioned conjugate gradient, block method of Lanczos and method of iterations of the subspace, explicit and implicit difference schemes of integration: central differences, Newmark, Wilson and Crank - Nicholson, finite-element and superelement schemes, etc.) and developed service provision are implemented.

The main quantitative possibilities for the version of the complex on IBM-compatible PCs (the main programming language of the computational modules is Intel Visual Fortran) are presented in Table 1.

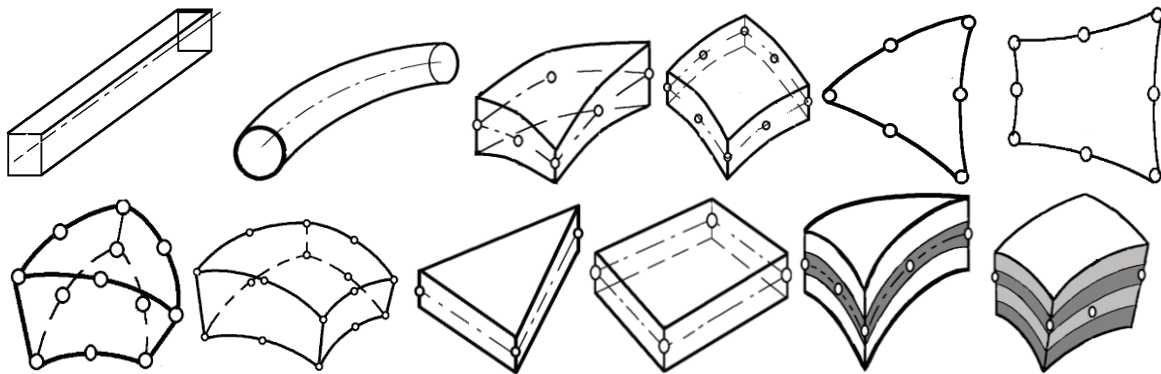


Figure 1. Some implemented types of finite elements.

Table 1. The main quantitative capabilities of “STADYO” software system.

Parameter for system / superelement	Limiting value
Degrees of freedom	24 000 000
Grid station	4 000 000
Finite elements	4 000 000
Fabric (environment)	4 000 000
Eigenfrequencies and forms	4 000

The hierarchy and degree of “nesting” of sub-structures (for the superelement “branch” of the software system) are arbitrary.

2. COMPOSITION OF THE “STADYO” SOFTWARE SYSTEM

“STADYO” software system consists of the following program modules (Figure 2):

- *STADYO-STATN* – solution of stationary problems of thermal conductivity, filtration, flow of an ideal fluid, and other problems of field theory with arbitrary boundary conditions;
- *STADYO-NSTATN* – solution of non-stationary problems of field theory (with the possibility of memorizing and using results for the subsequent solution of elastic problems);
- *STADYO-STAT* – linearly elastic calculation for stationary loads (volumetric, surface, linearly distributed, concentrated forces and moments, temperature, initial strains and stresses), including multiple-choice calculations, taking into account the stage-by-stage construction;
- *STADYO-FORM* – solving a particular problem of eigenvalues: determining the significant part of the spectrum of eigenfrequencies and vibration modes, critical loads and forms of stability loss;
- *STADYO-SEISM* – quasi-static “normative” spectral calculation for seismic actions specified by the acceleration spectra, with the determination of displacements, accelerations, forces and stresses using alternative methods;
- *STADYO-VIBR* – estimation of parameters of forced steady-state oscillations of systems without damping, with orthogonal and non-orthogonal damping;
- *STADYO-SPEC* – spectral dynamic calculation of linear-elastic systems (and systems with local nonlinearities) for force and kinematic effects with decomposition according to the modes of natural oscillations;
- *STADYO-DYN* – direct integration over time of the equations of motion of linear-elastic systems for given thermo-force and kinematic influences (including «non-platform» excitation schemes) with determination of accelerograms and response spectra;

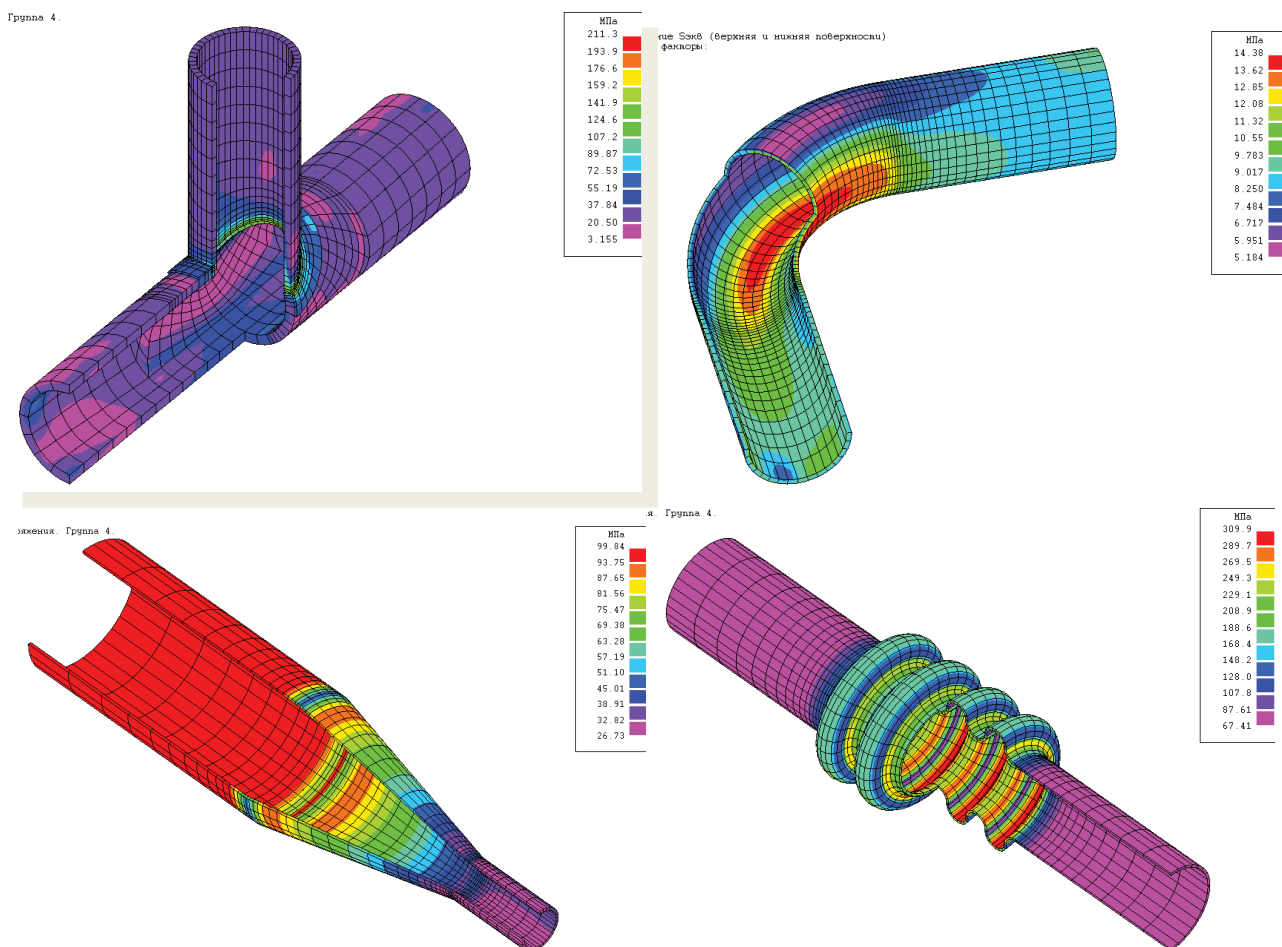


Figure 2. Stress state of pipeline parts (STADYO-ASTRA), 2005.

- *STADYO-NLIN* – solution of nonlinear problems (large displacements, finite deformations, plasticity, viscoplasticity and heredity effects for metals, concretes and soils, unilateral joints, seams and cracks with friction) for given static and dynamic thermal and kinematic influences;
- *STADYO-FRACT* – solving the linear problems of fracture mechanics, including the determination of stress intensity factors and J-integrals in bodies with cracks;
- *STADYO-SNIP* – the estimated evaluation of the strength and longitudinal reinforcement of reinforced concrete structures for the first and second limit states in accordance with the provisions of the current Russian building codes and regulations (SNiPs);
- *STADYO-ASTRA* – an object-oriented subsystem of refined automated calculation of the spatial temperature and stress-strain state, estimation of static, seismic and cyclic strength (in accordance with the requirements of the PNAE standards G-7-002-86) of typical pipeline parts: tees, bends and knees, conical transitions, lens compensators and zones of welded joints;
- *STADYO-CABINET* – an object-oriented subsystem of automated seismic resistance calculation (in accordance with the requirements of nuclear energy norms) of standard instrument cabinets of nuclear power plants;
- *STADYO-PED* – object-oriented subsystem of automated calculation of vibration of submersible electric motors (multi-layer thick-walled orthotropic finned shells in a viscous liquid);

- *STADYO-WEU* – an object-oriented subsystem of automated static and dynamic calculation of typical orthogonal wind power plants.

$$[\bar{K}]\{u_b\} = \{\bar{F}\}, \quad (3.2)$$

where

$$[\bar{K}] = [K_{bb}] - [K_{bi}][K_{ii}]^{-1}[K_{ib}]; \quad (3.3)$$

$$\{\bar{F}\} = \{F_b\} - [K_{bi}][K_{ii}]^{-1}\{F_i\}. \quad (3.4)$$

3. METHODS OF SUBMODULATION AND DYNAMIC SYNTHESIS OF SUBCONTRACTIONS

3.1. Preliminary remarks.

Modern “heavy” finite-element software systems (ANSYS, NASTRAN, SIMULIA Abaqus) contain options for constructing and importing-exporting reduced impact matrices to ensure, in particular, joint development and accurate analysis of complex multiply connected statically and dynamically loaded engineering structures and buildings. Another side of the same process is submodelling, for example, refined three-dimensional nonlinear analysis of complex and / or originally solved structural components of rod systems (column support, connections of chord-diagonal members, parts of pipeline systems). In this paragraph, the main provisions and matrix relations of methods of dynamic synthesis of substructures (MDS) and submodelling implemented in “STADYO” product are given.

3.2. The concept of the method of superelements.

One of the most effective ways to increase the computational efficiency of finite element algorithms is the superelement method (SEM), also known as the sub-structure method. The key procedure of the SEM is static condensation [9, 10] — elimination of internal degrees of freedom of the substructure $\{u_i\}$, — leading the initial block system of equilibrium equations

$$\begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix} \begin{Bmatrix} u_i \\ u_b \end{Bmatrix} = \begin{Bmatrix} F_i \\ F_b \end{Bmatrix} \quad (3.1)$$

to the reduced system with respect to the components of the vector of the corresponding boundary unknowns $\{u_b\}$:

Accounting of the possible repeatability of substructures leads to significant savings both by reducing the number of arithmetic operations, and to a significant reduction in the amount of stored and processed information. The data structure is organized hierarchically, so that making local changes to the calculation scheme, i.e. the correction of a part of the numerical model of the structure can be performed independently for each substructure with minimal computational costs. Representation of the global design model of the system as a set of substructures is also very convenient for its description and creates the prerequisite for the creation (application) of effective pre- and postprocessor software [1].

3.3. Methods of dynamic synthesis of substructures.

The superelement algorithm is also extended to solving a system of linear equations at each step of the implicit scheme of direct integration of the equations of motion and at each iteration in the calculation of natural oscillations [1]. An alternative and more efficient approach is the construction of special superelement algorithms based on the direct condensation of the equations of motion and ideologically close to the method of dynamic synthesis of substructures.

Methods for the dynamic synthesis of substructures can be classified by the method of selecting the basis vectors of the Rayleigh - Ritz method for substructures and by the method of their joining. Most often, as a basis for substructures, its vibration forms are used under certain boundary conditions: R.R. Craig, M.C.C. Bampton [15] and [16] – for the fixed boundary, [20] – for the free boundary, [17] – for the partially fixed boundary of the subconstruction.

The most preferable from the computational and realization points of view among the three considered variants of the method is the first one, which uses the vibration modes of the basic system of the displacement method. In order to simplify the joining of substructures and to correctly take into account their rigid displacements and displacements of boundary points, the basis of the vibration modes is expediently supplemented by a set of static functions of the superelement form, each of which is a vector of elastic displacements of the substructure nodes caused by a single displacement in the direction of the corresponding fictitious bond for the basic system of the displacement method. Such an approach, in particular, is described in [18,19].

In the general case, in a dynamic synthesis any suitable system of linearly independent vectors can be used as a basis for substructures, not necessarily including oscillation modes. Thus, in [21] a certain specific basis is used, for the construction of which a solution of the particular eigenvalue problem for the subconstruction is not required.

The numerical solution of the resulting reduced system of nonlinear integro-differential equations is found by modified implicit difference schemes of the Newmark type with the use of a significant part of the natural frequencies

$$[\Omega] = \text{diag}(\omega_1, \omega_2, \dots, \omega_n)$$

and form $[\Phi]$ of the partial subsystems-substructures [14]:

$$\begin{aligned} & [\bar{M}]\{\ddot{u}_b\} + [\bar{C}]\{\dot{u}_b\} + [\bar{K}]\{u_b\} + \\ & + \sum [G][\Omega] \int_0^t \text{SIN}[\Omega(t-\tau)][G]^T \{u_b(\tau)\} d\tau = \\ & = \sum (\{\bar{F}(t)\} - [G][\Omega]^2 \{q(t)\}); \end{aligned} \quad (3.5)$$

$$\{u_b(0)\} = \{u_{0b}\}; \quad \{\dot{u}_b(0)\} = \{v_{0b}\}; \quad (3.6)$$

$$[\bar{M}] = [M_b] + \sum [\bar{M}]; \quad [\bar{C}] = [C_b]; \quad (3.7)$$

$$[\bar{K}] = [K_b] + \sum [\bar{K}]; \quad (3.8)$$

$$[\bar{M}] = [M_{bb}] - [M_{bi}][M_{ii}]^{-1}[M_{ib}]; \quad (3.9)$$

$$[\bar{K}] = [K_{bb}] - [K_{bi}][K_{ii}]^{-1}[K_{ib}]; \quad (3.10)$$

$$\{\bar{F}(t)\} = \{F_b(t)\} - [M_{bi}][M_{ii}]^{-1}\{F_i(t)\}; \quad (3.11)$$

$$[G] = [K_{bi}][\Phi][\Omega]^{-2} - [M_{bi}][\Phi]; \quad (3.12)$$

$$\begin{aligned} \{q\} = \{q(t)\} = & \text{COS}[\Omega t][\Phi]^T [M_{ii}] \{\bar{u}_{0i}\} + \\ & + [\Omega]^{-1} \text{SIN}[\Omega t][\Phi]^T [M_{ii}] \{\bar{v}_{0i}\} + \\ & + [\Omega]^{-1} \text{SIN}[\Omega t] * [\Phi]^T \{F_i(t)\}; \end{aligned} \quad (3.13)$$

$$\{\bar{u}_{0i}\} = \{u_{0i}\} + [K_{ii}]^{-1}[K_{ib}]\{u_{0b}\}; \quad (3.14)$$

$$\{\bar{v}_{0i}\} = \{v_{0i}\} + [K_{ii}]^{-1}[K_{ib}]\{v_{0b}\}; \quad (3.15)$$

$$\text{SIN}[\Omega t] = \text{diag}[\sin(\omega_1 t), \sin(\omega_2 t), \dots, \sin(\omega_n t)]. \quad (3.16)$$

Craig-Bampton approach [5,7,13,16] realized in universal “commercial” products (NASTRAN, ANSYS, ADAMS, ...), is mentioned most popular, which reduces to the construction of reduced stiffness and mass matrices:

$$[\hat{K}] = [\hat{\Phi}]^T \begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix}; \quad [\hat{\Phi}] = \begin{bmatrix} \hat{K}_{CC} & 0 \\ 0 & \hat{K}_{NN} \end{bmatrix}; \quad (3.17)$$

$$[\hat{M}] = [\hat{\Phi}]^T \begin{bmatrix} M_{ii} & M_{ib} \\ M_{bi} & M_{bb} \end{bmatrix}; \quad [\hat{\Phi}] = \begin{bmatrix} \hat{M}_{CC} & \hat{M}_{NC} \\ \hat{M}_{CN} & \hat{M}_{NN} \end{bmatrix}, \quad (3.18)$$

where

$$[\hat{\Phi}] = \begin{bmatrix} I & 0 \\ \Phi_{iC} & \Phi_{iN} \end{bmatrix}; \quad (3.19)$$

Φ_{iC} – displacement of internal degrees of freedom for given single displacements of boundary (docking) nodes; Φ_{iN} – displacements of the internal degrees of freedom, corresponding to the retention of its own forms of oscillation, while limiting the degrees of freedom (sealing) of the boundary nodes; $[\hat{K}_{NN}]$ and $[\hat{M}_{NN}]$ – diagonal modal stiffness and mass matrices;

$$[\hat{K}_{CC}] \equiv [\bar{K}]; \quad [\hat{M}_{CC}] \equiv [\bar{M}]; \quad [\hat{M}_{NC}] = [\hat{M}_{CN}]^T \quad (3.20)$$

– filled matrix.

The same algorithm has been implemented since 1999 in the research “STADYO” software product [2]. The software implementation of matrix operations in the formation of $[\hat{K}]$ and $[\hat{M}]$ (and reduced load vectors $\{\hat{F}\}$), as well as possible affine transformations, is basically built on standard procedures contained in software package modules. Fulfilling the canonical finite(-super)-element assembly does not encounter any difficulties and allowance for consideration of $[\hat{K}]$, $[\hat{M}]$ and $\{\hat{F}\}$ in the general super-element model.

4. VERIFICATION AND EXPERIENCE OF USING THE STADYO SOFTWARE SYSTEM

The «STADYO» software system is verified in detail on a representative set of test and practical tasks, certified in Rostekhnadzor (2011, as part of ASTRA-STADYO [4]), documented in detail and implemented in leading design and research organizations “Atomenergoproject” and “Hydroproject”, Special Design Bureau “Hydropress”, All-Russian Research Institute of Atomic Engineering, All-Russian Research Institute of Electromechanics, Central Research Institute of Construction Structures, Design Bureau “Krasnaya Zvezda”, JSC “Melaks”, etc.). There is more than 41 years of experience with the complex in calculating studies of the spatial temperature and stress-strain state, strength and reliability of pipelines and their elements, technological, electrical equipment, machines and mechanisms, building structures and “foundation – construction” systems of particularly responsible industrial facilities and civil construction, taking into account the normatively regulated combinations of temperature, static, special dynamic (wind, seismic shock, wave, vibra-

tion, etc.) and emergency actions (Figure 3-14), such as:

- protective shells, reactor compartments, computer rooms, special buildings, diesel-generating standby electric power stations, cooling towers and waste storages of nuclear power plants (Armenian, Kursk, Smolensk, Chernobyl, Ignalina, Leningrad, Bilibinskaya, Novovoronezhskaya, Kalininskaya, Rostovskaya, Balakovo, Lovisa (Finland), Kozloduy (Bulgaria), Busher (Iran), Kudankulam (India), nuclear power plants of the new generation, etc.);
- dams, buildings of hydroelectric power plants and underground hydraulic structures, taking into account the stage-by-stage construction, nonlinear effects in seams and macro cracks, soil rheology, hydrodynamic effects of reservoirs (Volga, Kama, Inguri, Khudoni, Kurpsai, Nurek, Rogun, Namakhvani, Katun, Gekhi, Kapanda, Teri, Tang-E-Duk, Zagorskaya hydroelectric pumped storage power plant, etc. [3]);
- wind power plants of various types;
- unique buildings and typical civil engineering constructions (roof slab of the Big sports arena of Luzhniki, the monument to the 300th anniversary of the Russian Navy, the high-rise complex of the Moscow International Currency of Moscow Central Stock Exchange, the underground parking on the Manege Square shopping complex, the Aquadrome and Transvaal Park, the Basmanny Building market, the indoor swimming pool of the “Iskra” sanatorium (Sochi), multistorey panel and prefabricated monolithic block sections and their “assemblies” with an assessment of stability in emergency situations, high-altitude multifunctional complexes, etc. [6]);
- aboveground and underground pipelines of heating systems, main oil and gas pipelines, oil and gas processing and petrochemical industries, oil and gas exploration complexes;
- floating structures, ice-resistant platforms for oil and gas production on the shelf (Chayvo-1 and others);

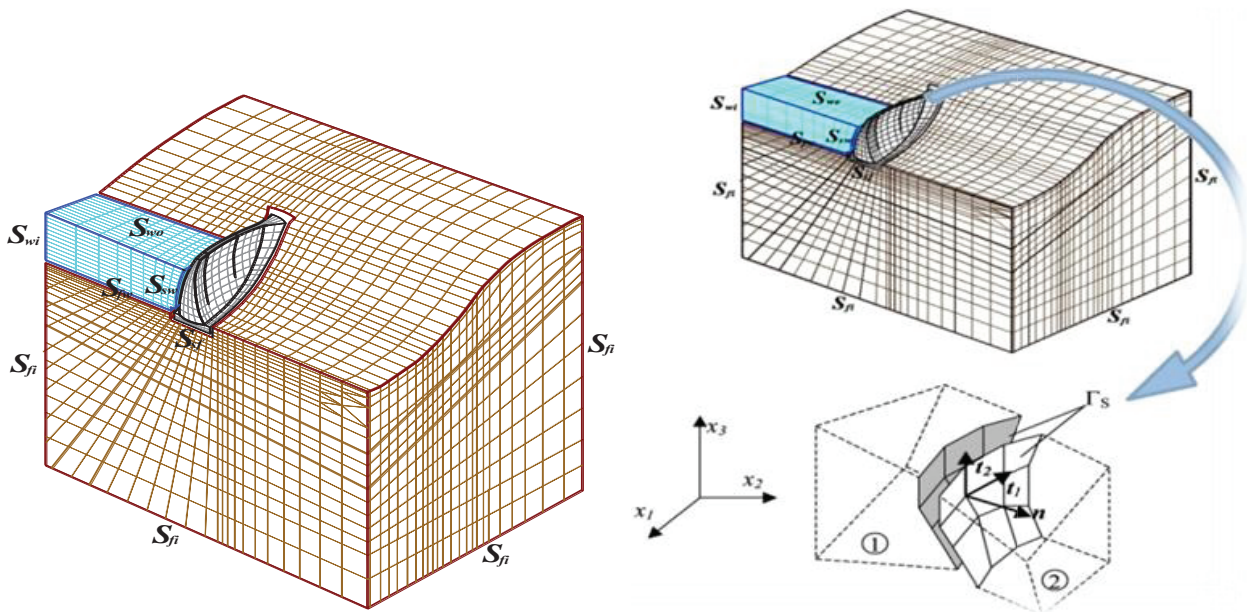


Figure 5. Typical superelement model of the “foundation – dam – reservoir” system.

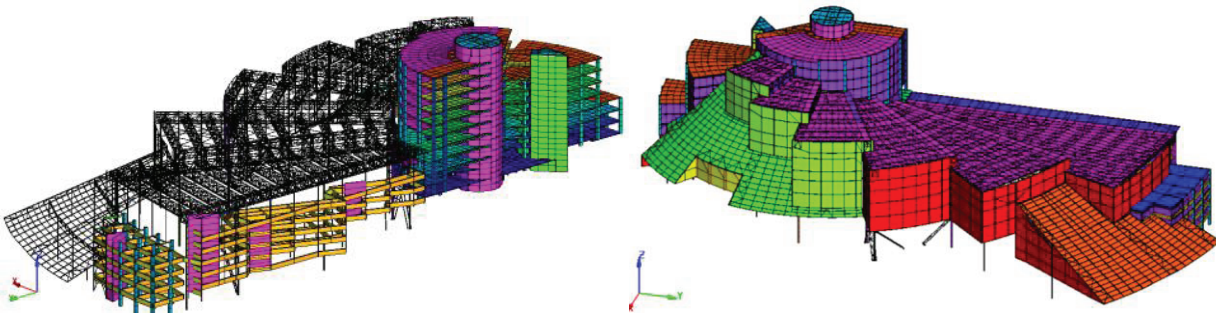


Figure 6. Super-element model of the sports complex “Aquadrom”.

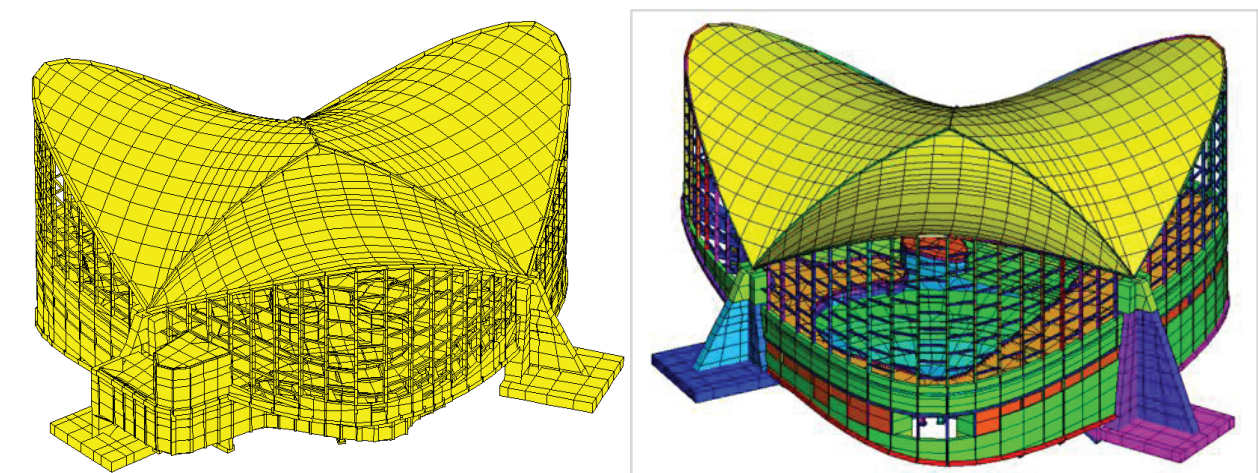


Figure 7. Super-element model of the indoor pool of the “Iskra” sanatorium (Sochi).

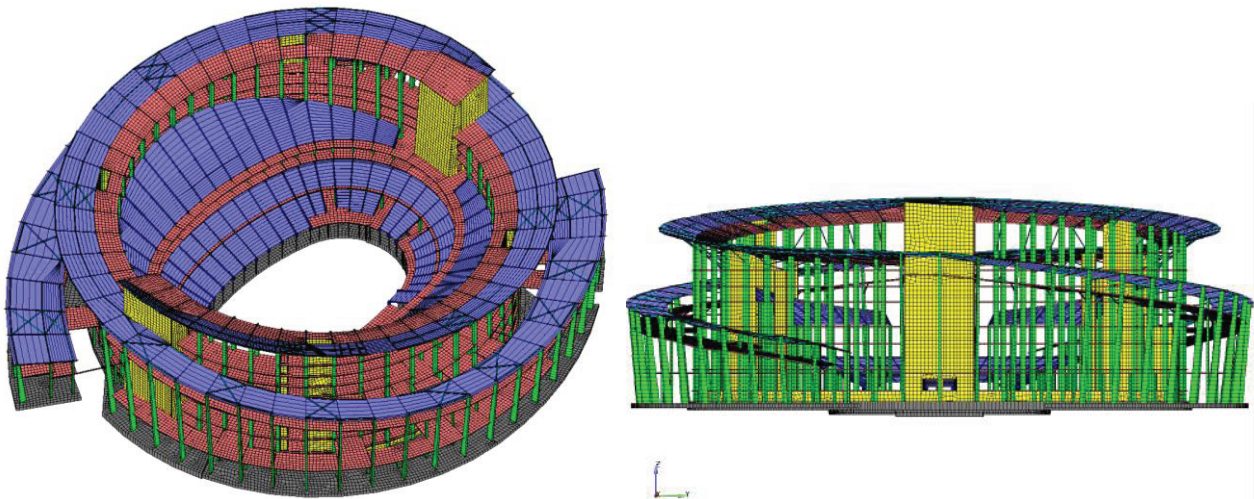


Figure 8. Superelement model of the ice palace of sports on Khodynka.

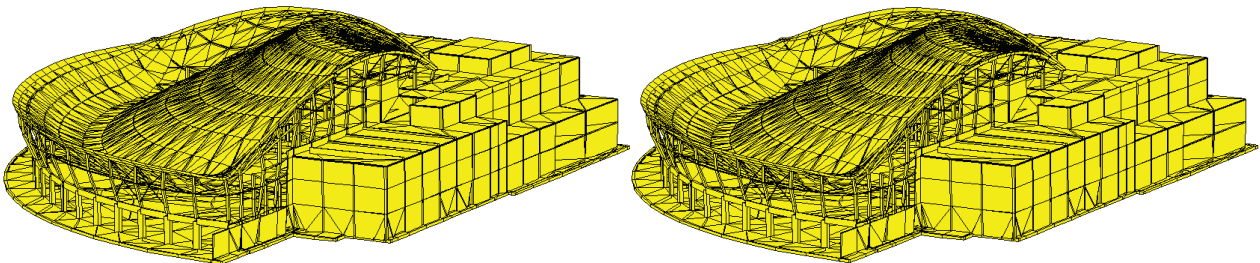


Figure 9. Deformed states of the “construction – foundation” system of the Transvaal-Park sports and fitness center with estimated load combinations (2005).

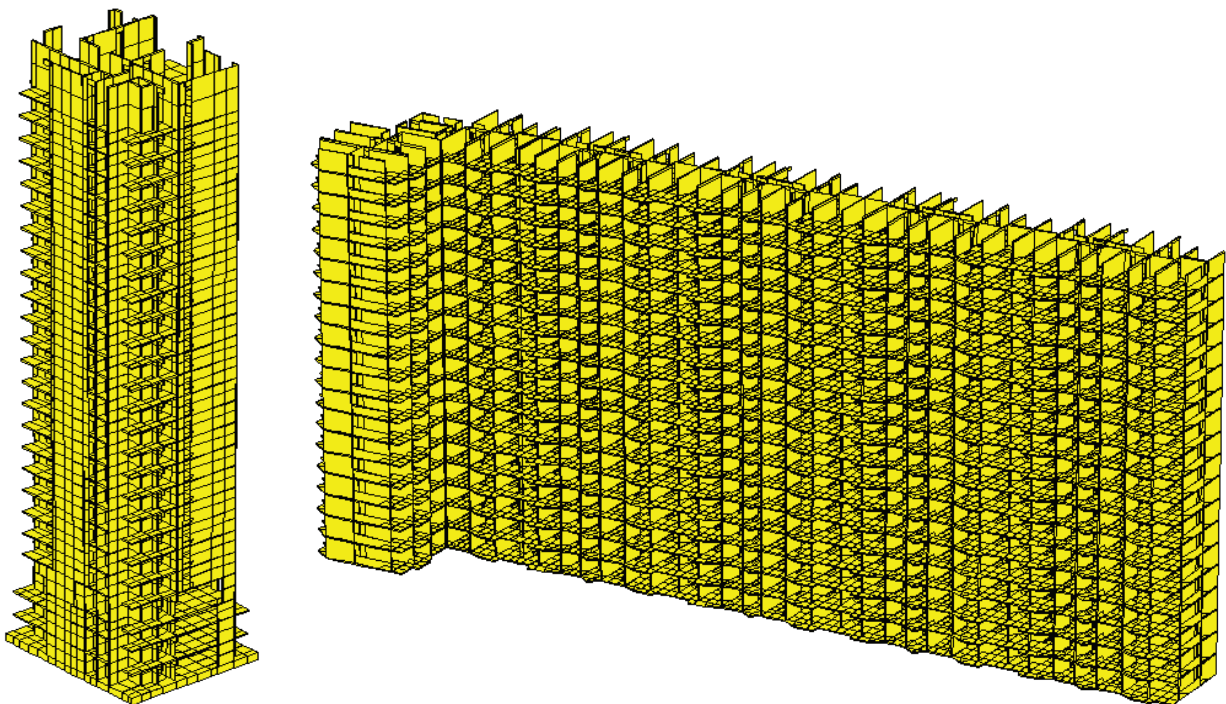


Figure 10. Superelementary models of single- and multisectional houses of typical series.

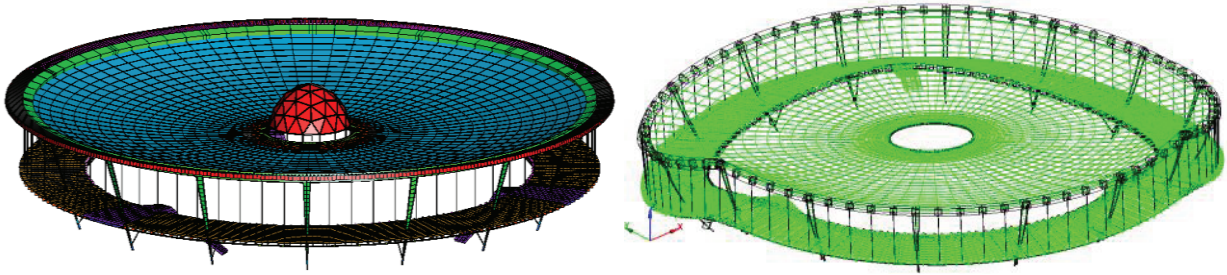


Figure 11. Superelement model and deformed state of the Basmany market (2007).

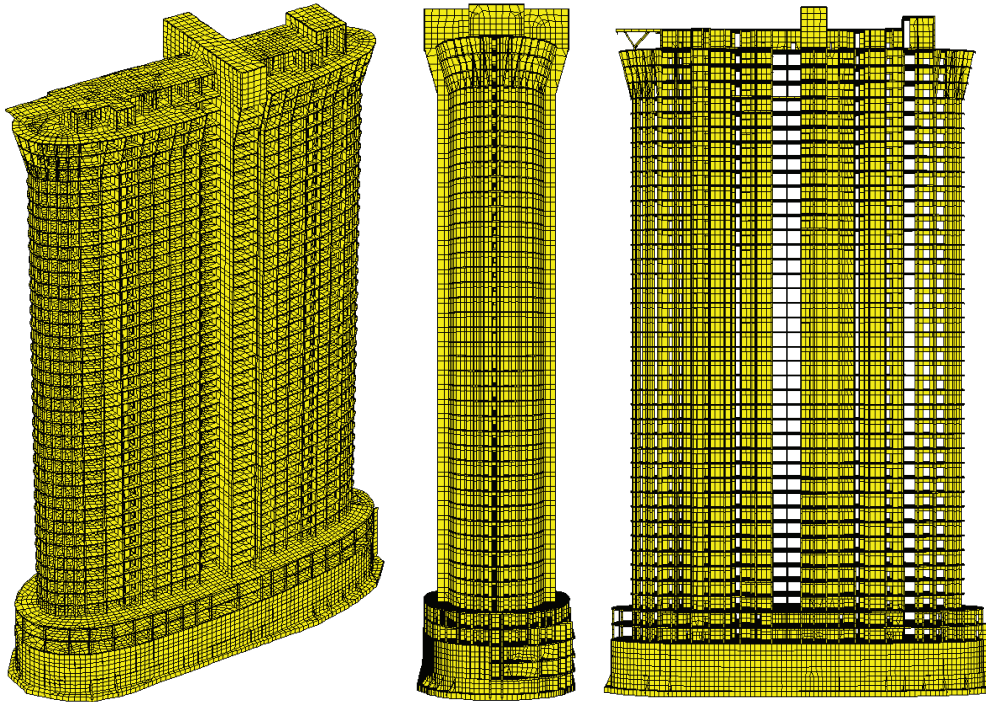


Figure 12. Model of multifunctional high-rise (44 floors) of the complex.

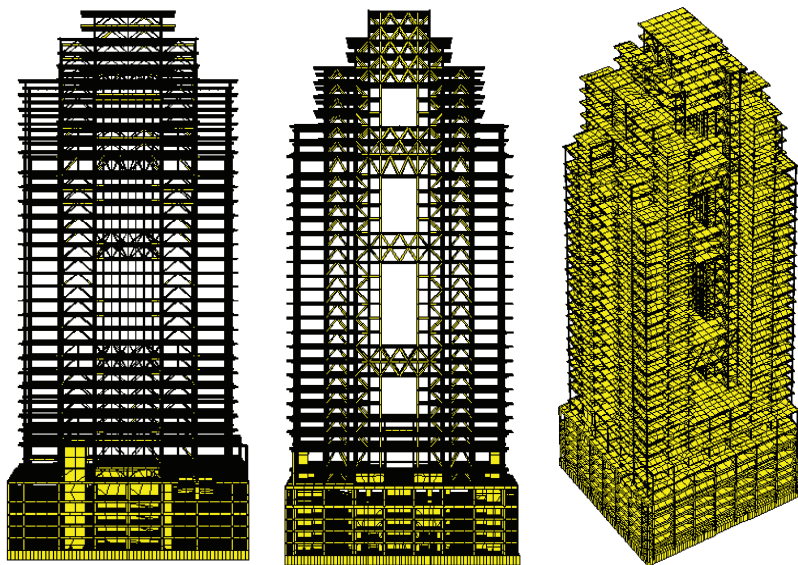


Figure 13. Model of a high-rise building of the Interbank Currency Exchange (5 underground and 31 above-ground floors).

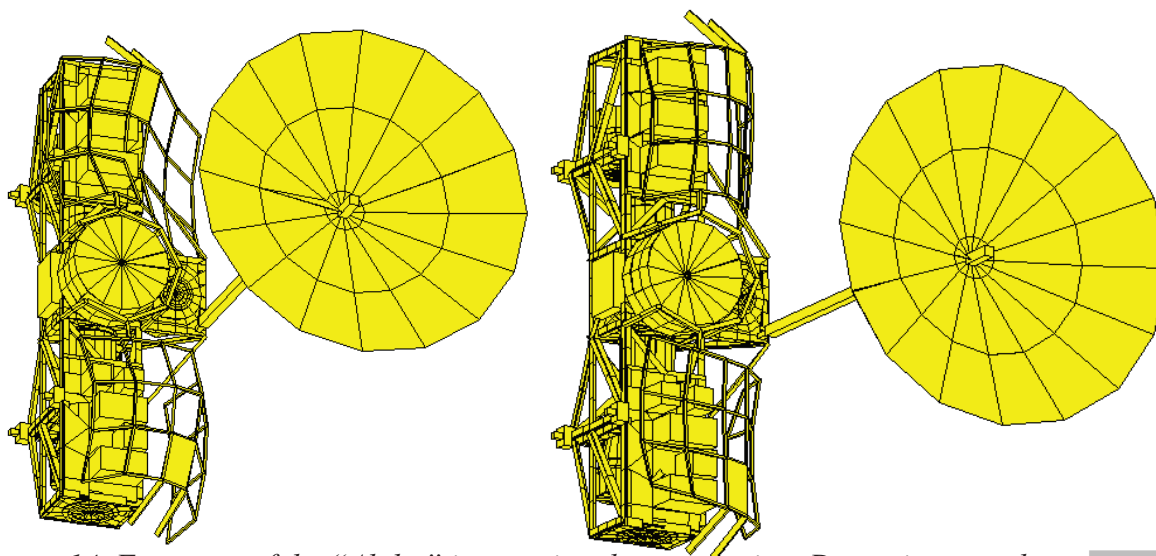


Figure 14. Fragment of the “Alpha” international space station. Dynamic superelement model. Lower own vibration modes (1999).

- complex machine-building structures, parts, machines and mechanisms (aerospace systems, transport, ship and power engineering, ferrous and non-ferrous metallurgy, household appliances, etc.).

Among the solved pioneering tasks is the modeling of the dynamics and strength of the “Russian segment – drive units – solar cells” system of the International Space Station “Alpha” at the stages of transportation, launch and orbital activity.

It should be noted that the outrunning development and use of the method of dynamic synthesis of substructures in the aerospace industry — this is evidenced by the above-mentioned fundamental theoretical works [12, 16, 17-19, 21] — seems historically and substantively substantiated. Indeed, dozens of design firms in different countries independently develop and optimize dynamically linked main subsystems (blocks and nodes), and the «price of the issue» is extremely high.

5. HORIZONS FOR THE DEVELOPMENT OF THE «STADYO» SOFTWARE SYSTEM

Among the planned tasks for the development of “STADYO” software system we can name

the following, reinforcing its status of a universal research software system [11]:

- development of an interface with “heavy” CAD systems for import-export models;
- development of a complete verification report reflecting the new capabilities of the software package;
- increase the computational dimension of the solved problems due to the new possibilities for creating geometric and FE models, the implementation of algorithms for parallel computations [12];
- further development of software implementations of alternative algorithms for dynamic synthesis of substructures;
- realization of a discrete-continual version of the finite element method as a competitive alternative and complement to the traditional FEM schemes;
- allowance for plastic deformation of metals (associated flow law with isotropic and kinematic hardening) for an adequate assessment of the static strength of pipeline parts, equipment and metal structures.

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