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ABOUT MULTISCALE MODELING WITH THE USE OF HAAR DISCRETE WAVELETS FOR TARGETED DATA EXTRACTION IN PROBLEMS OF STRUCTURAL MECHANICS

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Abstract: The distinctive paper is devoted to multiscale modelling of functions with the use of Haar discrete wavelets. Numerical implementation is realized with the use of “Fortran” programming language (“Intel Visual Fortran” software). Particularly several sample functions are under consideration. Three basic parts of corresponding algorithm include full wave reconstruction (implementing the entire Hilbert space of the wave); extraction of wave components constructed solely by wavelet functions; isolation and visualization of the final wavelet layer. Modeling wave behavior from multiple perspectives enables the extraction of highly valuable information, which can reveal defects in mechanical or structural systems. The Haar discrete wavelet, due to its basis and step-like wavelet functions, is particularly suitable for numerical implementation (programming, computation), featuring straightforward algorithms.

Keywords: Haar wavelet, fault detection, modeling, structural analysis

О МНОГОУРОВНЕВОМ МОДЕЛИРОВАНИИ С ИСПОЛЬЗОВАНИЕМ ДИСКРЕТНЫХ ВЕЙВЛЕТОВ ХААРА ДЛЯ ЦЕЛЕВОГО ИЗВЛЕЧЕНИЯ ДАННЫХ В ЗАДАЧАХ СТРОИТЕЛЬНОЙ МЕХАНИКИ

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Аннотация: В настоящей статье рассматривается многоуровневое моделирование функций с использованием дискретных вейвлетов Хаара. Численная реализация выполнена на основе применения языка программирования “Fortran” (соответствующее программное обеспечение “Intel Visual Fortran”). В частности, рассматриваются несколько примеров функций. Три основные части соответствующего алгоритма включают полную реконструкцию соответствующей волны (реализация всего гильбертова пространства волны); извлечение волновых компонент, построенных исключительно на основе вейвлет-функций; выделение и визуализацию конечного вейвлет-слоя. Моделирование поведения волны с разных точек зрения позволяет извлекать ценную информацию, которая может выявлять дефекты в механических или конструктивных системах. Дискретный вейвлет Хаара исключительно удобен для численной реализации (программирования, вычислений), характеризуется соответствующими простыми алгоритмами.

Ключевые слова: вейвлет Хаара, выявление дефектов, моделирование, расчеты строительных конструкций

INTRODUCTION

Wavelet analysis is a relatively recent mathematical tool that emerged in the late 1980s and is now widely applied to fundamental and practical problems, including engineering. Notable applications include image processing [1-3]. This analysis offers flexible signal processing techniques as an alternative to classical Fourier analysis [4-5].

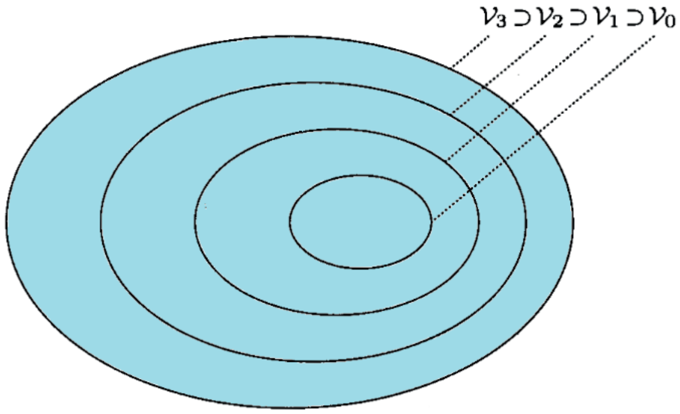


Figure 1. Decomposition of a function's Hilbert space into subspaces V

Thus, it enables solutions to problems where traditional Fourier methods are ineffective. A key advantage of wavelet analysis is its ability to detect highly localized variations in signals (waves), including jumps and irregularities. In contrast, Fourier analysis lacks the resolution to track rapid, minute changes, as its coefficients reflect wave behavior over an infinite range, making it inefficient for analyzing fast-varying signals or noise.

Wavelet analysis is a powerful tool for hierarchical mathematical decomposition, representing a wave's overall space as a combination of low- and high-frequency components. In this study, we employ Visual Fortran [6] to examine sample functions using Haar wavelet analysis [7-11], presenting their mathematical space in three distinct models. The presented modeling of the mathematical space of a wave makes it possible to examine the behavior of that wave from different perspectives and monitor its changes, whereas conventional wave observation often fails to capture expected data.

1. WAVE SPACE DECOMPOSITION WITH THE USE OF WAVELET THEORY

If a wave resides in a Hilbert space ($L_2(R)$), it can be constructed using simpler subspaces (Figure1):

$$L_2(R) = \bigoplus_{j \in Z} V_j \,. \tag{1}$$

The accuracy of the reconstructed wave model depends on the completeness of the selected subspace V_j. This modeling requires subspaces V_j to satisfy the following conditions:

$$V_0 \subset V_1 \subset V_2 \subset \ldots \subset L_2(R) \,. \tag{2}$$

In wavelet analysis, the target space is partitioned into nested subspaces, transitioning to a base space and complementary orthogonal subspaces (Figure 2). Here, subspaces V_j and W_j must meet the following criteria:

$$L_2(R) = V_0 \oplus W_0 \oplus W_1 \oplus \ldots ; \tag{3}$$

$$V_{j+1} = V_j \oplus W_j , \quad j \in Z ; \tag{4}$$

$$\bigcap_{j \in Z} W_j = \{0\} , \tag{5}$$

where Z is the set of integers.

In wavelet-based wave reconstruction, subspaces are mutually orthogonal and non-overlapping. This unique property enables the extraction of critical information not available in other modeling techniques. Such information can reveal mechanical or structural defects [12-16], as healthy systems exhibit predictable harmonic waves, while defective ones introduce uncontrolled noise and anomalous frequencies.

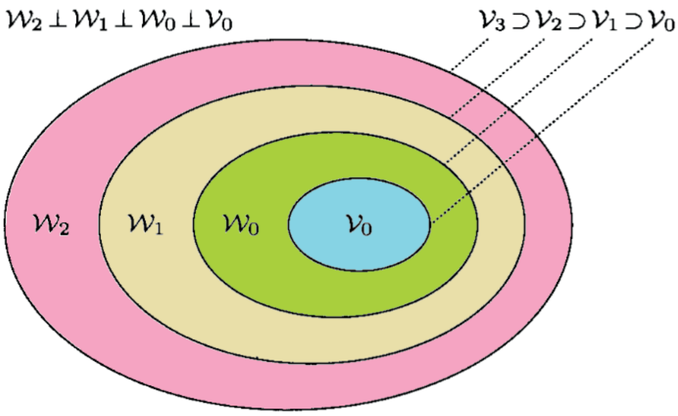


Figure 2. Decomposition of a function's Hilbert space into subspaces V_j and W_j

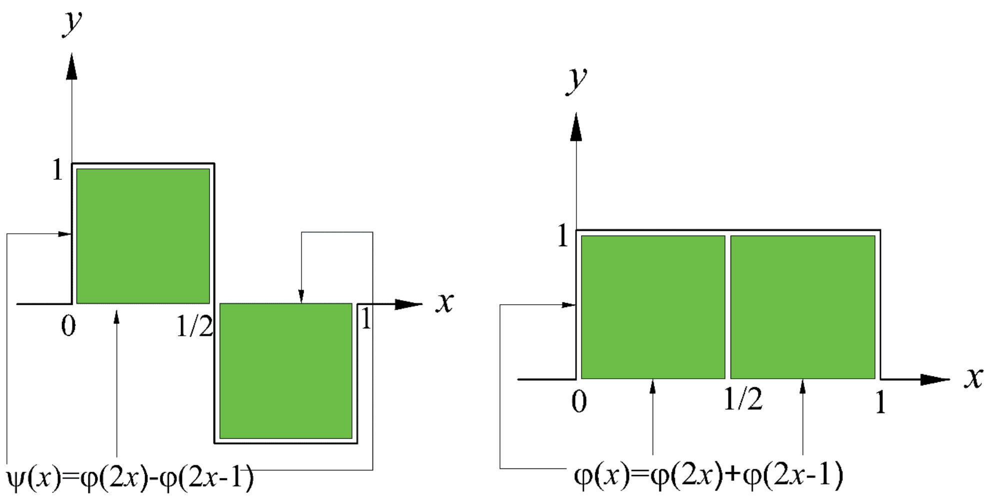


Figure 3. First two stages of the scaling function (φ) and wavelet function (ψ) in the Haar wavelet.

2. HAAR WAVELET

Mathematically, wavelet analysis permits diverse V_j and W_j subspaces for wave reconstruction, a distinctive feature of this method. The discrete Haar wavelet [17] is one of the simplest yet most effective wavelet tools, fully compatible with computational implementation and applicable to mechanical and structural problems [18-21] (Figure 3):

$$\phi(x) = \sqrt{2}\sum_{k=-\infty}^{\infty} h_k \phi(2x-k)\,; \qquad (6)$$

$$\psi(x) = \sqrt{2}\sum_{k=-\infty}^{\infty} \tilde{h}_k \phi(2x-k)\,; \qquad (7)$$

$$\tilde{h}_k = -(-1)^k h_{1-k}\,, \quad k \in Z\,. \qquad (8)$$

3. NUMERICAL MODELING WAVES DECOMPOSITION AND DATA EXTRACTION

Using "Intel Visual Fortran", we modeled and analyzed several sample functions via the Haar discrete wavelet algorithm. The modeling was implemented in three categories, each providing distinct wave insights:

- full wave reconstruction (implementing the entire Hilbert space);
- extraction of wavelet-only components;
- isolation of the final wavelet layer.

One application of this analysis is crack and defect detection in structures.

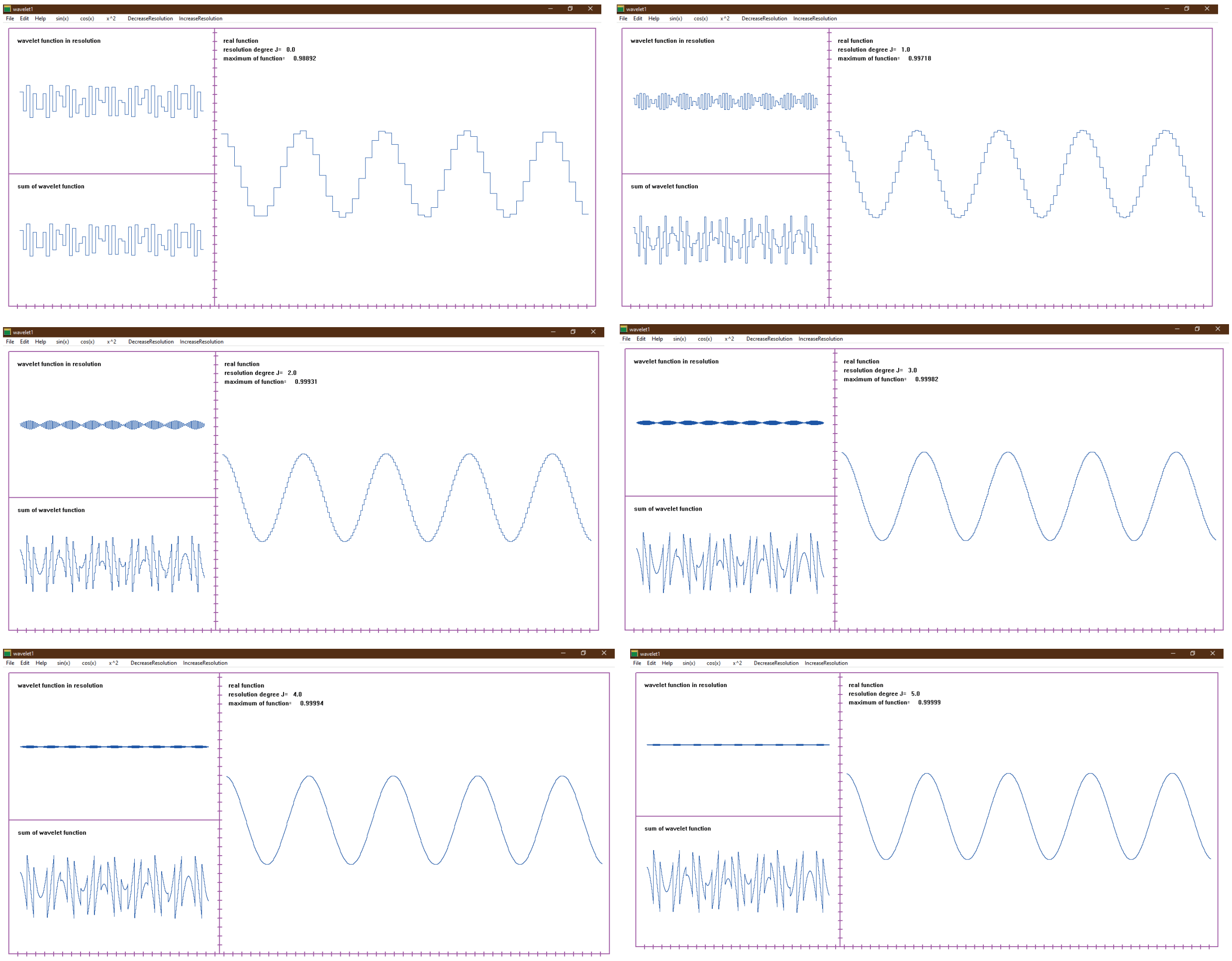


Figure 4. Implementation of sin(x)

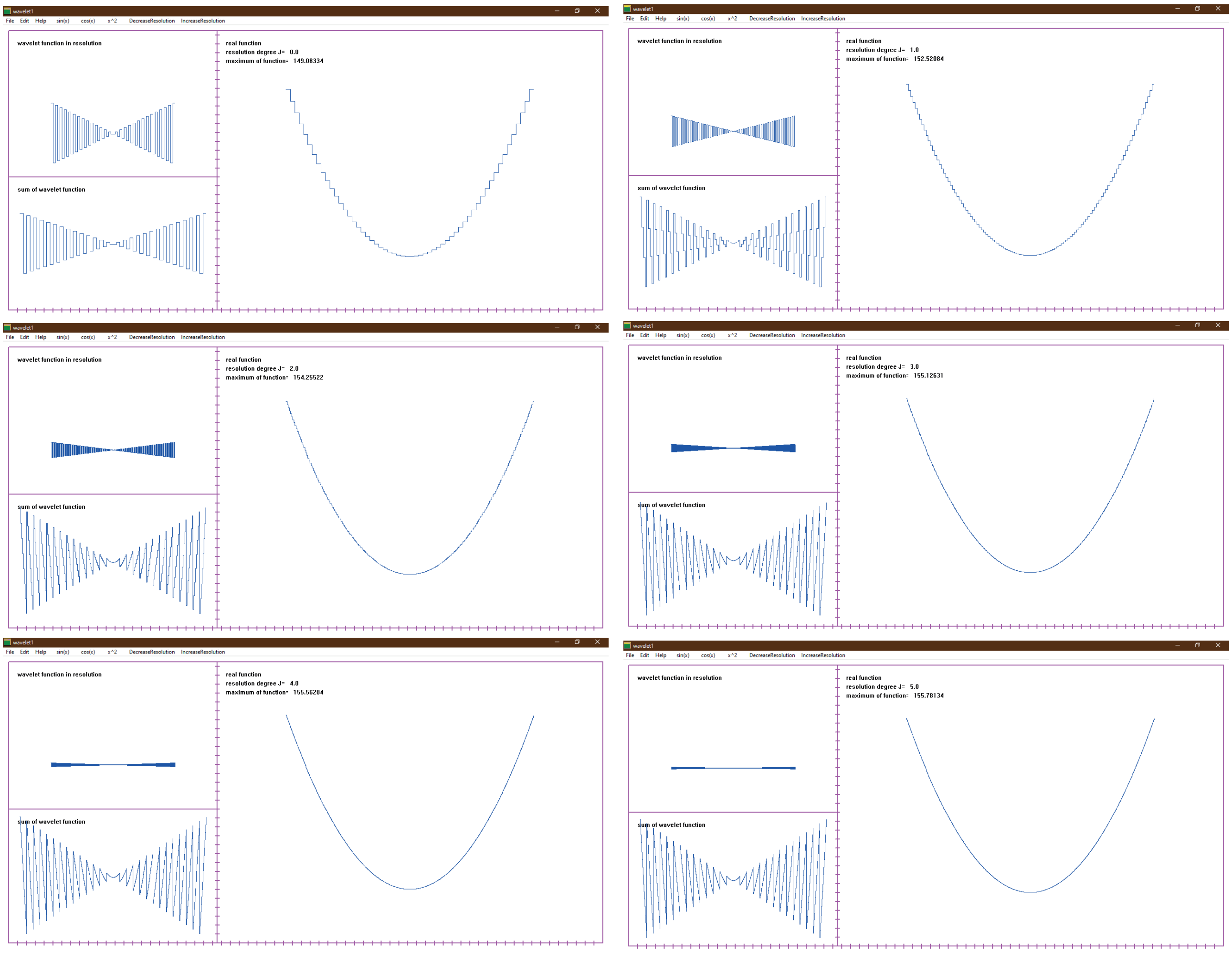
By deploying seismic wave controllers at various points, load-induced waves can be monitored and decomposed via wavelets to detect frequency deviations. The study tested the algorithm on sin(x) (Figure 4), cos(x), and x^2 (Figure 5). Results show rapid convergence with increasing wavelet resolution, a key advantage over Fourier analysis. The method accommodates both regular and irregular waves. Subspace-based models yielded data fundamentally different from the original wave, offering precise behavioral details. For random or harmonic waves, temporal variations were accurately captured.

CONCLUSION

This paper employed Visual Fortran to analyze functions using Haar wavelets in three distinct models. Multi-perspective wave modeling extracts unique, actionable insights, enabling defect detection in mechanical/structural systems. Healthy systems exhibit controlled harmonics, whereas defective ones introduce uncontrolled noise. The Haar wavelet's simplicity and numerical efficiency make it ideal for engineering applications.

Key Features of the Translation:

- formal academic tone with precise technical terminology;
- logical flow mirroring the original structure;
- figures and equations referenced per academic conventions;
- concise yet comprehensive rendering of complex concepts.



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