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THE BENDING CAPACITY OF THREE-LAYER PRECAST REINFORCED CONCRETE STRUCTURES ACCOUNTING FOR THE CONTINUOUS VARIATION IN COMPRESSIVE STRENGTH OF THE CONTACT LAYERS

Vu Dinh Tho [1], Pham Thi Hien [1], Elena A. Korol [2], Le Ngoc Lan [1], Pham Tuan Anh [1]

[1] University of Transport Technology, Hanoi, VIET NAM

[2] National Research Moscow State University of Civil Engineering, Moscow, RUSSIA

Abstract: This paper presents the flexural strength of precast three-layer reinforced concrete structures made from different types of concrete. The three-layer reinforced concrete structure consists of an outer layer of ordinary concrete with strength grades from B12.5 to B30 and an inner layer of lightweight polystyrene concrete. Experiments involving the fabrication of 150x150x150 mm concrete samples using two different materials – B25 normal concrete and B0.75 polystyrene concrete showed that when two layers are poured consecutively with a rest time of less than 2 hours, a contact layer forms between the two materials. This contact layer has a structure with decreasing density from the outer layer using normal concrete to the inner layer using lightweight concrete with low compressive strength. This paper proposes a method for calculating the flexural strength of precast three-layer reinforced concrete structures accounting for the continuous variation in the compressive strength of the contact layer. Calculation results for the load-bearing capacity of three-layer reinforced concrete beam specimens using traditional methods, proposed methods, and experiments have shown that considering the contact layer in the bending behavior of three-layer reinforced concrete beams yields results closer to experimental values than traditional methods. Increasing the thickness and characteristics of the contact layer increases the structure's load-bearing capacity by up to 1%. When the compressive strength of the outer concrete layer is increased from B15 to B25, the load-bearing capacity of the structural plate can increase by up to 59%. When the compressive strength of the inner concrete layer is increased from B5 to B15 while keeping the outer layer's concrete type unchanged, the load-bearing capacity of the three-layer structure can increase by up to 40.1%. The proposed method for calculating three-layer reinforced concrete structures with different materials, accounting for the material properties of the contact layer, accurately captures the phenomena observed during the practical fabrication of such structures.

Keywords: three-layer reinforced concrete structures; the contact layer; normal concrete; lightweight concrete; polystyrene concrete

НЕСУЩАЯ СПОСОБНОСТЬ ИЗГИБАЕМЫХ СБОРНЫХ ТРЁХСЛОЙНЫХ ЖЕЛЕЗОБЕТОННЫХ КОНСТРУКЦИЙ С УЧЁТОМ НЕПРЕРЫВНОГО ИЗМЕНЕНИЯ ПРОЧНОСТИ НА СЖАТИЕ БЕТОНА КОНТАКТНЫХ СЛОЁВ

Ву Динь Тхо [1], Фам Тхи Хиен [1], Елена А. Король [2], Ле Нгок Лан [1], Фам Туан Ань [1]

[1] Университет транспортных технологий, г. Ханой, ВЬЕТНАМ

[2] Национальный исследовательский Московский государственный строительный университет, г. Москва, РОССИЯ

Аннотация: В данной статье представлены данные о прочности на изгиб сборных трехслойных железобетонных конструкций, изготовленных из различных типов бетона. Трехслойная железобетонная конструкция состоит из внешнего слоя из обычного бетона с классами прочности от B12.5 до B30 и внутреннего слоя из легкого полистиролбетона. Эксперименты с изготовлением бетонных образцов размером 150x150x150 мм из двух различных материалов – обычного бетона класса B25 и полистиролбетона класса B0.75 – показали, что при последовательной заливке двух слоев с интервалом выдержки менее 2 часов, между двумя материалами образуется контактный слой. Этот контактный слой имеет структуру с уменьшающейся плотностью от внешнего слоя из обычного бетона к внутреннему слою из легкого бетона с низкой прочностью на сжатие. В данной статье предлагается метод расчета прочности на изгиб сборных трехслойных железобетонных конструкций, учитывающий непрерывное изменение прочности бетона контактных слоёв на сжатие. Результаты расчетов несущей способности трехслойных железобетонных балок с использованием традиционных методов, предложенных методов и экспериментов показали, что учет контактного слоя при изгибном поведении трехслойных железобетонных балок дает результаты, более близкие к экспериментальным значениям, чем традиционные методы. Увеличение толщины и характеристик контактного слоя повышает несущую способность конструкции до 1%. При увеличении прочности на сжатие бетона наружного слоя с B15 до B25, несущая способность балки может увеличиться до 59%. При увеличении прочности на сжатие внутреннего бетонного слоя с B5 до B15 и при сохранении неизменного типа бетона наружного слоя, несущая способность трехслойной конструкции может увеличиться до 40,1%. Предложенный метод расчета трехслойных железобетонных конструкций из различных материалов, учитывающий свойства материала контактного слоя, точно отражает явления, наблюдаемые при практическом изготовлении таких конструкций.

Ключевые слова: трехслойные железобетонные конструкции; контактный слой; конструктивный бетон; легкий бетон; полистиролбетон

1. INTRODUCTION

Multilayer reinforced concrete structures with different features are widely used in civil and industrial works [1], [2]. These multilayer structures are used extensively as enclosure structures to ensure load-bearing, sound insulation, and heat insulation requirements [3], [4]. The structure of three-layer reinforced concrete structures includes: the middle layer is made from ordinary concrete material with strength grade B12.5 to B30 [5], [6] and the inner layer is made from lightweight concrete types such as porous concrete [7–9], Lightweight aggregate concrete [10], arbolite concrete [11]. The thickness of the middle layer is determined based on the calculation of sound insulation, heat insulation or according to the desired performance requirements. The thickness of the outer layer is usually chosen to ensure load-bearing requirements and to ensure the concrete layer protects the steel reinforcement [12, 13].

Currently, there are many methods for calculating multi-layered reinforced concrete structures. In the studies by the authors [14], [15] this method for calculating multi-layered reinforced concrete structures was presented by converting cross-sections of various materials into I-shaped cross-sections of the same material, based on the elastic modulus ratio of the layers. This calculation method does not account for the distinct strength and strain characteristics of the various types of concrete used in multi-layered reinforced concrete structures. Therefore, the calculation results using this method often have significant errors.

The stress and strain state of multilayer reinforced concrete under load is a complex issue influenced by many factors. It has been studied by many authors. In publications [16], [17] the authors studied the influence of the mechanical properties of concrete and the geometric parameters of the inner and outer layers of multilayer structures on the structure's stress and strain. In research [18,19], the authors studied the stress-strain state of multilayer structures considering the nonlinear properties of concrete. The inner structural layer of multilayer reinforced concrete is often made of lightweight, low-strength concrete. Under load, this structure can easily develop small cracks in the inner concrete layer. To address this issue, the author [20] studied the influence of cracks appearing in the middle layer on the stress and strain state of a three-layer structure.

To consider the influence of different mechanical properties of different types of concrete on the stress-strain state of three-layer reinforced concrete structures, the authors Korol E. A and colleagues developed a method for calculating three-layer structures [21, 22]. In this method, the authors based their hypothesis on the assumption that the strain in the cross-section at the contact point between two different materials is the same, but that the relationship between strength and strain in each layer is different. This method allows consideration of the different mechanical properties of different types of materials.

In the study [23, 24], based on experiments on test samples of two-layer concrete structures with different materials, the authors found that during the production of these structures, a contact layer forms between the two layers. This bonding layer is formed on the basis of the penetration of aggregate particles of the outer layer into the inner layer and vice versa, the penetration of aggregate particles of the inner layer into the outer layer. Therefore, several solutions have been developed to analyze multilayer reinforced concrete structures with different materials. Authors V. I. Andreev and colleagues [25, 26] presented a method for calculating three-layer reinforced concrete structures with a thin contact layer, homogeneous material with elastic or plastic properties. This method for calculating multi-layer structures does not accurately reflect the material properties of the bonding layer and their working state under the action of loads.

In studies [27, 28], the stress-strain state of three-layer reinforced concrete beams was analyzed using the finite element method (on ANSYS). However, these studies were mainly based on the assumption that the bonding layer was either ignored or treated as a homogeneous material. This is clearly limited. In studies [14], [23, 24], [29] Tho VD and colleagues presented a method for calculating three-layer reinforced concrete structures that accounts for the contact layer formed between different material layers. The mechanical properties of the contact layer were determined by assuming an average of the concrete mixtures from the inner and outer layers.

However, these studies have not yet captured the nature of contact layer formation, which arises from the penetration of aggregate particles from the high-strength concrete layer into the lower-strength concrete layer, and vice versa. Consequently, the compressive strength of the concrete in the contact layer decreases continuously from the high compressive strength of the outer concrete layer to the low compressive strength of the inner concrete layer. Therefore, studying the formation of the contact layer and proposing a solution for calculating the stress-strain behavior of three-layer reinforced concrete structures, taking into account the continuous variation in the compressive strength of the contact layer, are necessary.

2. MATERIALS AND METHODS

2.1. Materials

In this study, in the outer layer of the three-layer reinforced concrete structure, heavy concrete with strength grade B12.5 – B30 was used [5], [6] and in the inner layer, lightweight polystyrene concrete with strength grade B0.75 [14] and lightweight concrete with strength grades B5, B10, and B15 [30] were used.

The stress-strain relationship diagram of concrete is described by equations (1), (2) & (3) and Figure 1a [14], [27], [30].

Algebraic expressions for the three-line diagram of the deformation of compressed concrete can be represented as:

When $0 \le \varepsilon \le \varepsilon_{b1}$, with $\sigma_b = E_{b,red}.\varepsilon_b$ (1)

When $\varepsilon_{b1} \le \varepsilon \le \varepsilon_{b0}$, with

$$\sigma_b = [\left(1-\frac{\sigma_{b1}}{R_b}\right)\frac{\varepsilon_b-\varepsilon_{b1}}{\varepsilon_{b0}-\varepsilon_{b1}}+\frac{\sigma_{b1}}{R_b}]R_b \quad (2)$$

(3)

When $\varepsilon_{b0} \le \varepsilon \le \varepsilon_{b2}$, with $\sigma_b = R_b$

Ứng suất $\sigma_{b1} = 0.6\, R_b$

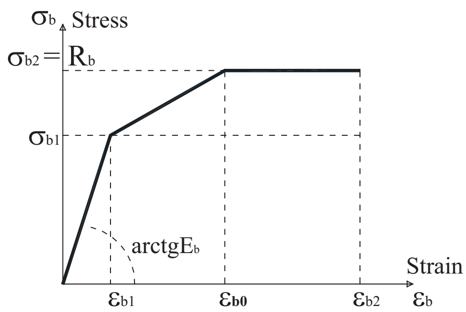
The stress-strain relationship diagram for the reinforcement is described by equations (4) and (5) and shown in Figure 1b [30].

When $0 \le \varepsilon \le \varepsilon_y$, with

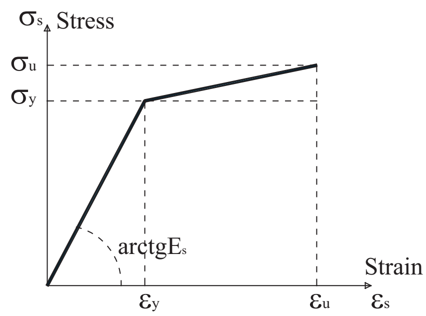
$$\sigma_s = E_s.\varepsilon_{s0} \quad (4)$$

The ε_{su} value is taken as 0.025.
When $\varepsilon_y \le \varepsilon \le \varepsilon_{su}$, with

$$\sigma_s = \sigma_y + \frac{\sigma_u - \sigma_y}{\varepsilon_u - \varepsilon_y}(\varepsilon_s - \varepsilon_y) \quad (5)$$



<u>Figure 1a.</u> Diagram of concrete compression
Where: σ_b – compressive stresses of concrete;
R_b – prismatic strength of concrete;
ε_b – the deformation of concrete;
ε_{b1}; ε_{b0}; ε_{b2} – the deformation of concrete corresponding to the stress $\sigma_{b1}=0.6*R_b$; $\sigma_{b0}=R_b=\sigma_{b2}$



*<u>Figure 1b.</u> Stress-*strain *diagram for reinforcement*
Where: σ_y – yield *strength* of steel;
σ_u – ultimate strength of steel;
ε_y – the deformation *of* steel when σ_y;
ε_u – the deformation *of* steel when σ_u.

2.2. A contact layer between layers of a three-layer concrete structure

In the research results [14], [23,24] the authors studied the formation of the contact layer between different types of materials: normal concrete B25 and lightweight polystyrene concrete B0.75. The concrete mix proportions are described in the following table 1 [23,24].

<u>Table 1</u>. Composition of concrete mixtures

Class of concrete	Density, kg/m^3	Cement (M400), kg	Water, l	Crushed limestone (sizes from 0.5 to 1 cm), kg	Sand (sizes from 0 to 2 mm), kg	Expanded polystyrene granules EPS (sizes from 2 to 5 mm), kg	Chemical additives, SilkRoad SR-5000F, g
B0.75	346	330	105	-	-	0.69	710
B25	2376	439	195	1121	621	-	-

Research results published [23, 24] have shown that, during the construction of concrete structures from different types of concrete with a waiting time of less than 2 hours, a bonding layer is formed between the different material layers, due to the movement of aggregate particles from the inner layer to the outer layer and aggregate particles from the outer layer moving into the inner layer, as shown in Figure 2. The thickness of this bonding layer is from 0 cm to 1 cm.

Because the contact layer is formed by the movement of aggregate particles of different types of concrete mixtures from the inner layer to the outer layer and vice versa, determining the mechanical properties of the concrete bond layer is very complex. Based on microstructural analysis, the authors in publications [23, 24] also showed that the aggregate particles in the heavy concrete mixture from the outer layer penetrating into the inner layer gradually decrease with the thickness of the bond layer and conversely, the aggregate particles in the polystyrene concrete mixture in the inner layer penetrating into the outer layer also gradually decrease with the thickness of the bond layer. This also forms the basis for the authors' proposal of a model for calculating structures with this type of bond under the action of a load. Without loss of generality, we can assume that the mechanical properties of the bonded layer between two different types of materials are as follows: Compressive strength of concrete R_b^*, Compressive strength of concrete R_{bt}^* and initial elastic modulus E_b^*; thickness of the bonded concrete layer h^*.

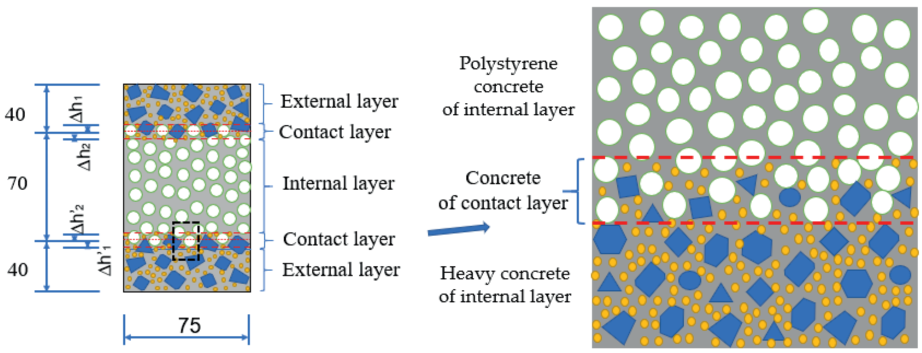


Figure 2. The *contact zone of layers of multilayer reinforced concrete structures*

2.3. Bending capacity of three-layer reinforced concrete beams

2.3.1. Bending capacity of three-layer reinforced concrete beams (without considering the contact layer)

In the studies [14], [24], [29], the authors presented a method for calculating the strength and deformation of a three-layer reinforced concrete structure with a monolithic section. The main provisions of TCVN 5574-2018 [30] can be applied after reducing the composite section to an I-beam. The stress-strain state of three-layer bent layers with a monolithic connection of sections normal to the longitudinal axis is determined by the positions of the elements' neutral axes under load. Then, the stress and strain calculation diagram of the structure before failure is shown in Figure 3 [14].

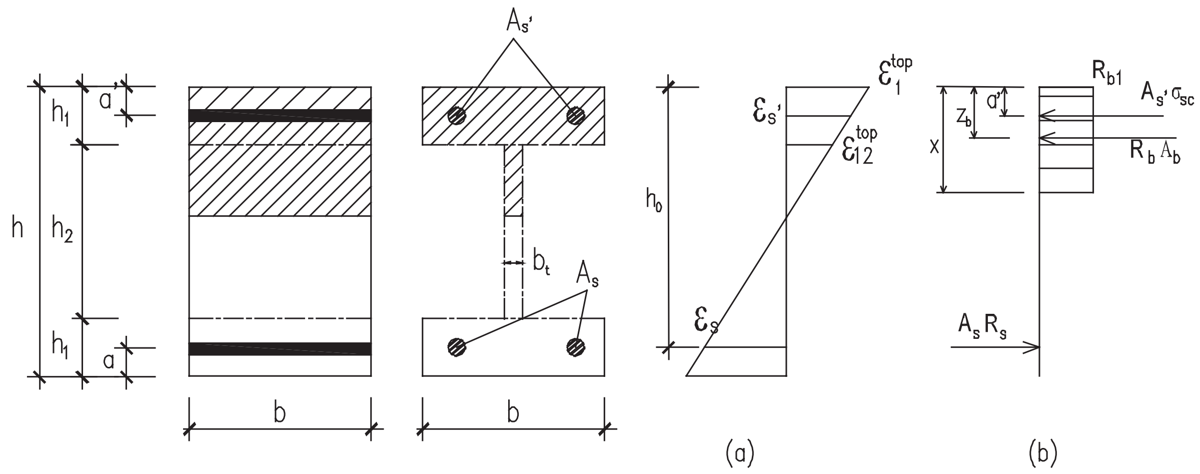


Figure 3. Scheme 1 for calculating the stress and strain of a three-layer reinforced concrete structure

The bearing capacity of the structure is calculated on the following conditions:

$$M \le M_{ult} \qquad (6)$$

With M_{ult} – Limit bending moment of the member;
M – Bending moment from loads.
Moment M_{ult} determined by the formula:

$$M_{ult} = R_{b1}bh_1(h_0 - \frac{h_1}{2}) + R_{b1}b\frac{E_{b2}}{E_{b1}}(x-h_1)(h_0 - h_1 - \frac{x-h_1}{2}) + \sigma_{sc}A_{s'}(h_0 - a') \qquad (7)$$

The height of the compression zone of concrete is determined by the condition:

$$R_sA_s = R_{b1}bh_1 + R_{b1}b\frac{E_{b2}}{E_{b1}}(x-h_1) + \sigma_{sc}A_s^{'} \qquad (8)$$

2.3.2. Bending capacity of three-layer reinforced concrete beams (Considering the contact layer, which is hypothesized to be a homogeneous layer of material)

The research results of authors Tho V. D, Korol E. A and colleagues have developed a method for calculating three-layer reinforced concrete structures with a middle layer of polystyrene concrete, taking into account the bond layer formed between two different material layers [23, 24]. The above studies are based on the hypothesis that the mechanical properties of the bond layer are those of a homogeneous material layer. Then, the stress and strain calculation scheme for the structure before failure is shown in Figure 4 [14]. The initial elastic modulus of the concrete of the contact layer is determined:

$$R_{b*} = \frac{R_{b1} + R_{b2}}{2}$$

With: h* is the thickness of the bonding layer; R_{b*} is the compressive strength of the bonding layer; R_{b1} is the compressive strength of the outer concrete layer; R_{b2} is the compressive strength of the inner concrete layer.

The thicknesses of the outer and inner material layers are changed as follows: $h_1^{'} = h_1 - 0.5h_*$ and $h_2^{'} = h_2 - h_*$

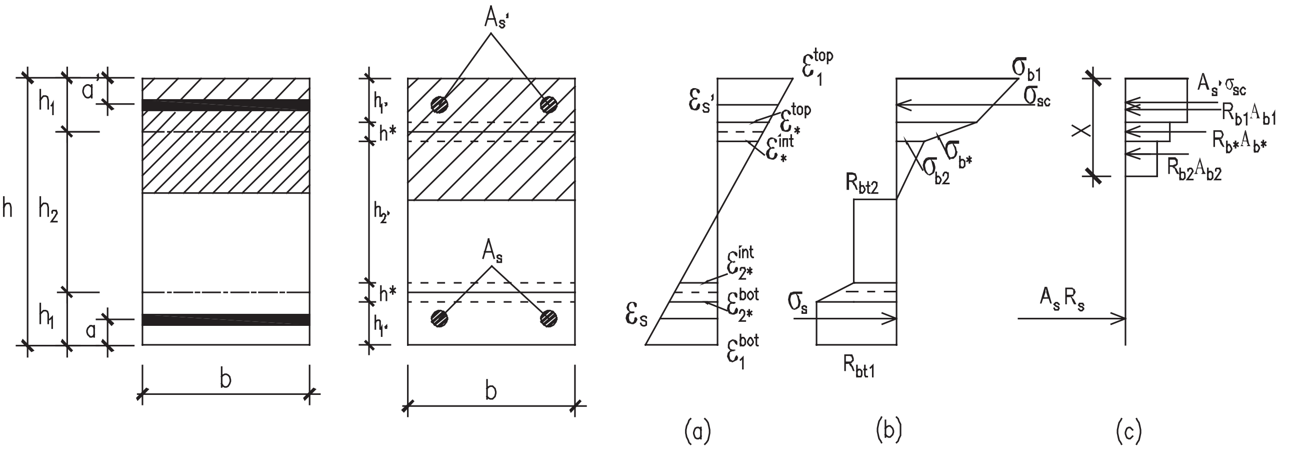


Figure 4. Scheme 2 for calculating of stress and strain of a three-layer reinforced concrete structure, during destruction (with contact layers)

The bearing capacity of the structure is calculated on the following conditions:

$$M \le M_{ult} \tag{9}$$

With M_{ult} – Limit bending moment of the member;
M – Bending moment from loads.
Moment M_{ult} determined by the formula:

$$\begin{aligned} M_{ult} &= R_{b1}bh_{1'}[h_0 - \frac{h_{1'}}{2}] + R_{b*}bh^*[h_0 - h_{1'} - \frac{h^*}{2}] \\ &+ R_{b2}b[x - h_{1'} - h^*][h_0 - h_{1'} - h^* - \frac{x - h_{1'} - h^*}{2}] + \sigma_{sc}A_{s'}(h_0 - a') \end{aligned} \tag{10}$$

The height of the compression zone of concrete is determined by the condition:

$$R_sA_s = R_{b1}bh_{1'} + R_{b*}bh^* + R_{b2}b[x - h_{1'} - h^*] + \sigma_{sc}A_{sc} \tag{11}$$

2.3.3. Development of a method for calculating multi-layer reinforced concrete structures considering the contact layer (Considering the contact layer, which is hypothesized to be a layer of material with continuously changing properties)

Experimental results [23, 24] have shown that this material layer undergoes continuous changes from the outer layer to the inner layer. Therefore, in this study, the authors hypothesize that the compressive and tensile strengths of the concrete in the bonding layer vary continuously with height h*, with these mechanical properties gradually decreasing from the outer to the inner layer.

$$R_{b*} = R_{b2} + \int_0^{h_*} \frac{(R_{b1} - R_{b2})}{h_*} dz$$

The thicknesses of the outer and inner material layers are changed as follows: $h_1^{'} = h_1 - 0.5h_*$ and $h_2^{'} = h_2 - h_*$.

Then, the stress and strain calculation scheme for the structure before crack formation and before failure is shown in Figure 5.

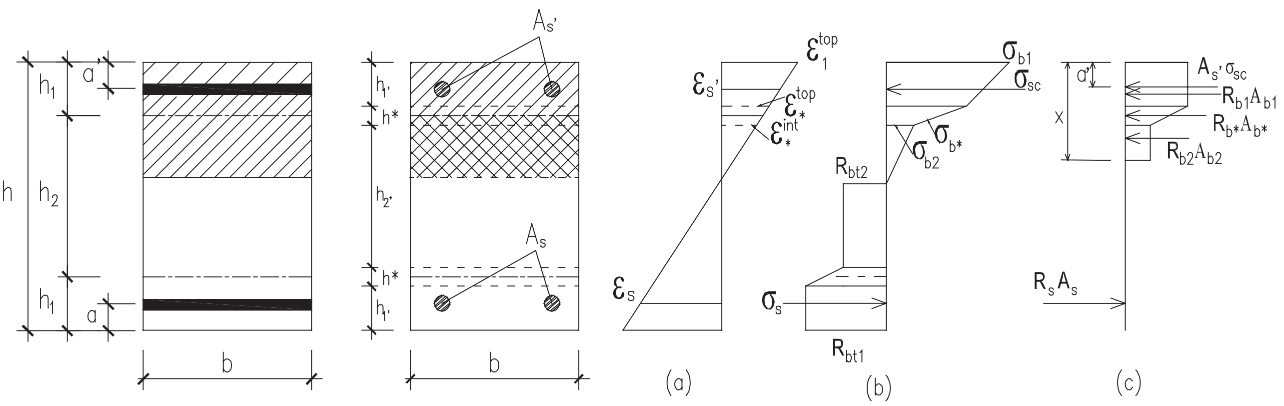


Figure 5. Scheme 3 for calculating the stress and strain of a three-layer reinforced concrete structure with contact layers

The bearing capacity of the structure is calculated on the following conditions:

$$M \le M_{ult} \tag{12}$$

With M_{ult} – Limit bending moment of the member; M – Bending moment from load.

Moment M_{ult} determined by the formula:

$$M_u = R_{b1}bh_{1'}(h_0 - \frac{h_{1'}}{2}) + R_{b2}bh_*(h_0 - h_{1'} - \frac{h_*}{2}) + \frac{1}{2}(R_{b1} - R_{b2})bh_{1*}(h_0 - h_{1'} - \frac{h_*}{3})$$
$$+R_{b2}b(x - h_{1'} - h_*)(h_0 - h_{1'} - h_* - \frac{x - h_{1'} - h_*}{2}) + \sigma_{sc}A_{s'}(h_0 - a') \quad (13)$$

The condition determines the height of the compression zone of concrete:

$$R_sA_s = R_{b1}bh_{1'} + R_{b2}bh_{1*} + \frac{1}{2}(R_{b1} - R_{b2})bh_{1*} + R_{b2}b(x - h_{1'} - h_*) + \sigma_{sc}A_{sc} \quad (15)$$

3. RESULTS AND DISCUSSION

3.1. Experimental and theoretical results on the load-bearing capacity of three-layer reinforced concrete structures and a comparison of analytical results

The results of stress-strain state analysis of multi-layer reinforced concrete structures using calculation methods according to Scheme 1, Scheme 2, the proposed Scheme 3, and the experiment were carried out on a beam sample (B-1) of a three-layer reinforced concrete beam with width *b* = 200mm, height *h* = 200mm, and length *l* = 2200mm. The structural and material parameters are shown in Figures 6 and 7 and in Table 1 [14].

Parameters of reinforcement used in the test beam: diameter 8mm, type of steel in test beam CB400-V, shown in Table 2 [14].

Table 1. Geometric and material parameters of a three-layer reinforced concrete beams

Test beam	Thickness of outer layers, mm	Class of concrete of the outer layer	Thickness of inner layers, mm	Class of concrete of the inner	Thickness of contact layers, mm
B1 (B1-1; B1-2; B1-3)	50	B25	100	B0.75	*h**

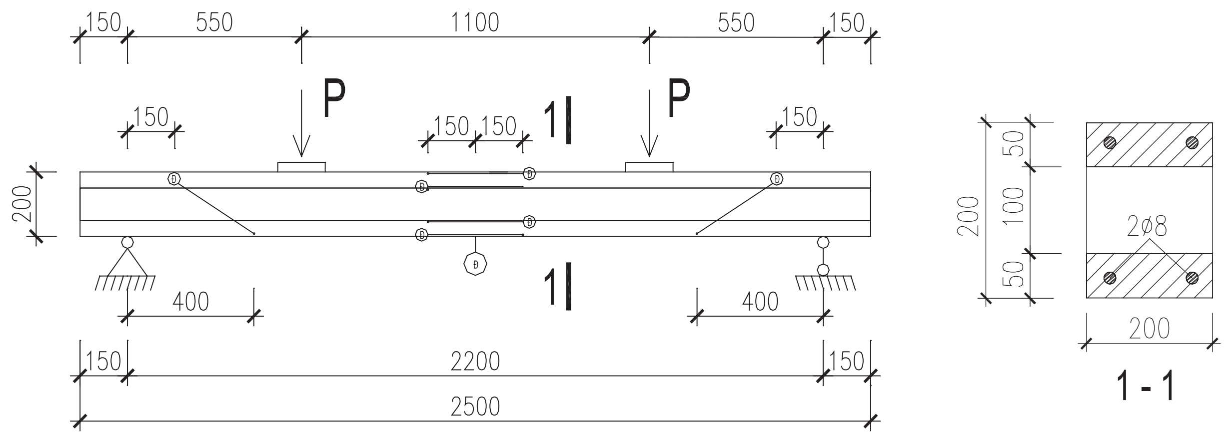


Figure 6. Parameters and dimensions of the test beam

Figure 7. The scheme of the layout instruments for bending testing

Table 2. Characteristics of steel in experimental tests

Diameter (mm)	f_y, MPa	f_u, Mpa	E_s 10^{-3}, Mpa
8	390	560	206

Table 3. Characteristics of concrete in experimental tests

Class of concrete	Density, kg/m^3	Compressive strength at different ages of 28 days R_b, MPa	Ultimate tensile strength in bending at the age of 28 days R_{bt}, MPa	Young's modulus E_b, MPa
B0.75	346	0.75	0.23	570
B25	2376	18.06	2.56	33200

The experimental results of the relationship between moment and deflection of test beam B1 under the action of load are shown in Figures 8, 9 [14] [24] and Table 4.

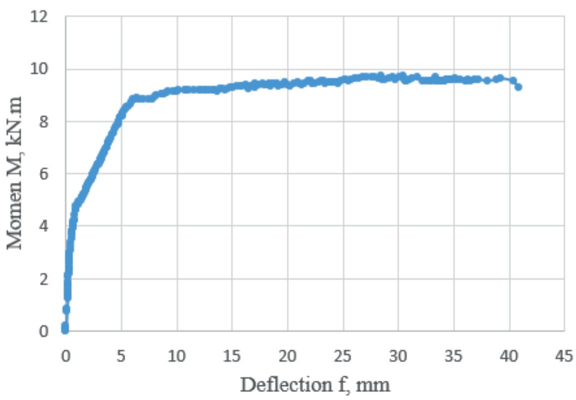


Figure 8. The relationship between the moment and the deflection of test beam B1

Figure 8. Views of the destruction of the tested beam samples

Table 4. The results of the destructive bending moment

Nº	**Test beam**	Destructive bending moment, kN.m							
		The test results [14]	Average value of test results	By using the scheme 1 [14]		By using the scheme 2 [14]		By using the scheme 3 (c)	
				When stress in steel					
				$1,2\sigma_y$	σ_u	$1,2\sigma_y$	σ_u	$1,2\sigma_y$	σ_u
1	B1-1	9.25	9.55	8.43	9.92	8.48	9.94	8.50	9.95
2	B1-2	9.56							
3	B1-3	9.84							

From the results obtained in Table 5, it has been shown that the destructive bending moment by the test beam is larger than the value calculated according to scheme 1 (converted to the equivalent cross-section of the letter I), scheme 2 and scheme 3, respectively, by 13.2%, 12.66% and 12.4% (when stress in steel is $1,2\sigma y$). Based on the results obtained, we make the following observations: Using the proposed model to calculate the strength and strain of multi-layer reinforced concrete beams yields results consistent with the beams' actual behavior.

3.2. Numerical analysis of the load-bearing capacity of a three-layer reinforced concrete beam considering the contact layer

Based on the analysis of experimental results and applied theoretical models regarding the performance of three-layer reinforced concrete structures, the author finds that the model proposed in section 2.3.3 can be applied to calculate the load-bearing capacity of multi-layer reinforced concrete structures made of different materials, taking into account the continuously changing characteristics of the bonding layer between the material layers. The authors analyzed the load-bearing capacity of three-layer reinforced concrete beams using beam specimens, as described in section 3.1, accounting for variations in the concrete types of the inner and outer layers. In particular, the authors also considered variations in the thickness of the contact layer formed when constructing two layers of different concrete types. The reinforcing steel bars used in the analytical beam are 2 ∅ 14, types of CB400V [30]. The material properties and parameters of the test beam are shown in Tables 6 & 7.

Table 5. Characteristics of concrete in beam tests

STT	Type of concrete	Class of concrete	Compressive strength R_b, MPa	Ultimate tensile strength in bending R_{bt}, MPa	Young's modulus E_b, MPa
1	Normal concrete	B15	8.5	0.75	24000
2		B20	11.5	0.9	27500
3		B25	14.5	1.05	30000
4	Lightweight concrete	B5	2.8	0.37	5000
5		B10	6	0.56	8000
6		B15	8.5	0.75	10500

Table 6. Parameters of three-layer beams

Beam	Thickness of outer layers, mm	Concrete of the **outer** layer		Thickness of inner layers, mm	Concrete of the inner layer		Ratio of R_{b1}/R_{b2}
		Class of normal concrete	Compressive strength R_{b1}, MPa		Class of lightweight concrete	Compressive strength R_{b2}, MPa	
B2-1	50	B25	14.5	100	B5	2.8	5.18
B2-2	50	B20	11.5	100	B5	2.8	4.11
B2-3	50	B15	8.5	100	B5	2.8	3.04
B3-1	50	B25	14.5	100	B10	6	2.42
B3-2	50	B20	11.5	100	B10	6	1.92
B3-3	50	B15	8.5	100	B10	6	1.42
B4-1	50	B25	14.5	100	B15	8.5	1.71
B4-2	50	B20	11.5	100	B15	8.5	1.35
B4-3	50	B15	8.5	100	B15	8.5	1.00

The results of the load-bearing capacity analysis of three-layer reinforced concrete beams under varying concrete types in the inner and outer layers, and with varying thickness of the bonding layer, are shown in Table 7.

Table 7. The results of the destructive bending moment of a three-layer beams when changing the thickness of the contact layer

*	Destructive bending moment Mu, kN.m								
	B2-1	B2-2	B2-3	B3-1	B3-2	B3-3	B4-1	B4-2	B4-3
0	25.875	20.714	16.277	26.235	24.428	21.869	26.33	24.831	22.81
0.4	25.878	20.717	16.278	26.232	24.429	21.87	26.331	24.832	22.81
0.8	25.883	20.722	16.28	26.241	24.431	21.872	26.333	24.834	22.81
1	25.885	20.731	16.282	26.243	24.433	21.874	26.335	24.836	22.81
Ratio of R_{b1}/R_{b2}	5.18	4.11	3.04	2.42	1.92	1.42	1.71	1.35	1

The moment of failure values of the beam samples described in Table 7 indicate that, as the thickness of the connection layer increases, the load-bearing capacity of the beam increases. However, the difference in the beam's load-bearing capacity when considering the thickness of the connection layer versus not considering it is negligible (less than 1%).

The results shown in Table 7 and Figure 9 indicate that the compressive strengths of concrete in the outer and inner layers affect the load-bearing capacity of a three-layer reinforced concrete beam.

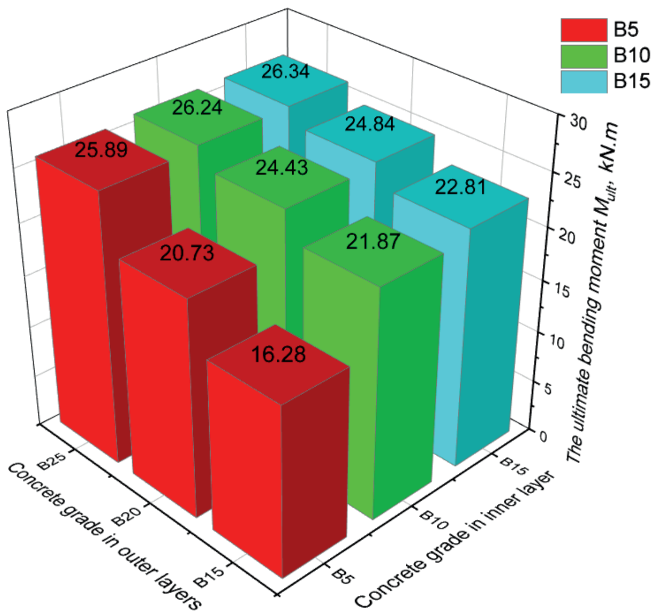


Figure 9. Change of the ultimate bending moment of a three-layer beam when changing the types of concrete in the outer layer and in the inner layer

With the same concrete grade of outer layer, when the grade of lightweight concrete in the internal layer increases from B5 to B15, the ultimate bending moment of the test beam can be increased from 16.28 kN.m to 22.81 kN.m (increase by 40.1%). With the same concrete grade of inner layer, when the grade of normal concrete in the outer layer increases from B15 to B25, the ultimate moment of the test beam can be increased from 16.28 kN.m to 25.89 kN.m (increase by 59%).

When the concrete grade of the inner concrete layer is B15, changing the concrete grade of the outer concrete layer results in the least change in the ultimate moment of the three-layer structure. This can be understood to mean that when the difference in compressive strength between the outer and inner concrete layers is small, the change in the ultimate bending moment is small.

4. CONCLUSIONS

Based on the results analyzed above, the author draws the following conclusions:

- This paper has proposed a method for calculating the load-bearing capacity of three-layer reinforced concrete beams, taking into account the continuous variation of the strength of the contact layer. The results of the load-bearing capacity analysis of three-layer reinforced concrete beams using the proposed method closely match the experimental results of beam samples.
- The load-bearing capacity of a three-layer reinforced concrete beam in cases where the thickness of the connection layer is taken into account differs insignificantly from the case where the thickness of the contact layer is not taken into account. The difference in limiting moments in the analysis of three-layer reinforced concrete beams in this study is less than 1%.
- The compressive strength of the outer and inner concrete layers significantly affects the change in the load-bearing capacity of a three-layer reinforced concrete beam. In the structural analysis of this study, the difference in load-bearing capacity can reach up to 59% when varying the compressive strength of the structure's outer concrete layer. For a change in

the inner concrete layer of the structure, the load-bearing capacity of the three-layer beam can differ by up to 40.1%.

The research results have contributed to the development of the theory of calculating the structural design of three-layer reinforced concrete structures using different types of materials. Considering the properties of the bonding layers helps to more accurately describe the actual phenomena occurring during the construction of these concrete structures. However, this study only examined a few specific cases. To make the research more practically significant, a comparative analysis with many other experimental cases is needed.

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Vu Dinh Tho – PhD in Technical Sciences, Teacher of the Department of Construction of civil engineering and industrial engineering, University of Transport Technology (UTT), (Vietnam), 100000, 54-Trieu Khuc, Thanh Liet, Ha Noi, Vietnam; e-mail: thovd@utt.edu.vn; 0000-0001-7374-9424.

Elena A. Korol – doctor of Technical Sciences, Professor, Professor of the Department "Housing and Communal Complex", National Research Moscow State University of Civil Engineering" (NIU MGSU); 129337, 26, Yaroslavskoye Shosse, Moscow, Russia; e-mail: KorolEA@mgsu.ru; 0000-0002-5019-3694.

Pham Tuan Anh – PhD in Technical Sciences, Associate Professor, Department of Construction of civil engineering and industrial engineering « University of Transport Technology (UTT)» (Vietnam), 100000, 54-Trieu Khuc, Thanh Liet, Ha Noi, Vietnam; ; e-mail: anhpt@utt.edu.vn; 0000-0001-9484-1916.

Pham Thi Hien – Master of Engineering, Teacher of the Department of Construction of civil engineering and industrial engineering, University of Transport Technology (UTT), (Vietnam), 100000, 54-Trieu Khuc, Thanh Liet, Ha Noi, Vietnam; e-mail: hienpt89@utt.edu.vn

Le Ngoc Lan – PhD in Technical Sciences, Teacher of the Department of Construction of civil engineering and industrial engineering, University of Transport Technology (UTT), (Vietnam), 100000, 54-Trieu Khuc, Thanh Liet, Ha Noi, Vietnam; e-mail: lanln@utt.edu.vn; 0009-0001-1361-471X

Ву Динь Тхо – канд. техн. наук, преподаватель кафедры "Промышленное и гражданское строительство", Университет транспортных технологий; 100000, 54- Чиеу Хук, Тхань Лиет, Ха Ной, Вьетнам; e-mail: thovd@utt.edu.vn; 0000-0001-7374-9424.

Король Елена Анатольевна – доктор технических наук, профессор, профессор кафедры «Жилищно-коммуналый комплекс», Федеральное государственное бюджетное образовательное учреждение высшего образования «Национальный исследовательский Московский государственный строительный университет» (ФГБОУ ВО «НИУ МГСУ»); 129337, г. Москва, Ярославское шоссе, д. 26; e-mail: KorolEA@mgsu.ru.; 0000-0002-5019-3694.

Фам Туань Ань – канд. техн. наук, доцент, преподаватель кафедры "Промышленное и гражданское строительство", Университет транспортных технологий; 100000, 54- Чиеу Хук, Тхань Лиет, Ха Ной, Вьетнам; e-mail: anhpt@utt.edu.vn; 0000-0001-9484-1916.

Фам Тхи Хиен – магистр, преподаватель кафедры "Промышленное и гражданское строительство", Университет транспортных технологий; 100000, 54- Чиеу Хук, Тхань Лиет, Ха Ной, Вьетнам; e-mail: hienpt89@utt.edu.vn;

Ле Нгок Лан – канд. техн. наук, преподаватель кафедры "Промышленное и гражданское строительство", Университет транспортных технологий; 100000, 54- Чиеу Хук, Тхань Лиет, Ха Ной, Вьетнам; e-mail: lanln@utt.edu.vn; 0009-0001-1361-471X.