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ABOUT DEVELOPMENT OF COMPUTATIONAL SCHEMES OF GROUP TARGETED KINEMATIC DEVICES FOR SOME ELASTIC SYSTEMS

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Abstract: For some elastic systems with a finite number of degrees of freedom of mass, in which the directions of mass movement are parallel, methods have been developed for creating additional kinematic devices, the introduction of each of which targetly reduces the value of only one natural frequency to a given value, while not changing any of the other natural frequencies and not one of the forms of natural oscillations. If it is necessary to precisely reduce the values of several natural frequencies, then this requirement can be realized by creating an appropriate number of separate targeted kinematic devices. The computational scheme of each of the individual targeted devices should include racks installed at the mass application nodes and directed along the trajectory of their movement. In some cases, separate targeted constraints can be independently installed in the source system. In most cases, on the basis of individual targeted kinematic devices, a computational scheme of a single group targeted device is formed, which reduces all the intended frequencies to the set values without changing any of the other natural frequencies and not one of the forms of natural oscillations. The distinctive paper is devoted to the method for forming a matrix of coefficients of additional inertial forces, which corresponds to a group targeted kinematic device. An algorithm for the creation of group targeted kinematic devices is proposed. Algorithm is tested with the use of “SCAD” and “Lira” software products.

Keywords: natural oscillation frequency, natural oscillation shape, natural modes, separate targeted kinematic device, group targeted kinematic device, coefficients of additional inertial forces

О ФОРМИРОВАНИИ РАСЧЕТНЫХ СХЕМ ГРУППОВЫХ ПРИЦЕЛЬНЫХ КИНЕМАТИЧЕСКИХ УСТРОЙСТВ ДЛЯ НЕКОТОРЫХ УПРУГИХ СИСТЕМ

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Аннотация: Для некоторых упругих систем с конечным числом степеней свободы масс, у которых направления движения масс параллельны, разработаны методы создания дополнительных кинематических устройств, введение каждого из которых прицельно уменьшает величину только одной собственной частоты до заданного значения, не изменяет при этом ни одну из остальных собственных частот и ни одну из форм собственных колебаний. Если необходимо прицельно уменьшить величины нескольких собственных частот, то это требование можно реализовать созданием соответствующего количества отдельных прицельных кинематических устройств. Расчётная схема каждого из отдельных прицельных устройств должна включать стойки, установленные в узлах приложения масс и направленные по траектории их движения. В некоторых случаях отдельные прицельные связи могут автономно устанавливаться в исходной системе. В большинстве случаев на основе отдельных прицельных кинематических устройств формируется расчётная схема единого группового прицельного устройства, которое уменьшает все намеченные частоты до заданных значений, не изменяя при этом ни одну из остальных собственных частот и ни одну из форм собственных колебаний. В данной статье рассмотрен способ формирования матрицы коэффициентов дополнительных инерционных сил, которой соответствует групповое прицельное кинематическое устройство. Предложен алгоритм формирования групповых прицельных кинематических устройств. Первичная пробация (тестирование) алгоритма была реализована с использованием программных продуктов “SCAD” и «Лира».

Ключевые слова: частота собственных колебаний, форма собственных колебаний, отдельная прицельное кинематическое устройство, групповое прицельное кинематическое устройство, коэффициенты дополнительных инерционных сил

1. INTRODUCTION

For some elastic systems with a finite number of degrees of freedom of the masses, in which the directions of motion of the masses are parallel, methods have been developed for creating additional kinematic devices, which introduction specifically reduces the magnitude of only one natural frequency to a given value, without changing any of the other natural frequencies or any of the natural oscillation modes [2, 7, 10-13]. If it is necessary to specifically reduce the values of several natural frequencies, this requirement can be realized by creating an appropriate number of separate targeted kinematic devices [1]. The computational scheme [3, 4] for each of the separate targeted devices must include posts installed in the nodes of mass application and directed along the trajectory of their motion. In some cases, individual targeted constraints can be independently installed in the original system. In most cases, a computational scheme for a single group targeted device [8] is formed on the basis of individual targeted kinematic devices, which reduces all the intended frequencies to given values, without changing any of the other natural frequencies or any of the natural oscillation modes. The distinctive paper is devoted to the method of forming of matrix of additional inertial force coefficients corresponding to a group kinematic targeted device. An algorithm [9, 16-21] for forming group kinematic targeted devices is proposed. Initial testing of the algorithm was carried out with the use of “SCAD” and “Lira” software products [18].

2. ALGORITHM FOR CREATION OF COMPUTATIONAL SCHEME OF A GROUP TARGETED KINEMATIC DEVICE

The following algorithm for creation of computational scheme for a group targeted kinematic device is used:

1) Individual targeted kinematic devices are generated; each of them reduces one of the intended natural frequencies to a specified value;
2) All individual targeted kinematic devices are placed at the nodes of the original system;
3) In areas where several posts are aligned, only one is left;
4) Hinges are introduced at the intersections of the rods.

During the first step of the algorithm, the natural frequency spectrum of the original system with n degrees of freedom is considered

$$\omega[1],\ \omega[2],\ ...,\ \omega[q],\ ..,\ \omega[q+j],\ \omega[q+(j+1)],\ ...,\ \omega[n].$$

It is necessary to specifically clear the frequency interval $\omega[q]$, .., $\omega[q+j]$ from j frequencies, reducing their values to values $\omega[s],\ ..,\omega[s+j-1]$.

3. NUMERICAL EXAMPLE OF IMPLEMENTING OF ALGORITHM OF CREATION OF COMPUTATIONAL SCHEME OF GROUP TARGETED KINEMATIC DEVICE

Let us demonstrate the implementation of the algorithm described in the previous paragraph using the example of a system with five degrees of freedom from [2]. The beam section is a composite I-beam (Figure 1). The modulus of elasticity of the system material is $E = 206000$ MPa. The mass values are equal to

$$m[1] = m[5] = 400\ \text{kg};$$
$$m[2] = m[4] = 800\ \text{kg};\ m[3] = 1200\ \text{kg}.$$

The frequencies and modes of the natural oscillations of the original system are determined by the displacement method, in which the unknowns are the displacements in the direction of mass motion. The basic system of the displacement method is shown in Figure 2.

The coefficients of the displacement method equations and the values of the nodal masses form matrices

$$A = \|a[i,\ k]\|;\quad M = \|m[i]\|. \tag{1}$$

The roots of the equation

$$\left|A - \omega^2 M\right| = 0 \tag{2}$$

define the frequency spectrum of the system's natural oscillations. The frequencies and modes of natural oscillations of the original system are presented in Table 1.

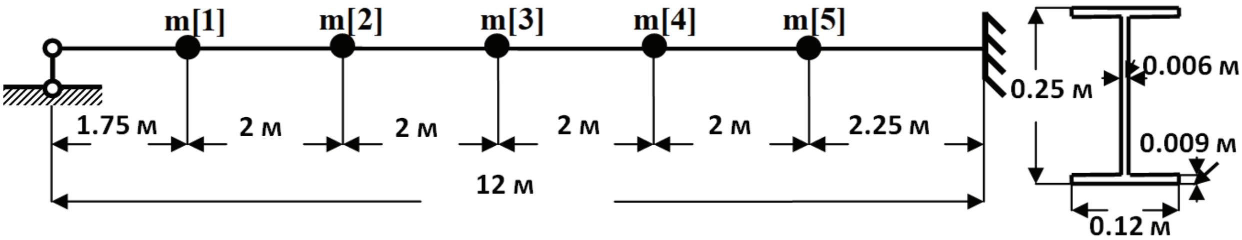

Figure 1. Considering structure (numerical example)

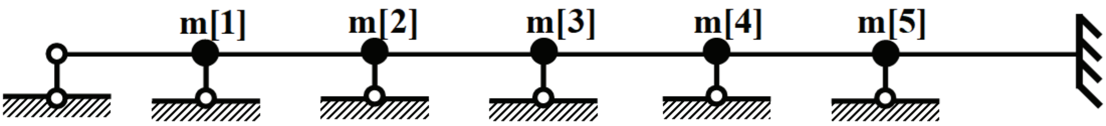

Figure 2. The basic system of the displacement method (numerical example)

Table 1. Frequencies and modes of natural oscillations of the original system

Number k	Формы собственных колебаний					$m[k]$
	for frequency 14.07953	for frequency 53.0752	for frequency 109.5337	for frequency 195.0757	for frequency 226.8870	
0	0.0000	0.0000	0.0000	0.0000	0.0000	0
1	-0.315087	-0.502865	-0.594382	-0.73793	-0.369613	400
2	-0.565080	-0.520103	-0.174580	0.370387	-0.264938	800
3	-0.606213	0.103822	0.432120	-0.086169	0.212601	1200
4	-0.432097	0.574986	-0.359747	-0.137055	-0.383591	800
5	-0.164932	0.367738	-0.547801	0.540420	0.775147	400
6	0.0000	0.0000	0.0000	0.0000	0.0000	0

In order to form individual targeted kinematic devices, it is necessary to determine the forces that should appear in the device's posts [2, 5-7, 10-15]

$$R_0[i,\ k] = m[i]v[i,\ k], \qquad (3)$$

where $i,\ k$ are the numbers of the components of the k-th mode of natural oscillations, respectively.

If in the considering example it is required to reduce the value of the first natural oscillation frequency from 14.0715 sec^{-1} to 10 sec^{-1}, and the second natural oscillation frequency from 53.0769 sec^{-1} to 11 sec^{-1}, then

$$R_0[i,\ 1] = m[i]v[i,\ 1]; \quad R_0[i,\ 2] = m[i]v[i,\ 2].$$

The values are given in Table 2.

Table 2. Values of $R_0[i,\ k]$

i	1	2	3	4	5
$R_0[i,\ 1]$	63.04	226.24	364.32	173.28	33.08
$R_0[i,\ 2]$	-101.52	-210	62.88	232.16	74.24

Table 3. Frequencies and modes of natural oscillations of the modified system

Number k	Формы собственных колебаний					$m[k]$
	for frequency 10.0095	for frequency 53.0752	for frequency 109.5337	for frequency 195.0757	for frequency 226.8870	
0	0.0000	0.0000	0.0000	0.0000	0.0000	0
1	0.315087	-0.502865	-0.594382	-0.73793	-0.369613	400
2	0.565080	-0.520103	-0.174580	0.370387	-0.264938	800
3	0.606213	0.103822	0.432120	-0.086169	0.212601	1200
4	0.432097	0.574986	-0.359747	-0.137055	-0.383591	800
5	0.164932	0.367738	-0.547801	0.540420	0.775147	400
6	0.0000	0.0000	0.0000	0.0000	0.0000	0

In order to construct a computational model for a kinematic targeted device that reduces $\omega[q]$ to $\omega[s]$, a matrix of additional inertial forces is formed:

$$M_0 = M_{m0} M_m; \qquad (4)$$

$$M_m = \| m_0[i,k] \|_{i,k=1}^{n}; \qquad (5)$$

$m_0[i,\ k] = m[i]m[k]v_\omega[i,\ q]v_\omega[k,\ q]$ or

$$m_0[i,\ k] = R_0[i,\ q]R_0[k,\ q]; \qquad (6)$$

$$M_{m0} = \frac{\sum_{i=1}^{n}\sum_{k=1}^{n}(a[i,\ k] - (\omega[s])^2 m[i,\ k])v_\omega[i,\ q]v_\omega[k,\ q]}{\sum_{i=1}^{n}\sum_{k=1}^{n}(\omega[s])^2 m_0[i,\ k]v_\omega[i,\ q]v_\omega[k,\ q]}. \qquad (7)$$

After forming the matrix, the following equation is solved:

$$\left| A - \omega^2 (M + M_{m0} M_m) \right| = 0. \qquad (8)$$

The solution results determine the frequency spectrum and natural oscillation modes of the modified system.

For systems in which the directions of mass motion during natural oscillations lie in the same plane, the method of creation of computational model is based on the properties of a rope polygon constructed from the forces $R_0[i,\ k]$. For the initial system considered in the distinctive paper, such an approach is given, for example, in [2]. There we have $R_0[i,\ 1] = m[i]v[i,\ 1]$ and corresponding kinematic targeted device is formed,

that reduces the first natural oscillation frequency from 14.0715 sec^{-1} to 10 sec^{-1}. The frequencies and forms of natural oscillations of the modified system are presented in Table 3.
The general view of this kinematic targeted device is shown in Figure 3. We have the following lengths of posts:

$$l_p[1] = 2.6934\,\text{m};\quad l_p[2] = 0.9406\,\text{m};$$
$$l_p[3] = 0.2000\,\text{m};\quad l_p[4] = 1.0879\,\text{m};$$
$$l_p[5] = 2.7498\,\text{m}.$$

The magnitude of the additional mass is determined by the relationship (see, for example, [2])

$$M_{add} = M_{mo}(N[j])^2, \tag{9}$$

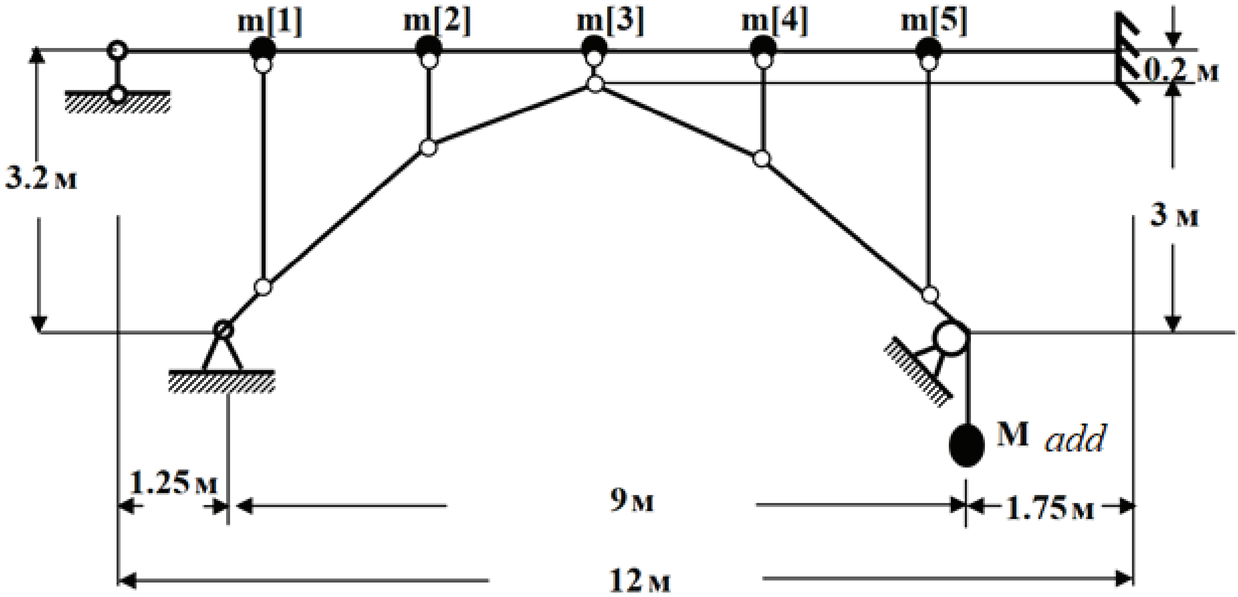

Figure 3. The general view of computational scheme of targeted kinematic device

Table 4. Coordinates of the belt nodes

Number of node	0	1	2	3	4	5	6
Coordinate of node	4.0557	2.9725	1.6217	6.4406	9.4115	5.5631	4.0557

Table 5. Frequencies and modes of natural oscillations of the modified system

Number k	Формы собственных колебаний					$m[k]$
	for frequency 11.0062	for frequency 14.07953	for frequency 109.5337	for frequency 195.0757	for frequency 226.8870	
0	0.0000	0.0000	0.0000	0.0000	0.0000	0
1	-0.502865	-0.315087	-0.594382	-0.73793	-0.369613	400
2	-0.520103	-0.565080	-0.174580	0.370387	-0.264938	800
3	0.103822	-0.606213	0.432120	-0.086169	0.212601	1200
4	0.574986	-0.432097	-0.359747	-0.137055	-0.383591	800
5	0.367738	-0.164932	-0.547801	0.540420	0.775147	400
6	0.0000	0.0000	0.0000	0.0000	0.0000	0

where $N[j]$ is the force in the outermost post of the device's belt, to which the additional mass is attached.

The additional mass for the formed device is equal to $M_{add} = 1586.5229$ kg. The cross-sectional areas of the kinematic device rods are identical; their values can be chosen arbitrarily. A cable runs from the lower hinge of fifth post through the block to the ground.

Similarly we have $R_0[i,\ 2] = m[i]v[i,\ 2]$ and a matrix of additional inertial forces is formed based on the forces, reducing the second natural frequency from 53.0769 sec^{-1} to 11 sec^{-1}, and a corresponding computational model for the targeted kinematic device is created. A general view of the computational model for this targeted kinematic device is shown in Figure 4.

The additional mass is equal to

$$M_{add} = 6301.7206915414\,\text{kg}.$$

The frequencies and modes of natural vibrations of this modified system are presented in Table 5.

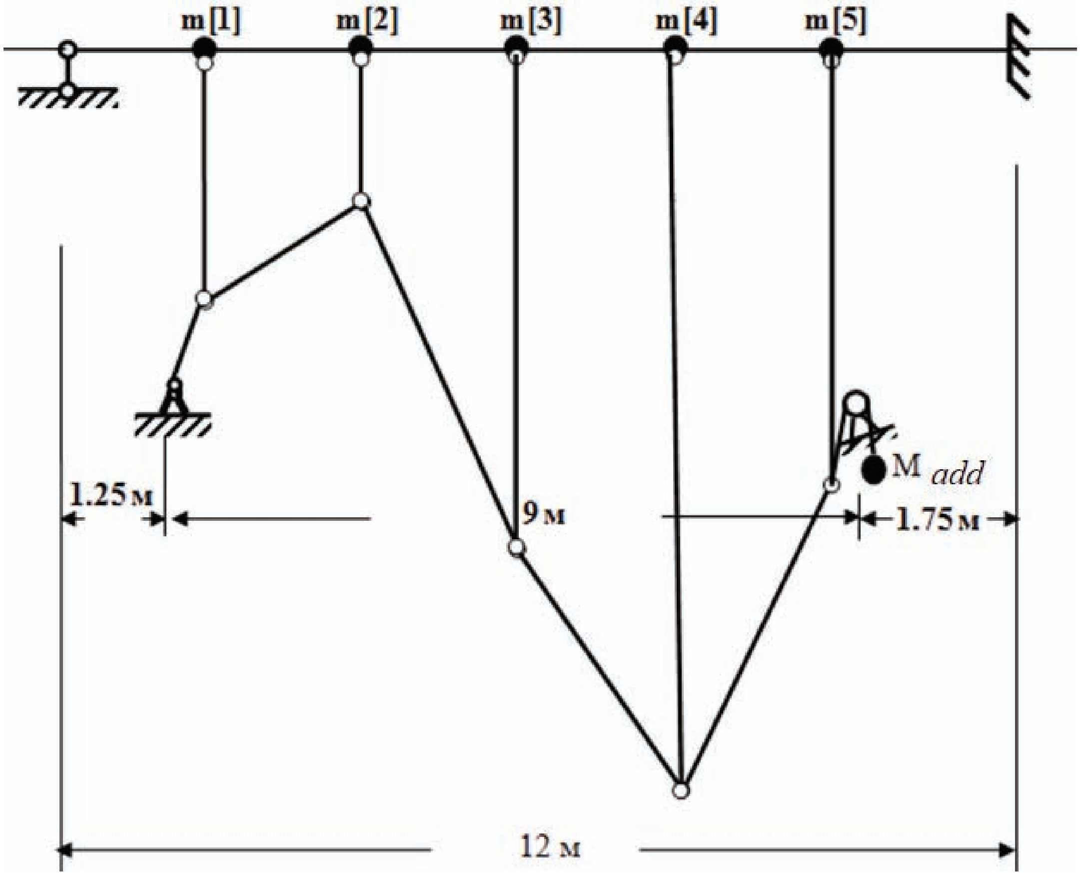

Figure 4. The general view of computational scheme of targeted kinematic device

Table 6. Coordinates of the belt nodes

Number f o node	0	1	2	3	4	5	6
Node coordinate (upper chord)	3.2000	2.6934	0.9406	0.2000	1.0879	2.7498	3.2000
Node coordinate (lower chord)	4.0557	2.9725	1.6217	6.4406	9.4115	5.5631	4.0557

After creation of individual kinematic targeted devices necessary for solving the problem, a group matrix of additional inertial force coefficients can be formed. In order to do this it is required simply sum the coefficients of the additional inertial force matrices of the individual kinematic devices.

By implementing the second step of the algorithm described above, we form a group targeted kinematic device. In order to do this, the generated computational schemes for the individual targeted kinematic devices are placed at the nodes of the original system. The general view of this device is shown in Figure 5.

We have the following additional masses:

$$M_{add1} = 1586.5229\,\text{kg}, \quad M_{add2} = 6301.72069\,\text{kg}.$$

In this version of the group targeted kinematic device, both additional masses are attached to the rightmost links of the individual kinematic devices' belts.

In some cases, it may be necessary to attach additional masses to different sides of the group targeted kinematic device. Implementing this requirement does not complicate the procedure of creation of the group targeted device. In this case, in the individual kinematic devices used, the corresponding numbers j of the outermost belt links are selected when determining the values (9).

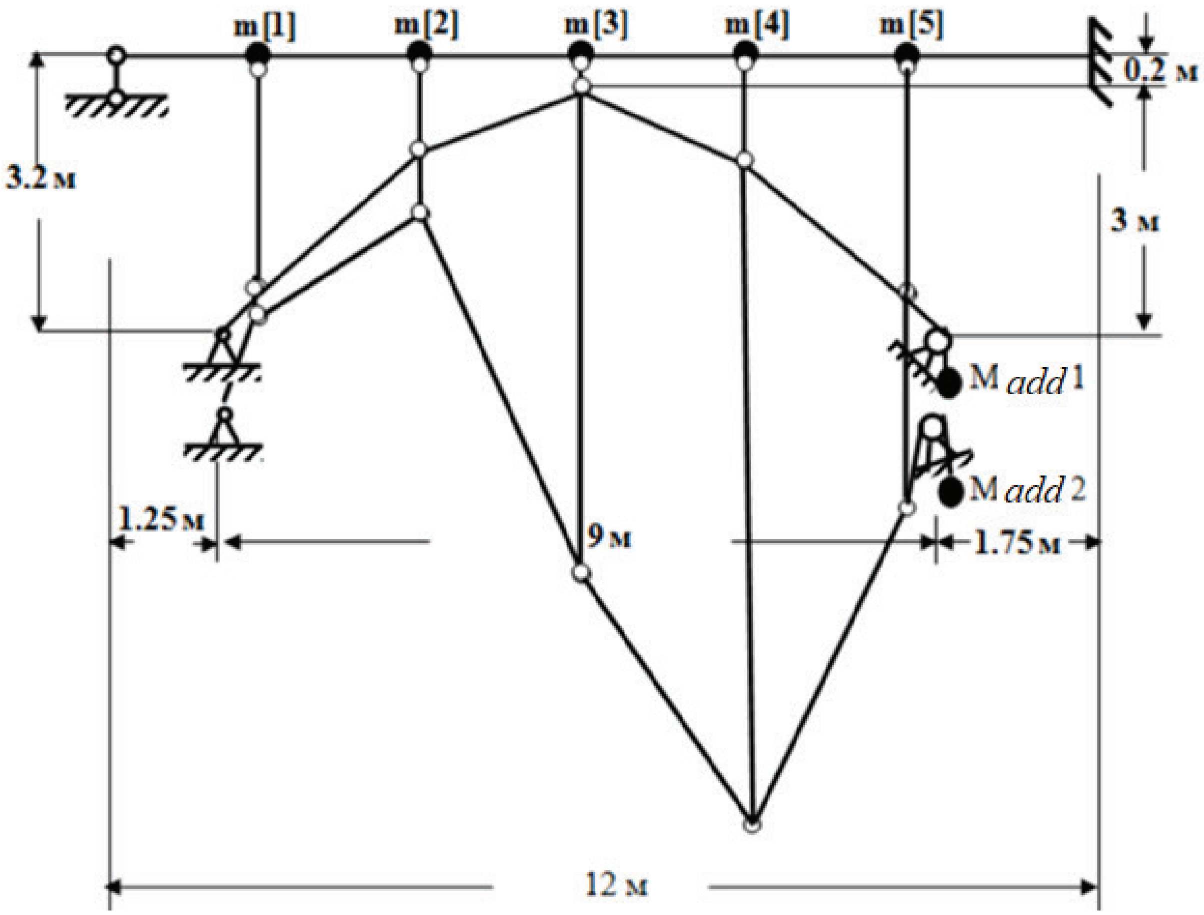

Figure 5. The general view of the device

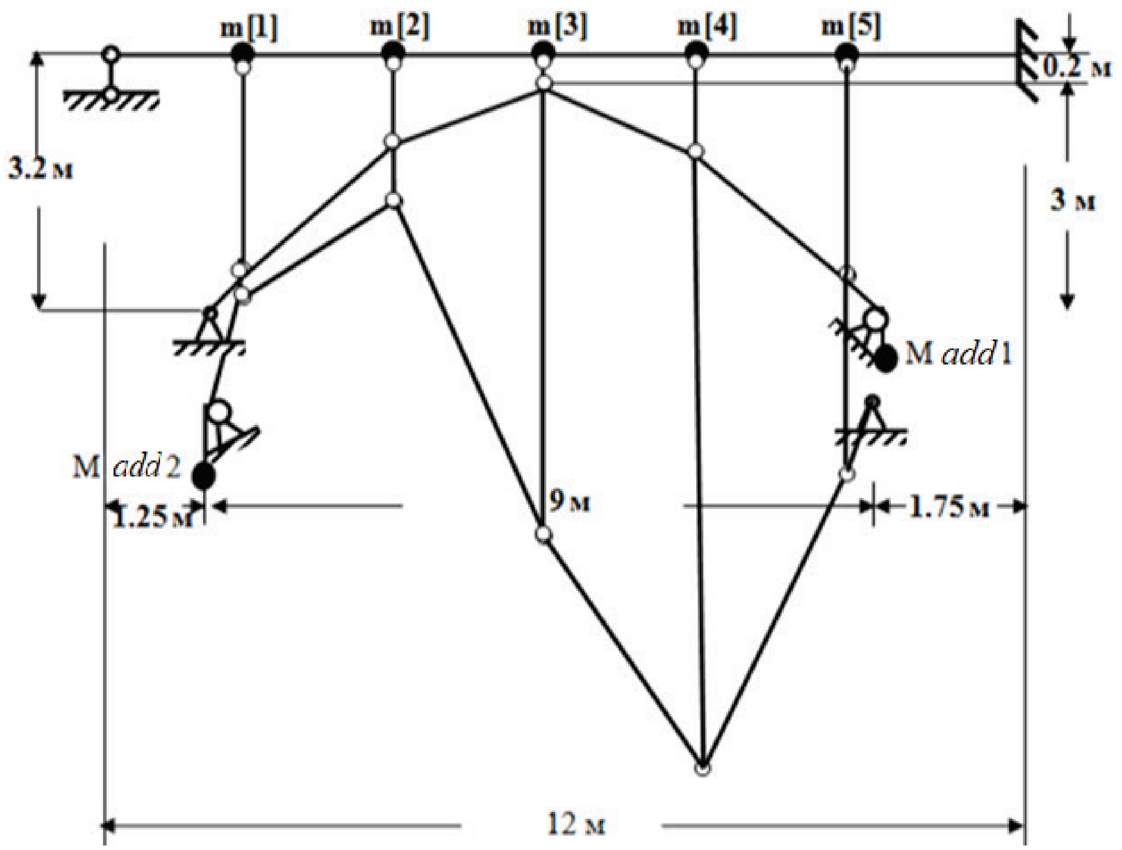

Figure 6. The variant of group targeted kinematic device.

Table 7. Frequencies and modes of oscillation of both variants of the group targeted kinematic device

Number k	Формы собственных колебаний					$m[k]$
	for frequency 11.0062	for frequency 14.07953	for frequency 109.5337	for frequency 195.0757	for frequency 226.8870	
0	0.0000	0.0000	0.0000	0.0000	0.0000	0
1	-0.502865	-0.315087	-0.594382	-0.73793	-0.369613	400
2	-0.520103	-0.565080	-0.174580	0.370387	-0.264938	800
3	0.103822	-0.606213	0.432120	-0.086169	0.212601	1200
4	0.574986	-0.432097	-0.359747	-0.137055	-0.383591	800
5	0.367738	-0.164932	-0.547801	0.540420	0.775147	400
6	0.0000	0.0000	0.0000	0.0000	0.0000	0

Table 8. Comparison of the natural frequencies of the original system with the results of testing using the "SCAD" and "LIRA" software products

	$\omega[1]$	$\omega[2]$	$\omega[3]$	$\omega[4]$	$\omega[5]$
Proposed pproach	14.0715	53.0769	109.5339	195.0757	226.8873
"SCAD" and "LIRA" software packages	14,0820	53,4030	111,5070	201,4150	237,1590

A version of the group sighting kinematic device in which the additional masses are attached to different sides is shown in Figure 6.

The frequencies and modes of natural oscillations of both variants of the group targeted kinematic device are identical and are presented in Table 7.

The posts of the individual targeted kinematic devices, mounted at the mass application nodes and directed along their trajectory, are aligned. Forces $R_0[i,\ k] = m[i]v[i,\ k]$ arise in the aligned posts, each of which determines one of the individual targeted kinematic devices.

The absence of mutual influence between the forces determining the various individual kinematic devices is justified by the properties of the forces $R_0[i,\ k]$. The forces $R_0[i,\ k_1]$ determining the targeted device, which reduces the natural frequency $\omega[k_1]$, do not perform work on the displacements of the natural oscillation mode of any other natural frequency, that is,

$$\text{if } k_1 \neq k_2 \text{ we get } \sum_{i=1}^{n} R_0[i,\ k_1]v[i,\ k_2] = 0. \quad (10)$$

Relationship (10) determines the absence of mutual influence between the forces $R_0[i,\ k]$ arising in the posts of the group targeted device.

The results presented in Table 7 confirm that the introduction of the group targeted kinematic device reduces only the first two selected natural oscillation frequencies to the intended values and does not lead to a change in the natural modes and values of the remaining frequencies. The results presented in Tables 1, 2, 3, 5, 7 were obtained by the finite element method (FEM) [18] by dividing (discretizing) the length of the original beam into 24 sections. A comparison of the natural vibration frequencies of the original system with the test results of the “SCAD” and “LIRA” software products is presented in Table 8. Considering the difference in the calculation method for Tables 1, 2, 3, 5, 7 between the “SCAD” and “LIRA” software products, the agreement between the results can be considered fairly close, confirming the reliability of the original data.

Table 9. Testing the group targeted kinematic device

	$\omega[1]$	$\omega[2]$	$\omega[3]$	$\omega[4]$	$\omega[5]$
Initial	14,0820	53,4030	111,5070	201,4150	237,1590
Group	10,8000	11,1400	111,5200	201,4200	237,1600

We will now test the group targeted kinematic device using the “SCAD” and “LIRA” software products. The results are presented in Table 9.

The data presented in Table 9 prove that the testing performed convincingly demonstrates a reduction exclusively in those frequencies targeted by the group kinematic device, while the other natural oscillation frequencies remained unchanged.

In summary, the distinctive paper proposes an algorithm of creation of group targeted kinematic devices. Besides, a method of creation of a matrix of additional inertial forces corresponding to a group kinematic targeted device is considered. We provide a rationale for the absence of mutual interference between the forces $R_0[i,\ k]$ arising in the group kinematic device posts, and tests the proposed algorithm using the “SCAD” and “LIRA” software products.

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