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PROBABILISTIC DYNAMIC ANALYSIS OF A SINGLE-MASS VIBRATION ISOLATION SYSTEM

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Abstract: The article considers the problem of probabilistic calculation of a single-mass oscillatory system intended for solving problems of force or kinematic vibration isolation. The solution of three problems for the mode of free and forced oscillations is considered depending on the element, the probabilistic characteristics of which are determined. The first problem includes the case of a known probabilistic distribution of the load and constant system parameters. The second problem includes the case of a known probabilistic distribution of the system parameters under a constant external influence. The third problem will combine known probabilistic characteristics for both the external influence and the system parameters. The purpose of solving these problems is to obtain the distribution of the system response under the specified types of influences, as well as to test the methodology for their numerical solution using the Monte Carlo method.

Keywords: vibration isolation, probabilistic calculations, randomness, force vibration protection, kinematic vibration protection, single-mass system

ВЕРОЯТНОСТНЫЙ РАСЧЁТ ОДНОМАССОВОЙ ВИБРОЗАЩИТНОЙ СИСТЕМЫ

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Аннотация: В статье рассматривается задача вероятностного расчёта одномассовой колебательной системы, предназначенной для решения задач силовой или кинематической виброизоляции. Рассматривается решение трёх задач для режима свободных и вынужденных колебаний в зависимости от элемента, вероятностные характеристики которого определены. К первой задаче относим случай известного вероятностного распределения нагрузки и постоянными параметрами системы. Ко второй – случай известного вероятностного распределения параметров системы при постоянном внешнем воздействии. Третья задача будет объединять известные вероятностные характеристики как для внешнего воздействия, так и для параметров системы. Целью решения данных задач является получение распределения отклика системы при указанных видах воздействий, а также апробация методики численного их решения, используя метод Монте-Карло.

Ключевые слова: виброизоляция, вероятностные расчёты, случайность, силовая виброзащита, кинематическая виброзащита, одномассовая система

1. INTRODUCTION

The single-degree-of-freedom (SDOF) mechanical model is widely used as the simplest model for analyzing oscillations and vibration isolation of various objects. The equation of motion of such a linear system is written as [1]

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = F_1(t)\,, \tag{1}$$

where m is the mass of the object, kg; c is the coefficient of viscous damping, Ns/m; k is the dynamic stiffness of the elastic element, N/m; and $F_1(t)$ is the external force, N.

The natural frequency of the system could be determined as $\omega_0 = \sqrt{k/m}$, while the relative damping coefficient is defined as $\zeta = \dfrac{c}{2\sqrt{km}}$.

The deterministic approach to vibration isolation calculation assumes that the system parameters (m, c, k) and the external loading $f(t)$ are precisely known and constant. However, in real conditions, both factors could vary significantly, being random in nature due to manufacturing tolerances, changes in material properties and unpredictability of external loads [2, 3]. Ignoring this random variability leads either to unreasonably high safety margins (and an increase in the mass and cost of vibration isolation system) or to reduced reliability if the actual deviations exceed the calculated tolerances [3, 4]. Considering the probabilistic characteristics of both the load and system parameters allows for a more realistic assessment of the dynamic response of the vibration-isolated object and the reliability of vibration protection [1]. Modern design standards for structures and machines are increasingly based on the concepts of reliability and the probabilistic approach, especially for critical structures under severe dynamic loads (seismicity, explosion, road disturbances, etc.) [3].

For vibration isolation systems, the statistical approach means determining the probability distribution of the response (displacements, and/or| accelerations) instead of a single deterministic value. In this way, it is possible to estimate the probability with which critical vibration levels will be exceeded and to select the system parameters that ensure a given level of reliability under random impacts [3]. In practice there are problems in which the external impact on the system could be represented as a random variable (or random process) with known distribution parameters. A large body of work is devoted to the study of this issue, including the monographs by Bolotin [5], Biderman [6], Mondrus [7], Raiser [8], etc. Such types of random loads include wind loads [9, 10], transport loads [11, 12], pedestrian loads [13, 14], microseismic activity [15, 16], and many others. There is also a class of problems in which the system parameters (bearing capacity, deformation characteristics) could be expressed as random variables with known distribution parameters [18 – 20].

This article is aimed at the assessment of the response probabilistic characteristics for a simple SDOF system subjected to an external load, provided that either the load is a random variable with a known distribution, or the system parameters have known probabilistic characteristics. Since the materials used in vibration isolation systems for buildings, structures, and various units have a certain manufacturing error: either geometric dimensions or mechanical parameters, this problem is of high interest. In addition, the forces acting on vibration isolation system often vary greatly over time, which leads to the need to use probabilistic approaches in their calculation justification.

In connection with the above, we consider problems of three types:

1. Random characteristics of system parameters.
2. Random load with a known distribution.
3. Combination of tasks of types 1 and 2.

2. MODEL

We consider the SDOF model for the vibration isolation system shown in Figure 1, in which the isolated body of mass *m* rests on the Kelvin-Voigt element, which has an elastic element of stiffness *k* and a viscous element *c*. The body *m* is subject to either an external load of the type $F_1(x,t)$ (to simplify the problem, we will assume that its value does not depend on the displacement *x* of mass *m*), or a kinematic disturbance of the base of the type $F_2(t) = z(t)$ is applied.

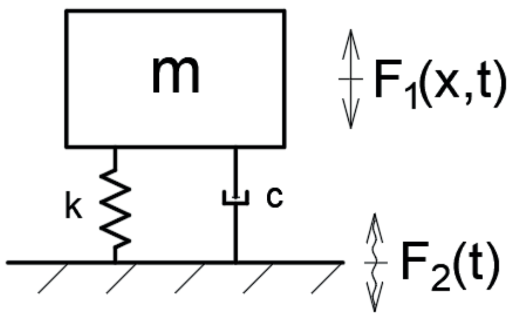


Figure 1. Calculation scheme of a single-mass oscillatory system

The equation for an oscillating mass m under the action of an external load $F_1(t)$ is described by equation (1) subjected to the initial conditions

$$x(0) = x_{00}, \qquad \dot{x}(0) = x_{10} . \tag{2}$$

In the case of an external motion of the base $\ddot{z}(t)$, equation (1) is reduced to the form:

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{z} . \tag{3}$$

The quality of the vibration isolation is assessed by the value of the transfer function:

$$\Delta L = 20\log\sqrt{\frac{1+\eta^2\left(\dfrac{f}{f_0}\right)^2}{\left(1-\left(\dfrac{f}{f_0}\right)^2\right)^2+\eta^2\left(\dfrac{f}{f_0}\right)^2}}, \tag{4}$$

where f is the external load frequency, Hz; f_0 is the natural frequency of the system, Hz, $\omega_0 = 2\pi f_0$; and $\eta = 2\zeta$ is the loss coefficient of the system.

To account for the spread of parameters in calculations of single-mass systems, both numerical Monte Carlo methods and analytical approaches are used [21, 22]. The direct Monte Carlo method involves multiple modeling of the system response with different realizations of random load parameters or stiffness/damping (generated according to a known distribution law) and subsequent statistical analysis of the response (assessment of the amplitude distribution, probability of exceeding threshold values, etc.). This approach is universal and is easily implemented for the SDOF model [23]. However, it does not provide an explicit analytical understanding of the influence of parameter uncertainty and may require large computational costs when it is necessary to estimate small failure probabilities. Therefore, analytical methods have been developed in the literature that make it possible to obtain response characteristics without directly enumerating a set of realizations. One of the classical results is the so-called "damping effect" in stochastic oscillators: it was shown that random fluctuations of the natural frequency ω_0 lead to a decrease in the average resonant response as if the system had more damping, smoothing out the resonance [24]. Adhikari and Pascual [1] considered a linear oscillator with a given distribution of the natural frequency (uncertainty in k for fixed m, c) and obtained an analytical expression for the mean square response in the vicinity of resonance. In particular, for the case of ω_0 uniform distribution, a closed formula for the equivalent damping coefficient was derived, explaining the decrease in the amplitude of the average response compared to a deterministic system [24]. In addition, they developed a hybrid analytical-numerical method (the Laplace method) for calculating not only the average, but also the variance of the response function (frequency characteristic) of the SDOF system for various specified parameter distributions (normal, lognormal, gamma distribution, etc.) [25, 26]. The obtained approximate formulas were successfully verified by comparison with direct simulation using the Monte Carlo method [27]. An important conclusion was that standard small parameter methods (series expansion in small parameter variance) or polynomial chaos could give large errors near resonance [28]. This is due to the nonlinear dependence of the forced vibration amplitude on the natural frequency near the resonant point. Thus, special approaches that take the stochastic nature of resonance into account are required for a reliable estimate of the maximum response. Since for the vibration isolation system its operational range is usually far from resonance, the Monte Carlo method was used for the calculations in further analysis.

2.1. Problem I

In real vibration isolators, the system parameters – primarily the elastic element stiffness (k) and the damping coefficient (c) – are not strictly constant values, but have a spread due to such technological features as material degradation over time, variations in temperature and humidity conditions, and so on [29]. This leads to the fact that the resonant frequency ω_0 and the quality factor of the system also become random variables. If the external influence (e.g., harmonic force or acceleration of the foundation) is fixed and defined, then the system response (oscillation amplitudes, transmitted forces) will be random due to the uncertainty in k, c (or m) [29]. Such a scatter in properties is incidental for elastomeric vibration isolators and rubber mounts: the actual elastic moduli and damping properties may differ from sample to sample. In addition to the discrepancies between the nominal and actual parameters due to manufacturing tolerances, additional changes occur during operation (e.g., rubber aging leads to changes in stiffness and damping). The spread of stiffness and loss factor values is usually described by statistical distributions determined based on experiments. There are various hypotheses about the distribution law of such parameters in literature. In the absence of sufficient experimental data, some researchers assume a uniform spread of ±5% from the nominal value or use a normal distribution with a variation coefficient of about 10–25% [30]. However, recent studies indicate that actual distributions could differ significantly from the normal one. For example, for rubber-metal building supports (seismic isolators), it was shown that the distributions of horizontal and vertical stiffness are better described by a lognormal law or a generalized extreme distribution, whereas the assumption of a “normal” distribution yields large systematic discrepancies [31, 32]. In Wu et al. [2], the characteristics of 224 rubber bearings were measured and statistical processing of different distributions was performed. Thus, based on the goodness-of-fit criteria, it was found that the lognormal and generalized extreme distributions (type I) provide the best fit to the experimental stiffness histograms, while the normal distribution deviates significantly [33, 34]. At the same time, the stiffness variation coefficients obtained in these tests lie in the range of ~1–5%, which reflects the relative homogeneity of serial isolators during controlled production [35, 36]. Nevertheless, even such a spread significantly affects the dynamic properties of the system. Vibration isolation systems often use vibration damping mats laid along the base of the foundation of the protected building, structure or machine. Let us assume a design situation in which a vibration damping mat with plan dimensions a x b and thickness t is laid along the base of the protected mass m. In this case, the system parameters have statistical characteristics presented in Table 1.

In the case of using vibration damping mats, their dynamic rigidity k_{dyn} could be determined by the formula:

$$k_{dyn} = \frac{E_{dyn} \cdot A_{surf}}{t}, \tag{5}$$

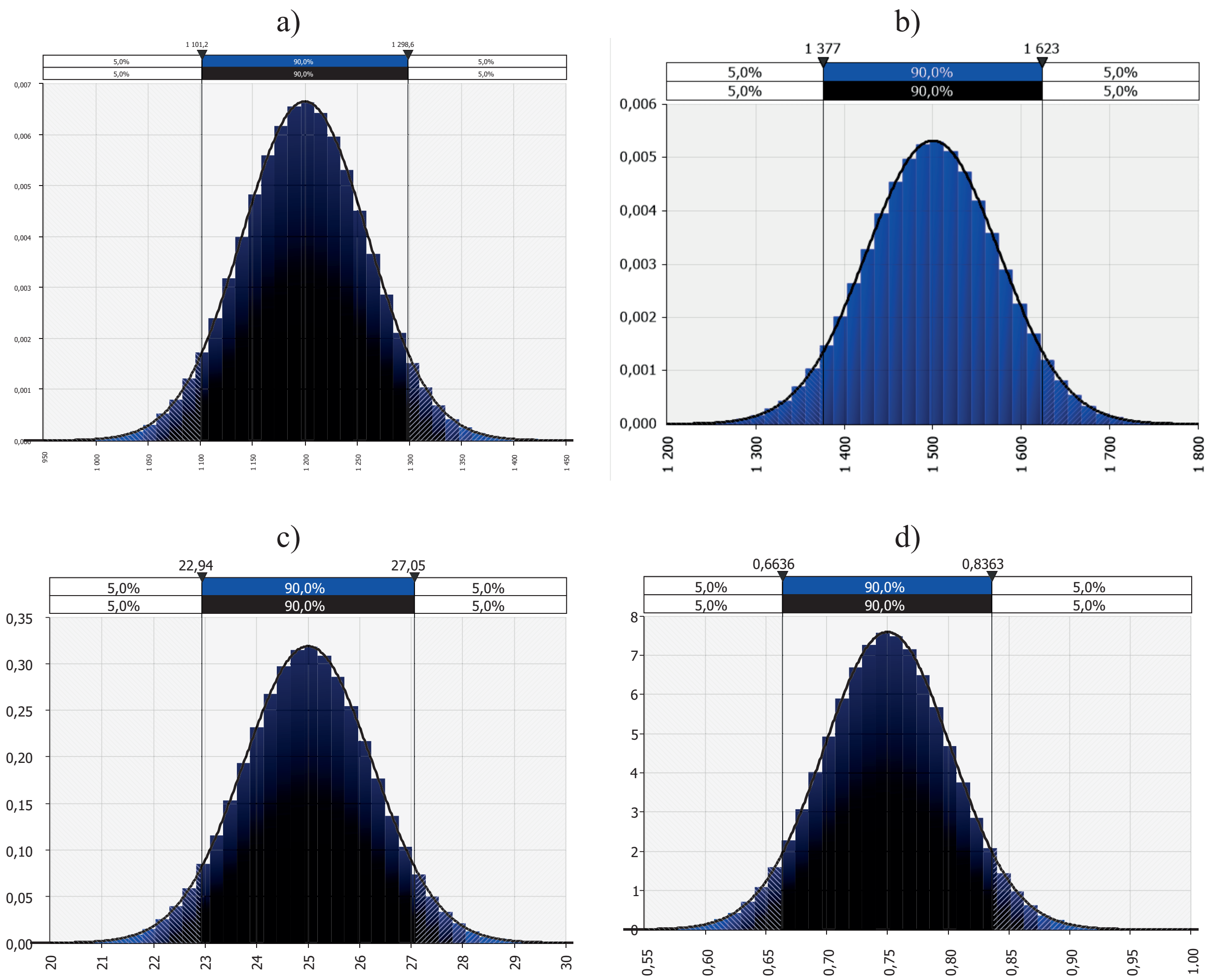
where E_{dyn} is the value of the dynamic modulus of elasticity corresponding to the specified load on the sample, the frequency of load application and its shape factor, N/m^2; A_{surf} is the area of the loaded sample, m^2; and t is the thickness of the loaded sample, m.

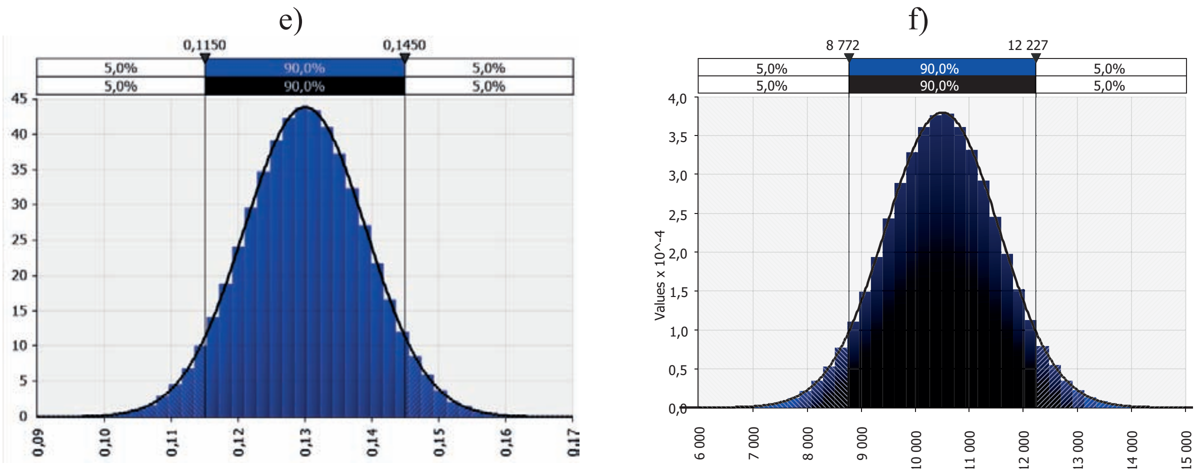
Assuming that the specified parameters have a normal distribution, it is possible to evaluate the quality of the specified vibration protection system and to estimate the sensitivity to a set of factors.

Figures 2 a – f show the normal distribution curves of the parameters used in the calculations. The curves were obtained by the Monte Carlo method (at least 100,000 tests) taking into account the distribution parameters according to Table 1.

Table 1. Statistical characteristics of vibration protection system elements

Parameter	Mean value	Dimension	Variance	Notes
Geometric dimensions of vibration damping mats				
a	1200	mm	5%	ISO3302-1:1999 (Class L3)
b	1500	mm	5%	
t	25	mm	5%	
Physical and mechanical characteristics of mats				
E_{dyn}	0.75	N/mm^2	7%	Author data
η	0.13	-	7%	
Isolated mass				
m	10500	kg	10%	Considering safety factors + errors in dimensions + errors in material density

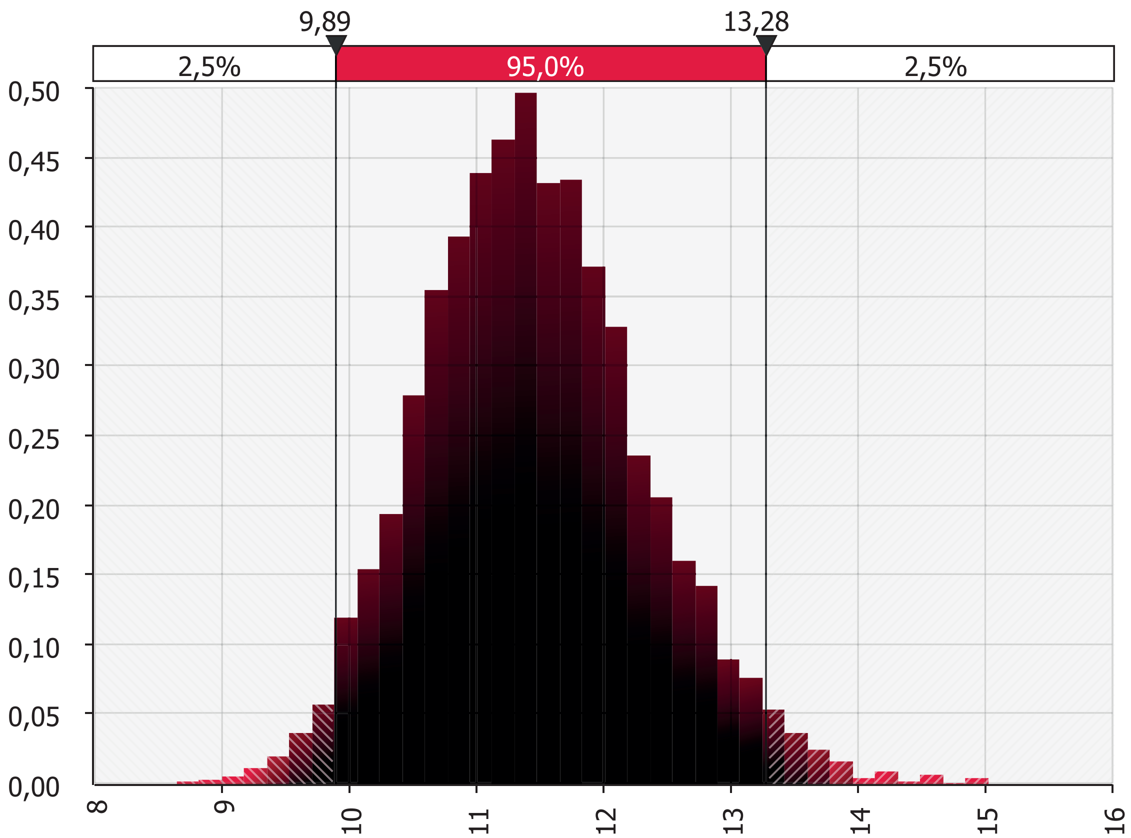




<u>Figure 2.</u> *Distribution of design parameters according to Table 1: a) length; b) width; c) thickness; d) Edyn; e) η; f) m.*

By setting the parameters in formulas (4) and (5) in the form of random variables having a normal distribution with the characteristics given in Table 1, it is possible to estimate the distribution of the calculation results, namely: the natural oscillation frequency of the vibration protection system, as well as the efficiency of vibration isolation in the frequency range from 1 to 100 Hz (a 1/3-octave spectrum was selected for calculation).

The distribution of the natural oscillation frequency of the vibration protection system is shown in Figure 3.



<u>Figure 3.</u> *Probability density of the natural oscillation frequency of the vibration protection system*

The calculation results show that the 95% confidence interval includes f_0 values from 9.89 to 13.28 Hz with the median value being 11.42 Hz. The standard deviation for f_0 is 0.86 Hz. The graph in Figure 3 illustrates a slight asymmetry relative to the median value, the asymmetry coefficient is 0.34 (i.e. the right tail of the distribution is larger than the left). When performing a deterministic calculation, f_0 = 11.41 Hz, which coincides with the median value.

To assess the efficiency of the vibration protection system, we will estimate the distribution of the transfer function (4). The results are presented in the form of a Whisker diagram in Figure 4.

a)

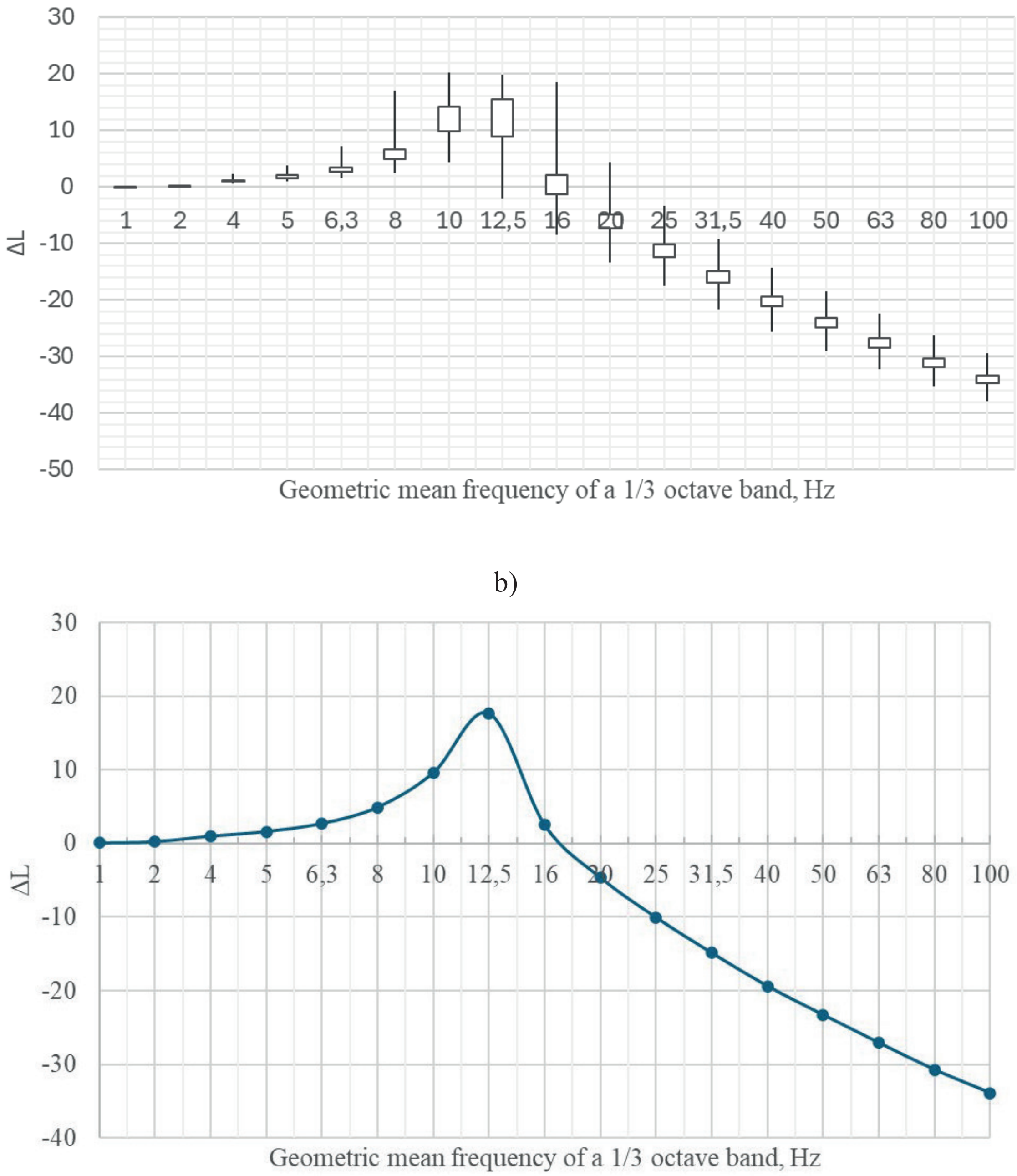


Figure 4. Distribution of calculation results for the ΔL function: a) with due account for the probability distribution; b) deterministic case

According to the graph in Figure 4a, it can be noted that, in contrast to the deterministic graph of the frequency response of the system (Figure 4b), the account for the statistical distribution of the parameters of the vibration isolation system leads to an expansion of the resonance zone (which corresponds to an increase in the spread of the natural oscillation frequency according to Figure 3 - from 9.89 to 13.28 Hz). The median value shown in Figure 4a repeats the graph in Figure 4b. But at the same time, the spread of the maximum and minimum values (end points of straight lines) in the resonance region has the greatest significance. In this case, the vibration isolation system is extremely sensitive to the reliability of the specified values. It could be emphasized that the pre-resonance mode has a distribution of results shown in Figure 5a - with a predominance of the right tail. i.e. the asymmetrical coefficient has positive values increasing to the resonance mode.

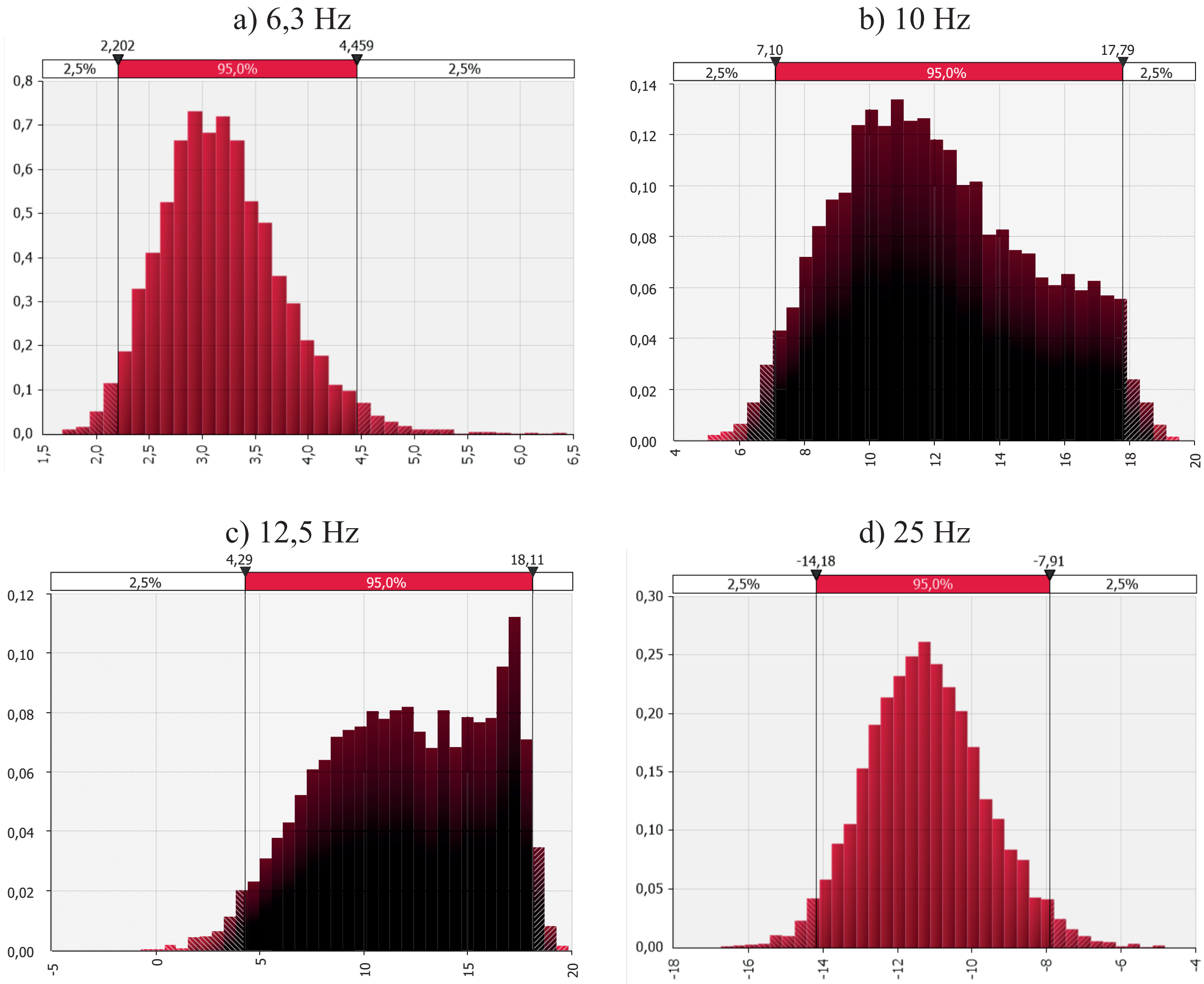


Figure 5. Distribution of the ΔL function depending on the geometric mean frequency of the 1/3-octave band.

In the resonance mode in the 1/3-octave band with the geometric mean frequency of 12.5 Hz, the asymmetry coefficient has negative values, as does the value of the standard deviation. The spread of the value of the resonance increase of oscillations has significant values from 2.5 to 4 dB.

2.2. Problem II

Another fundamental source of uncertainty in vibration protection problems is the random nature of the external dynamic impact. Many disturbing forces and impacts are random processes by their nature: for example, seismic shocks, turbulent wind gusts, uneven road surfaces under a car, technological vibrations from equipment with a chaotic operating mode, etc. Even if the system parameters (*m, c, k*) could be considered as constant and known, the time implementation of the load *f*(*t*) continuously changes and is not exactly predictable, although it could have statistical regularities (spectral density, probability distribution of amplitudes, etc.). For the second type of problem, we consider the solution of equation (1) for an external load with a known distribution [23] shown in Figure 6. Many studies have shown that logistic distribution allows the most reliable assessment of the dynamic impact created, for example, by the movement of rail trains [11, 37].

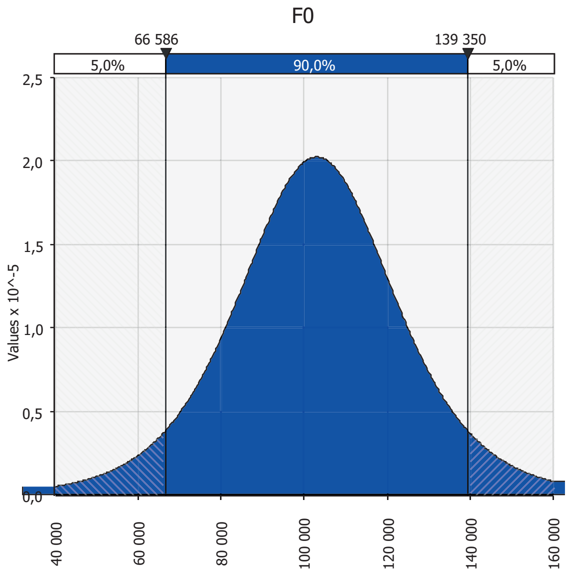


Figure 6. External load probability density

The solution for the response of the system (spectral displacement of the vibration-isolated mass) under the action of a known external force is

$$X = \frac{F_0}{\sqrt{\left(k - m\omega^2\right)^2 - \left(c\omega\right)^2}}. \tag{6}$$

By using the numerical Monte Carlo method, it is possible to consider various variants of distributions for the F_0 load, obtained, among other things, from the results of field tests. In particular, according to the results of the studies in [11], it is shown that in different octave bands, the form of the distribution function differs significantly (which is associated with different mechanisms of generating the specified vibration). Since the parameters of the system in the problem of the second type are deterministic, the distribution function will be a single curve, and not a family of realizations. In this case, of particular interest is the assessment of the spectral distribution for the mass displacement (6), which is shown in Figure 7 as a Whisker plot.

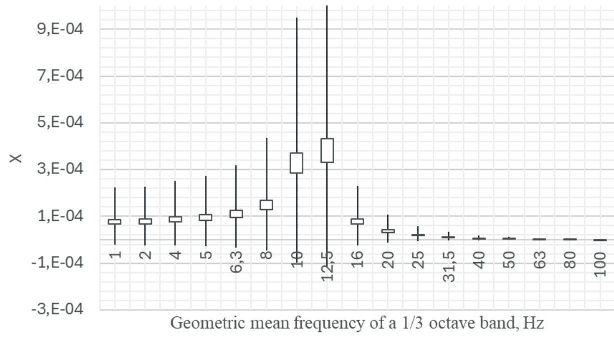


Figure 7. Response spectrum of the vibration protection system in the 2d type problem

The nature of the graph shown in Figure 7 (at least its median values) corresponds to the theoretical deterministic frequency response curve of a single-mass system, but differs in the magnitude of the spread. A significant spread is observed (between the maximum and minimum recorded values) in the pre-resonance region - one-third octave bands with a geometric mean frequency of 1 – 8 Hz. In this case, the largest values of the spread of the calculation results occur at resonance - one-third octave bands with geometric mean frequencies of 10 and 12.5 Hz. In the post-resonance region, the spread is reduced, thus, in one-third octave bands with geometric mean frequencies above 25 Hz it becomes of little influence. The 25% and 75% percentile, which are the extreme shelves of the boxes on the graph of

Figure 7, behaves similarly, however, the spread between these values is approximately an order of magnitude smaller than the spread between the maximum values.

2.3. Problem III

The most complex, but also the most realistic situation is when uncertainty is present both in the parameters of the vibration-isolated system and in the external dynamic impact. In this case, to adequately assess the response, it is necessary to conduct a joint probabilistic analysis that considers two sources of variability [5]. The problem is complicated by the fact that the response distribution is determined by the convolution (combination) of the distributions of random parameters and a random input process. A rigorous analytical approach is often impossible, and one must resort to either rough estimates (for example, assuming that the total effect of randomness is equivalent to some effective uncertainty) or large-scale numerical experiments. The Monte Carlo method was used in the problem precisely for this purpose. In the direct approach, a pair was generated: a random realization of a set of system parameters (*m, c, k*) and a random realization of the loading process *f*(*t*), after which the response was calculated using formula (6). Having repeated this process tens of thousands (or millions) times, the response distribution was empirically constructed. The calculation result is shown in Figure 8.

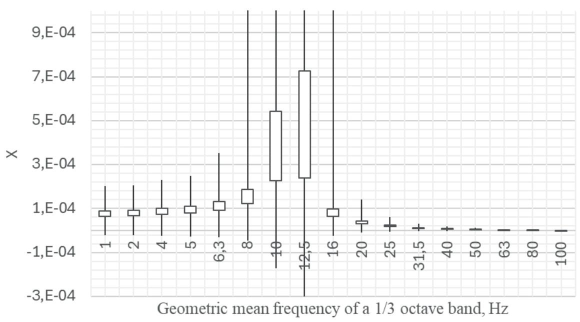


Figure 8. Response spectrum of the vibration protection system in the 3d type problem

The system response in the 3d type problem is characterized by the same principles as in the 2d type problem - repetition of the frequency response curve of a single-mass system. Comparison of the calculation results shown in Figures 7 and 8 reveals that the spread both in maximum values and in percentiles of 25% and 75% in the case of the 3d type problem is much higher. This is especially observed in the resonance region, where for the 3d type problem not only the external load contributes to the spread of values, but also the system parameters themselves (in this case, the natural frequency) fluctuate in a wide range of values (similar to the 1s type problem). Figures 9 a and b present comparative graphs of the mathematical expectation and the root mean square deviation (RMS), respectively.

The mathematical expectation (Figure 9a) in the problem of the 3d type, as predicted, is higher than in the problem of the 2d type by in average of 1–7% (larger values correspond to one-third octave bands near the resonance band), but at resonance this difference increases to 6.4 times. A similar observation could be made based on the results of the standard deviation analysis shown in Figure 9b. On average, the standard deviation in problems of the 3d type is higher than in problems of the 2d type by 14–79%, but at resonance this difference increases to 2000 times.

In this work, a comparison of the probability density of the response in characteristic one-third octave bands corresponding to the pre-resonance, resonant, and post-resonant operating modes was performed. The distribution curve for a problem of the 2d type is shown in red on the graphs in Figures 10a–c, and in blue – for a problem of the 3d type. The nature of the distribution density graph in the pre-resonance mode (in the one-third octave band with a geometric mean frequency of 6.3 Hz) for the problem of the 3d type partly repeats the nature of the frequency response distribution shown in Figure 5a with the predominance of the right tail of the distribution: the asymmetry coefficient is 0.63 versus 0.0088 for the problem of the 2d type. For the post-resonance region shown in Figure 10c, the behavior is identical to the pre-resonance one.

The largest and most significant difference between the calculation results is observed at resonance, as is seen in Figure 10b. For ease of display, the calculation results for problems of the 2d and 3d types are presented as a distribution function.

The resonance mode (Figure 10b) is characterized by a significant scatter in the measurement results, only 81.1% of the values for the problem of the 3d type fall within the displacement range from 0.0001 to 0.001 m, with 99.8% for the problem of the 2d type. The distribution for the 3d type problem has a large right tail – the asymmetry coefficient is 211 versus 0.01 for the 2d type problem, what indicates a higher probability of high displacement values of the vibration-isolated mass and a higher probability of exceeding the established criterion of the threshold displacement value. The influence of the model parameters on the calculation results is shown in Figure 11 for the resonance vibration mode.

Considering the results shown in Figure 11, it could be stated that the control of the dynamic state of objects depends significantly on their mass, namely, the accuracy with which this parameter is determined. In second place is the accuracy of determining the geometric dimensions of vibration damping materials and the spread of dynamic load values. Under the current restrictions shown in Table 1, in third place are the dynamic characteristics of vibration damping materials used in a specific vibration isolation system.

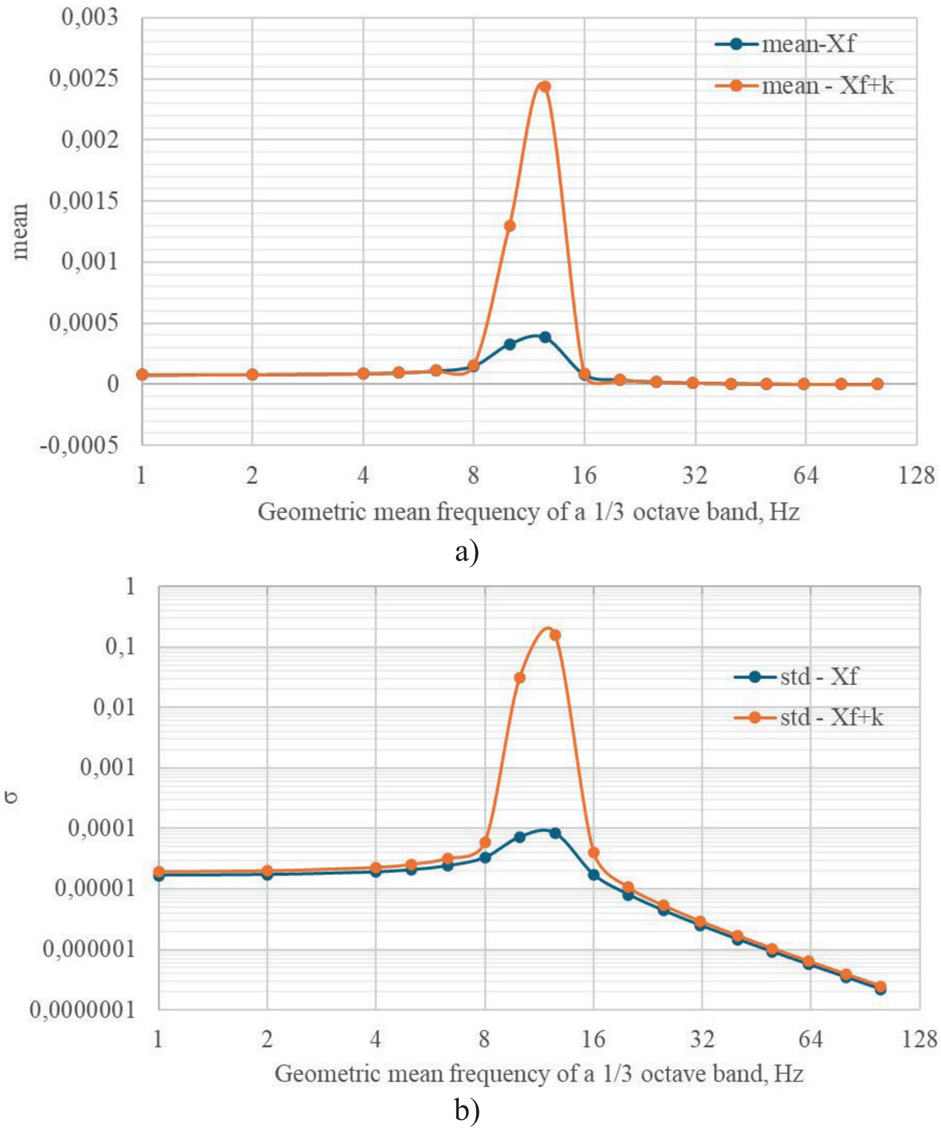


Figure 9. Spectra of mathematical expectation (a) and root mean square value (b) for problems of the 2d and 3d types

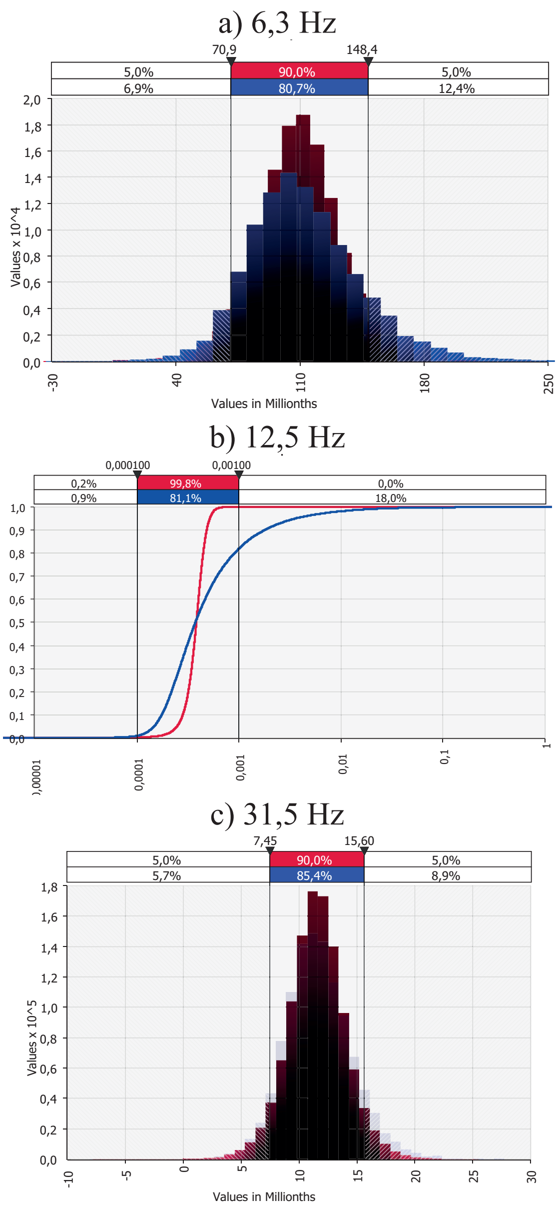


Figure 10. *Comparative graphs of the probability density distribution of the system response*

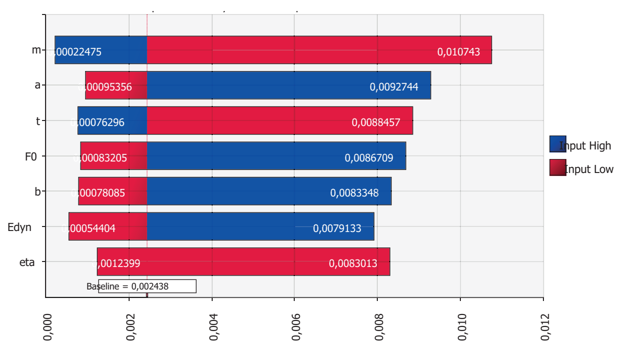


Figure 11. *Influence of model parameters on calculation results for the 3d type problem*

3. CONCLUSION

The conducted calculations of SDOF systems and their analysis demonstrate that accounting for random factors significantly alters the assessment of dynamic behavior in such systems. When system parameters are stochastic, distributions of resonant frequencies and amplitude-frequency characteristics (for example, within each band of a one-third octave spectrum) can be predicted to estimate the probabilistic range of vibration isolation efficiency. For random external excitations, advanced spectral and statistical analysis tools provide mean and extreme estimates of response considering actual load characteristics. Consideration of both types of uncertainties simultaneously is more complex and currently addressed by combining analytical and numerical methods, including Monte Carlo simulations. The approach developed in this work enables determination of probabilities of exceeding threshold values established for the isolated object using experimental results, while remaining within the linear operating region of vibration isolator components. Additionally, the Monte Carlo method numerically evaluates how element-to-element variability contributes to probabilistic response characteristics, allowing variation of these parameters as needed to increase accuracy. These approaches enable engineers to make more informed decisions when selecting vibration isolator parameters, balancing mass-stiffness trade-offs with required probability levels for reducing vibrations below safe thresholds. As reliability and durability requirements grow for machinery and structures, the probabilistic approach to vibration isolation transitions from scientific research into practical engineering calculation tools, ensuring safety and economic efficiency in vibration protection projects.

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