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STRESS FIELDS NEAR EARTHQUAKE EPICENTERS

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Annotation: The closed form expressions for stress fields are derived. The analysis of stress fields reveals several phenomena, which may be essential for the design of seismic protection of underground structures, namely (i) infinite peaks in both pressure and second stress invariant at the arrival of Rayleigh wave at any points on the free surface; (ii) finite values of stress invariant fields at any of the undersurface points; and (iii) the presence of an infinite peak in the displacement magnitude

Keywords: Bulk wave, v. Mises equivalent stress, plastic dissipation energy; seismic source

ПОЛЯ НАПРЯЖЕНИЙ ВБЛИЗИ ЭПИЦЕНТРОВ ЗЕМЛЕТРЯСЕНИЙ

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Аннотация: Выведены выражения в замкнутой форме для полей напряжений. Анализ полей напряжений выявляет несколько явлений, которые могут быть существенными для проектирования сейсмозащиты подземных сооружений, а именно: (i) бесконечные пики как давления, так и второго напряжения, инвариантного при приходе волны Рэлея в любые точки на свободной поверхности; (ii) конечные значения полей инвариантов напряжений в любой из точек под поверхностью; и (iii) наличие бесконечного пика в величине смещения

Ключевые слова: объемная волна, эквивалентное напряжение Мизеса, диссипация энергии; сейсмический источник

1. INTRODUCTION

Both analytical and numerical solutions for stress and displacement fields in an isotropic linearly elastic halfspace loaded with a surface delta-like force normal to the plane boundary are constructed and analyzed. Analytical solutions for stress fields are derived apparently for the first time using the Cagniar – de Hoop method, developed for the construction of a displacement field in a halfspace.

Instead of analyzing individual stress components, stress invariants are analyzed, revealing several interesting phenomena, (i) both first and second stress invariants, together with the associated pressure and v.Mises equivalent stress are finite at the internal points of the halfspace; (ii) both pressure and the equivalent v.Mises stress tend to infinity when approaching the free surface at the moment of arrival of the Rayleigh wave; (iii) near the free surface the pressure field suddenly changes sign at the moment of arrival of the Rayleigh wave; (iv) at any time both pressure and equivalent v.Mises stress decrease exponentially with depth; and (v) everywhere in the halfspace both stress invariants remain finite at the moments of arrival of bulk waves. These observations and, in particular, the observed abrupt change in the sign of the pressure field at the arrival of

Rayleigh wave, are important for the development of seismic protection systems for undersurface structures, since at the appearance of negative pressure, the soil loses its bearing capacity, even if the equivalent v.Mises or Tresca criteria are small [1-18].

The outer plane Lamb problem of displacement field in a halfspace or halfplane loaded with a concentrated dynamic force applied to the plane boundary is most often solved by applying different analytical and numerical methods.

(A) The Cagniar – de Hoop method, used to invert Fourier and Laplace integral transforms and allowing to obtain the closed form solutions for displacement fields. Most of the works in this group are confined to deriving displacement fields on a free surface only, however, several works relate to the displacement fields in the whole halfspace, e.g.. It should also be noted that quite a large number of works in this group is concerned with the inner Lamb problem for a delta-like force acting inside the halfspace).

(B) Self-similar solutions, known also as the functionally-invariant solutions. This method is based on an analogy between plane or axisymmetric problems of the linear elasticity and the complex potential method. The method of self-similar solutions is mainly used for constructing solutions of the inner Lamb problem, however, some plane problems, including ones related to the moving cracks, can also be considered; see. Another remark concerns the application of the self-similar method to solving both inner and outer Lamb problems for an anisotropic halfspace.

(C) The construction of a 1D hyperbolic equation for the Rayleigh wave in a plane outer Lamb problem is suggested in, enabling to exclude vertical coordinate from the governing equation, but retaining it in the pseudo-differential operator acting on the applied surface load. In this regard see also.

(D) Finite element (FE) approaches are usually combined with either Lax – Wendroff energy preserving explicit numerical scheme or Godunov energy preserving explicit scheme for time integration, where Lamb problems were studied by FE coupled with Lax – Wendroff scheme. Other FE formulations include spectral finite element methods, allowing to considerably decreasing dispersion errors associated with the Runge phenomenon at solving Lamb problems.

(E) Finite difference (FD) schemes are used much less frequently compared to FE approaches. In [1-3] the outer and inner Lamb problems were studied by the FD approaches combined with Godunov type schemes for time-integration.

In summary, most of the reviewed publications are concerned with the analysis of the displacement fields at the free surface of a halfspace or half-plane. Meanwhile, the displacement and stress fields in the interior of the halfspace or half-plane, appears to be at least as important as the displacement field on the free surface, especially taking into account various undersurface structures affected by the arrival of technogenic seismic waves.

2. PRINCIPAL EQUATIONS

Equation of motion for a linearly-elastic medium can be written in the form

$$\left(c_1^2\nabla_{\mathbf{x}}\mathrm{div}_{\mathbf{x}} - c_2^2\mathrm{rot}_{\mathbf{x}}\,\mathrm{rot}_{\mathbf{x}} - \mathbf{I}\partial_{tt}^2\right)\cdot\mathbf{u}(\mathbf{x},t) = 0$$

$$x_2 \equiv \mathbf{x}\cdot\boldsymbol{\nu} < 0, \qquad t \ge 0 \qquad (2.1)$$

where $\mathbf{u}$ is the displacement field; $\mathbf{x}$ is the spatial coordinate; t is the time; $\mathbf{I}$ is the unit 3×3-matrix; $\boldsymbol{\nu}$ is the unit outward normal to the plane boundary; and c_1, c_2 are respectively longitudinal and shear bulk wave velocities

$$c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}; \qquad c_2 = \sqrt{\frac{\mu}{\rho}} \qquad (2/2)$$

Herein, λ, μ are Lame’s constants

$$\lambda = \frac{E\nu}{(1-2\nu)(1+\nu)}; \qquad \mu = \frac{E}{2(1+\nu)} \qquad (2.3)$$

where E is Young's modulus; and, ν is Poisson's ratio.

The boundary plane is assumed to be traction-free, except one point, where a delta-like force is applied

$$\mathbf{t}_\nu(\mathbf{x},t)\big|_{\mathbf{x}\in\Pi_\nu} = -P_0\delta\left(\mathbf{x}\cdot(\mathbf{k}\times\boldsymbol{\nu})\right)\delta(t)\boldsymbol{\nu} \tag{2.4}$$

where Π_ν is the boundary plane; $\mathbf{k}\in\Pi_\nu$ is a unit vector, specifying the direction of the applied line-load; and, P_0 is an amplitude multiplier. The initial conditions correspond to the state of rest:

$$\mathbf{u}(\mathbf{x},t)\big|_{t=0} = 0; \qquad \partial_t\mathbf{u}(\mathbf{x},t)\big|_{t=0} = 0\,. \tag{2.5}$$

Conditions (2.4) and (2.5) should be supplemented by the Sommerfeld radiation condition

$$\begin{gathered}
\mathbf{u}(\mathbf{x},t)\big|_{\mathbf{x}\cdot\boldsymbol{\nu}\to-\infty} = 0;\\
\nabla_{\mathbf{x}}\mathbf{u}(\mathbf{x},t)\big|_{\mathbf{x}\cdot\boldsymbol{\nu}\to-\infty} = 0\\
\mathbf{u}(\mathbf{x},t)\big|_{|\mathbf{x}\cdot(\mathbf{k}\times\boldsymbol{\nu})|\to\infty} = 0;\\
\nabla_{\mathbf{x}}\mathbf{u}(\mathbf{x},t)\big|_{|\mathbf{x}\cdot(\mathbf{k}\times\boldsymbol{\nu})|\to\infty} = 0
\end{gathered} \tag{2.6}$$

Following Cagniar – de Hoop method, the desired equations for the displacement field in a halfspace can be represented in the form

$$\mathbf{u}(\mathbf{x},t) = \frac{P_0}{\pi c_1\mu R(\xi)}\times\operatorname{Im}\left(\mathbf{p}(\mathbf{x},t)\right) \tag{0.1}$$

where functions the Rayleigh function)

$$R(\xi) = \left(\gamma^2-2\xi^2\right)^2 + 4\xi^2\sqrt{\left(\gamma^2-\xi^2\right)\left(1-\xi^2\right)} \tag{2.8}$$

Herein,

$$\begin{gathered}
\gamma = \frac{c_1}{c_2} > 1; \qquad r = \sqrt{x_1^2+x_2^2}\\
x_1 = \mathbf{x}\cdot(\mathbf{k}\times\boldsymbol{\nu}); \qquad x_2 = \mathbf{x}\cdot\boldsymbol{\nu}
\end{gathered} \tag{2.9}$$

Equation (0.1) defines the displacement field everywhere in the considered halfspace.

Performing differentiation with respect to spatial variables in the infinitesimal Cauchy relations

$$\varepsilon(\mathbf{x},t) = \frac{1}{2}\left(\nabla_{\mathbf{x}}\mathbf{u}(\mathbf{x},t) + \left(\nabla_{\mathbf{x}}\mathbf{u}(\mathbf{x},t)\right)^T\right) \tag{0.2}$$

where T means transposition, yields strain tensor. Now, Hooke's law with account of plane strain condition

$$\boldsymbol{\varepsilon}\cdot\cdot\,\mathbf{k}\otimes\mathbf{k} = 0 \tag{0.3}$$

where double dots mean convolution with respect to two indices, gives

$$\boldsymbol{\sigma}(\mathbf{x},t) = \lambda\operatorname{Tr}\left(\boldsymbol{\varepsilon}(\mathbf{x},t)\right)\mathbf{I} + 2\mu\boldsymbol{\varepsilon}(\mathbf{x},t) \tag{0.4}$$

The behavior of individual stress components is less interesting than the behavior of the corresponding stress invariants

$$\begin{gathered}
I_\sigma = \operatorname{Tr}(\boldsymbol{\sigma}); \qquad II_\sigma = \frac{1}{2}\left(\operatorname{Tr}(\boldsymbol{\sigma})^2 - \operatorname{Tr}(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})\right)\\
J_\sigma = \frac{I_\sigma^2}{3} - II_\sigma; \qquad III_\sigma = \det(\boldsymbol{\sigma})
\end{gathered} \tag{0.5}$$

where J is second deviatoric invariant, from which the following two associated functions are of great importance in various applications

$$p = -\frac{I_\sigma}{3}; \qquad \sigma_i = \sqrt{3J_\sigma} \tag{0.6}$$

where p is the pressure; and, σ_i is the v.Mises equivalent stress.

Performing differentiation with respect to spatial variables in the infinitesimal Cauchy relations

$$\varepsilon(\mathbf{x},t) = \frac{1}{2}\left(\nabla_{\mathbf{x}}\mathbf{u}(\mathbf{x},t) + \left(\nabla_{\mathbf{x}}\mathbf{u}(\mathbf{x},t)\right)^T\right) \tag{0.7}$$

where T means transposition, yields strain tensor. Now, Hooke's law with account of plane strain condition

$$\boldsymbol{\varepsilon} \cdot\cdot \mathbf{k} \otimes \mathbf{k} = 0 \tag{0.8}$$

where double dots mean convolution with respect to two indices, gives

$$\boldsymbol{\sigma}(\mathbf{x},t) = \lambda \mathrm{Tr}\big(\boldsymbol{\varepsilon}(\mathbf{x},t)\big)\mathbf{I} + 2\mu\boldsymbol{\varepsilon}(\mathbf{x},t) \tag{0.9}$$

The behavior of individual stress components is less interesting than the behavior of the corresponding stress invariants

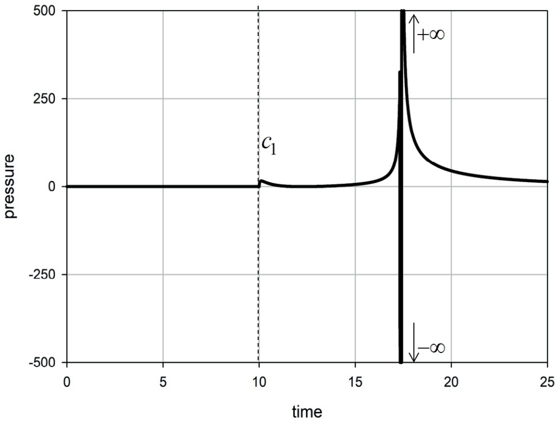
$$\begin{aligned} &I_\sigma = \mathrm{Tr}(\boldsymbol{\sigma}); \qquad II_\sigma = \frac{1}{2}\Big(\mathrm{Tr}(\boldsymbol{\sigma})^2 - \mathrm{Tr}(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma})\Big) \\ &J_\sigma = \frac{I_\sigma^2}{3} - II_\sigma; \qquad III_\sigma = \det(\boldsymbol{\sigma}) \end{aligned} \tag{0.10}$$

where J is second deviatoric invariant, from which the following two associated functions are of great importance in various applications

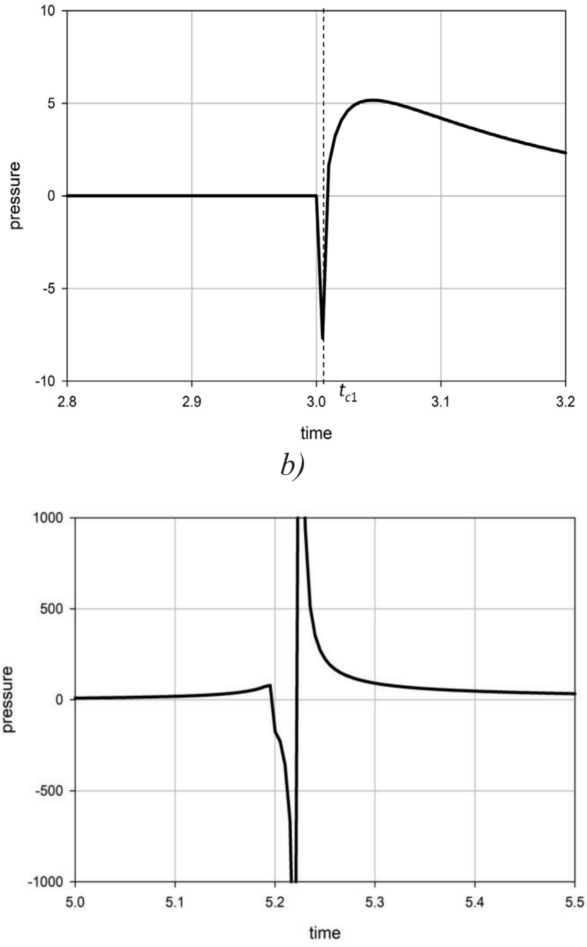
$$p = -\frac{I_\sigma}{3}; \qquad \sigma_i = \sqrt{3J_\sigma} \tag{0.11}$$

where p is the pressure; and, σ_i is the v.Mises equivalent stress.

The invariant stress fields are shown in Fig. 1, 2 and 3.



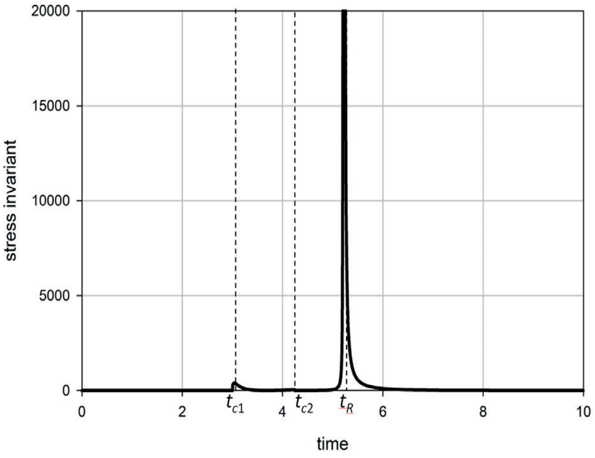
a)



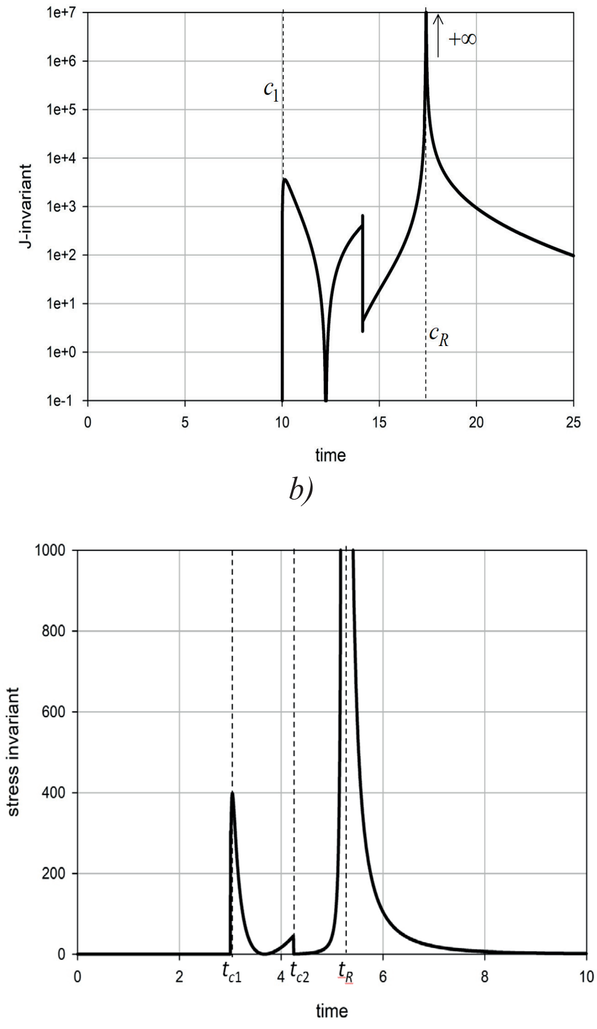
b)

c)

Figure 1. Pressure variation over time; a) overview; b) near P-wave arrival; c) near arrival of Rayleigh wave



a)



b)

c)

Figure 2. *J-invariant stress field over time; a) overview; b) near P-wave arrival; c) near arrival of Rayleigh wave*

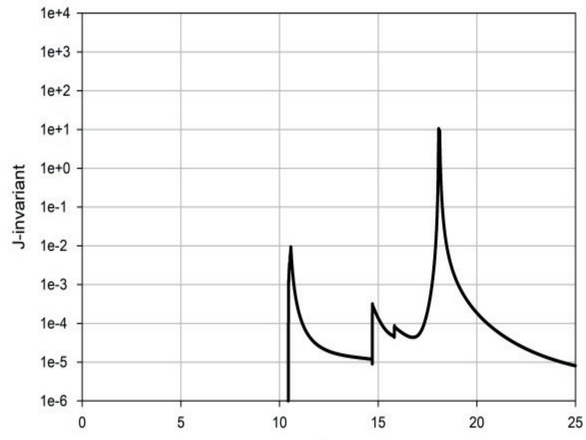


Figure 3. *The modified second invariant stress field over time: an overview*

These plots reveal,

(I) small peaks corresponding to the arrival of P-waves in both pressure and v.Mises equivalent stress, which is a square root of the second deviatoric invariant;

(II) infinite peaks related to the arrival Rayleigh waves at a point $\mathbf{x}_0$ located on the free surface; moreover,

(III) the pressure field exhibits a double discontinuity upon arrival of a Rayleigh wave;

(IV) the peaks associated with the arrival of Rayleigh waves, being finite at the inner points of the considered halfspace, still dominate over peaks associated with bulk waves, at least on depths not exceeding $\frac{1}{3}l$;

(V) the J_σ-invariant field and hence, the v.Mises equivalent stress, becomes infinite at the arrival of Rayleigh wave; and

(VI) at the inner points of the halfspace, the peak of the J_σ-invariant at the arrival of Rayleigh wave dominates over the peaks associated with bulk wave arrivals

3. CONCLUDING REMARKS

According to the obtained results, the outer Lamb problem for displacement and stress invariant fields caused by a vertical delta-like force, applied at the free surface, reveals several phenomena, some of which are observed apparently for the first time:

The displacement magnitude on the free surface exhibits an infinite discontinuity at the arrival of Rayleigh wave, while the arrival of the longitudinal P-wave produces a finite peak.

The displacement magnitudes in the inner points of the halfspace are finite, even at a close distance to the free surface; the peak associated with the arrival of Rayleigh wave dominates over bulk wave arrivals up to a depth $\frac{1}{3}l$.

The pressure field in the close vicinity of the free surface exhibits a double discontinuity upon arrival of a Rayleigh wave at $t \to t_R \pm 0$, where t_R is the time of the Rayleigh wave arrival. Moreover, at

$t \to t_R - 0$, $p \to -\infty$, while at $t \to t_R + 0$, $p \to +\infty$. In the interior points of the halfspace the double peaks corresponding to the arrival of Rayleigh waves dominate over peaks related to bulk wave arrivals, at depths $\le \frac{1}{3} l$. The appearance of infinite or large negative pressure values eventually means deterioration of soil properties.

The J-invariant field and the associated v.Mises equivalent stress defined by Eq. (0.6), reveal the infinite peak at the Rayleigh wave arrival and a finite peak associated with the arrival of bulk P-wave. Similarly to the preceding fields, at the inner points of the halfspace the peak associated with Rayleigh wave dominates over peaks related to the bulk waves, at least for the studied depths $\le \frac{1}{3} l$.

The observed negative pressure values associated with the Rayleigh wave arrival, being extremely dangerous for soil and the undersurface structures, stipulates the necessity in developing specific methods of seismic protection, primarily against short-duration Rayleigh waves, which create large negative pressure. In this regard, widely used seismic barriers, which are mainly designed against relatively long-period harmonic Rayleigh waves producing no (or small) negative pressure, could hardly be used in the considered case of short-duration waves [18-24].

The observed infinite peaks of the pressure and deviatoric stress invariant, associated with the arrival of Rayleigh wave, stipulate the development of seismic barriers against Rayleigh waves, which should be filled with elastoplastic materials, obeying (i) low cohesion to achieve dissipation of the deviatoric strain energy, which is proportional to the -stress invariant; (ii) either a small or preferably vanishing angle of internal friction to ensure energy dissipation at high pressure values; and, (iii) equal acoustic impedances (or similar) of the barrier material and the ambient soil for trapping wave energy within the barrier material, to prevent diffraction of Rayleigh wave into protected region; see [22-24] for heuristic suggestions on the construction of seismic barriers.

And the concluding remark concerns the assumed infinitesimality of strain fields, implying linear relations between displacement and strain fields. A natural extrapolation of these results could lead to applying both geometrically and physically nonlinear equations for finding displacement, strain and stress fields in the near epicenter regions.

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