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DEEP BEAM ANALYSIS WITH THE USE OF DISCRETE-CONTINUAL FINITE ELEMENT METHOD IN THE PRESENCE OF A VERTICAL CRACK

Marina L. Mozgaleva, Pavel A. Akimov
National Research Moscow State University of Civil Engineering, Moscow, RUSSIA

Abstract: Deep beam analysis with the use of discrete-continual finite element method in the presence of vertical crack is under consideration in the distinctive paper. The operational formulation within discrete-continual approach is given, examples of analysis are presented. Cases of different lengths of the corresponding vertical crack are analyzed. The presented examples demonstrate the advantages of the discrete-continual finite element method used in comparison with the conventional finite element method (the use of the latter is associated with a significant increase in the number of nodes in the grid domain and, consequently, with an increase in the order of the resolving system of equations).

Keywords: discrete-continual finite element method, numerical solution, deep beam, structural analysis, crack, finite element method

РАСЧЕТ БАЛКИ-СТЕНКИ С ИСПОЛЬЗОВАНИЕМ ДИСКРЕТНО-КОНТИНУАЛЬНОГО МЕТОДА КОНЕЧНЫХ ЭЛЕМЕНТОВ ПРИ НАЛИЧИИ ВЕРТИКАЛЬНОЙ ЩЕЛИ

М.Л. Мозгалева, П.А. Акимов
Национальный исследовательский Московский государственный строительный университет,
г. Москва, РОССИЯ

Аннотация: В настоящей статье рассматривается расчет балки-стенки с использованием дискретно-континуального метода конечных элементов при наличии вертикальной щели. Приведены операторная постановка задачи в рамках дискретно-континуального подхода, примеры расчета. Анализируются случаи различной длины соответствующей вертикальной щели. Представленные примеры демонстрируют преимущества используемого дискретно-континуального метода конечных элементов по сравнению с традиционным методом конечных элементов (использование последнего связано со значительным увеличением количества узлов сеточной области и, следовательно, с увеличением порядка разрешающей системы уравнений).

Ключевые слова: дискретно-континуальный метод конечных элементов, численное решение, балка-стенка, расчеты конструкций, щель, метод конечных элементов

INTRODUCTION

Deep beam analysis with the use of discrete-continual finite element method (DCFEM) has been already considered in [1-9, 13]. Corresponding systematic literature review has been presented in [1-9, 13] as well. In this regard, let us immediately move on to the main material of the distinctive paper in case of presence of a vertical crack.

1. FORMULATIONS OF THE PROBLEM

Let the constancy of the parameters of the problem be in the direction corresponding to x_2 (main direction). The operational formulation of the problem with the use of so-called method of extended domain [2], taking into account the selection of the main direction, is determined by the equation [3]:

$$\bar{z}' = \tilde{L}\bar{z} + \bar{\bar{F}},$$
$$0 \le x_1 \le \ell_1, \quad -\ell_2/2 \le x_2 \le \ell_2/2, \quad (1.1)$$

where we have

$$\tilde{L} = \left[\begin{array}{c|c} 0 & E \\ \hline L_{vv}^{-1}L_{uu} & L_{vv}^{-1}L_{uv} \end{array} \right]; \quad (1.2)$$

$$L_{uu} = \partial_1^* D_1^T C D_1 \partial_1; \quad (1.3)$$

$$L_{uv} = \partial_1^* D_1^T C D_2 - D_2^T C D_1 \partial_1; \quad (1.4)$$

$$L_{vv} = D_2^T C D_2; \quad (1.5)$$

$$E = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}; \quad (1.6)$$

$$D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; \quad D_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad (1.7)$$

$$C = \begin{bmatrix} 2\mu+\lambda & \lambda & 0 \\ \lambda & 2\mu+\lambda & 0 \\ 0 & 0 & \mu \end{bmatrix}; \quad (1.8)$$

$$\partial_k = \frac{\partial}{\partial x_k}, \quad \partial_k^* = -\partial_k, \quad k = 1,2; \quad (1.9)$$

$$\bar{\bar{F}} = \left[\begin{array}{c} 0 \\ \hline -L_{vv}^{-1}\bar{\bar{F}} \end{array} \right]; \quad \bar{z} = \begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix}; \quad (1.10)$$

$$\bar{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}; \quad \bar{v} = \partial_2 \bar{u} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}; \quad (1.11)$$

ℓ_1, ℓ_2 are corresponding dimensions of extended domain (linear dimensions of considering structure) [2]; $x = (x_1, x_2)$; x_1, x_2 are Cartesian coordinates; μ and λ are Lame parameters [14, 15]; Ω is the domain, occupied by structure;

$$\Omega = \{ (x_1, x_2): \; 0 < x_1 < \ell_1; \quad -\ell_2/2 < x_2 < \ell_2/2 \}; \quad (1.12)$$

$\theta(x_1, x_2)$ is characteristic function of domain Ω; $\delta_\Gamma = \delta_\Gamma(x_1, x_2)$ is the delta function of boundary $\Gamma = \partial\Omega$; $\bar{n} = [\, n_1 \;\; n_2 \,]^T$ is boundary normal vector; $\bar{F}$ is the load vector in domain Ω; $\bar{f}$ is the corresponding boundary load vector; $\bar{u}$ is the vector of displacements (unknown vector function),

The system of equations (1.1) is supplemented by boundary conditions, which are set in sections with coordinates

$$x_2^1 = -\ell_2/2; \quad x_2^2 = \ell_2/2.$$

The discrete component of the numerical solution is represented by direction along the axe x_1.

2. EXAMPLES OF ANALYSIS

2.1. The first example.

Let us consider the analysis of a deep beam, fixed along the lateral faces in both directions, under the action of an external load concentrated at a point $x_2 = 0$, directed along an axis parallel to x_1 (Figure 1).

Let us consider the following geometrical parameters of structure:

$$\ell_1 = 1\,\text{m}, \quad \ell_2 = 10\,\text{m}.$$

Let us consider the following parameters of the material of the structure:

$$E = 2650 \cdot 10^4 \text{ kN/m2}$$

is the elasticity coefficient;

$$\nu = 0.15$$

is the Poisson's ratio.

Let us consider the following external load

$$P = 100\,\text{kN}.$$

Let's consider the dependence of the results of analysis on the gap length.

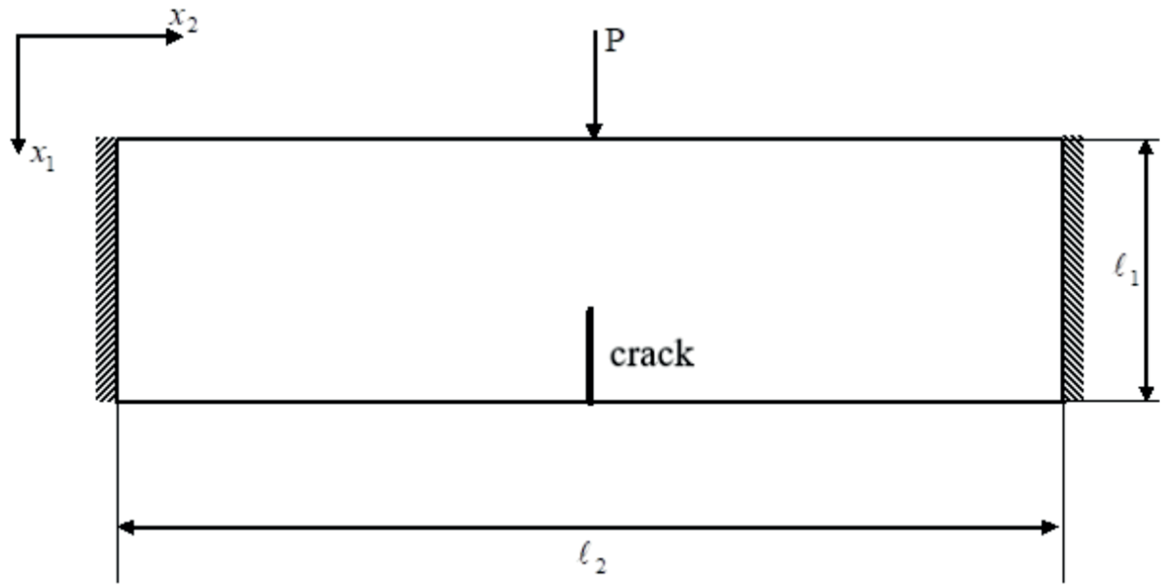

Figure 1. Example of analysis

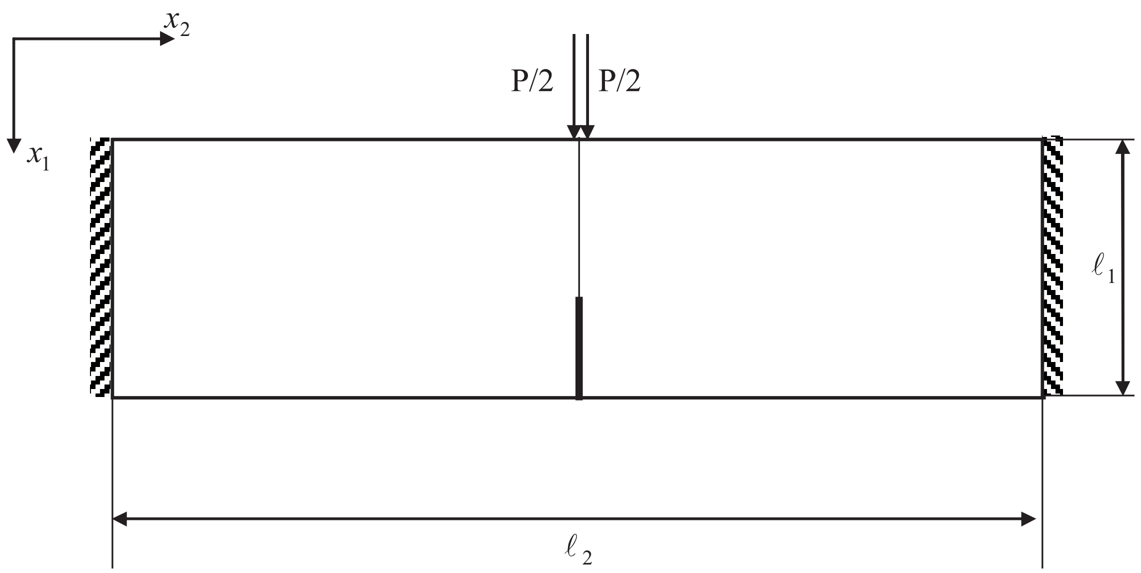

Figure 2. Design model

Let us consider the original problem in the form of a combination of two boundary problems (Figure 2).

The interface passes along the axis of the load action. In this case, the interface boundary equations at the nodes outside the gap line represent the conditions of ideal contact. The equations at the nodes in the gap zone represent the free edge conditions. The load is specified on both sides of the interface at a distance of $2\cdot 10^{-7}$ m. The discrete component includes $n = 33$ nodes (32 finite elements).

A graphical comparison of the numerical solution at the interface nodes for different values of the gap length dl ($dl = 0$ (no gap), $dl = \ell_1 / 8$, $dl = \ell_1 / 4$, $dl = \ell_1 / 2$). Besides, we consider the case when we have $L1 = \ell_1$ and $L2 = \ell_2$.

As can be seen, as the length of the gap increases, its opening increases (Figure 4), and the expansion stresses in the immediate vicinity of its beginning also increase (Figures 5, 6).

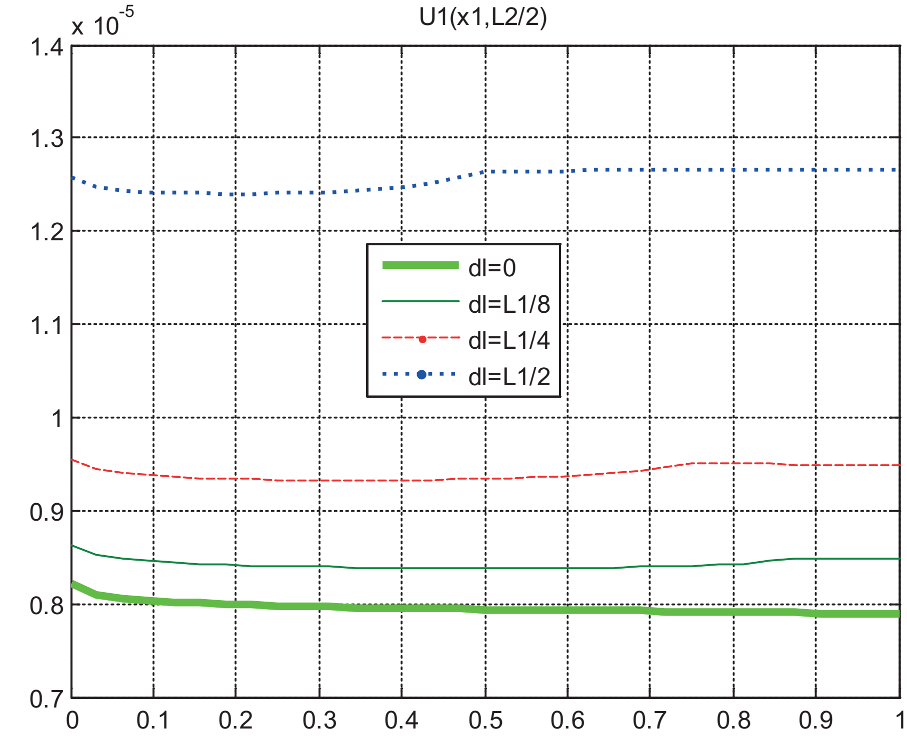

Figure 3. Displacements U1

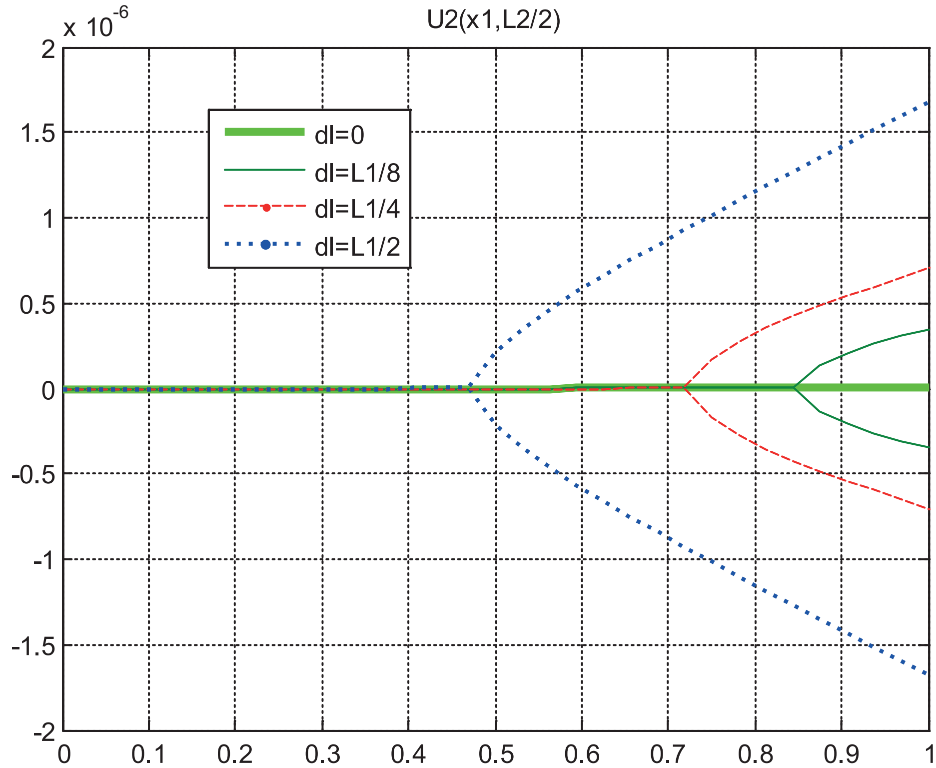

Figure 3. Displacements U2

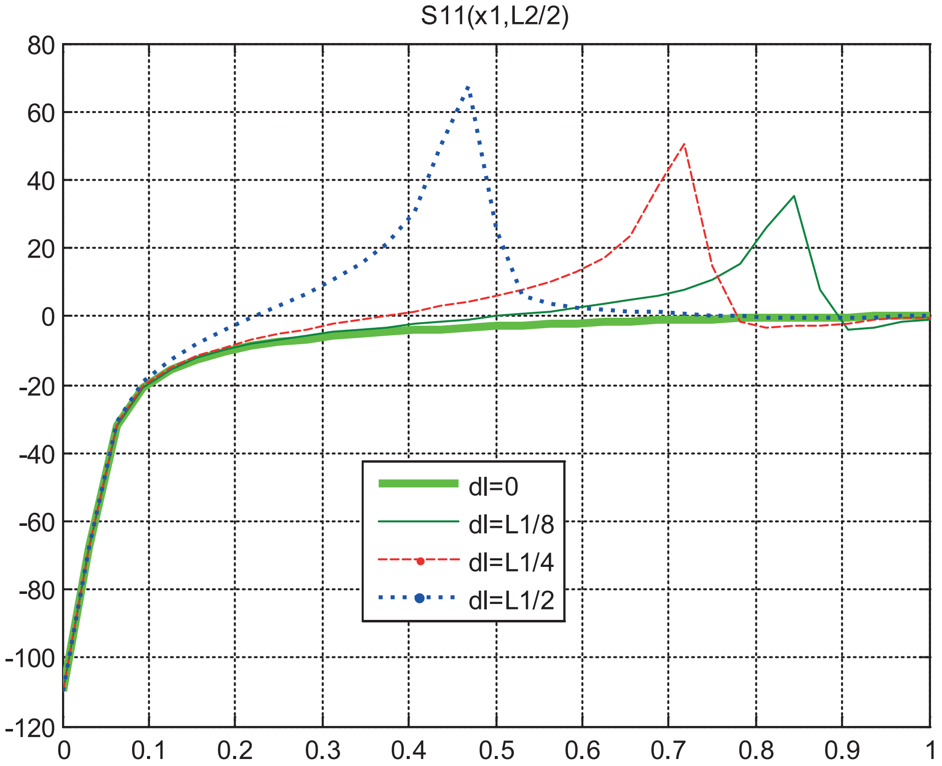

Figure 5. Stresses $\sigma 11$

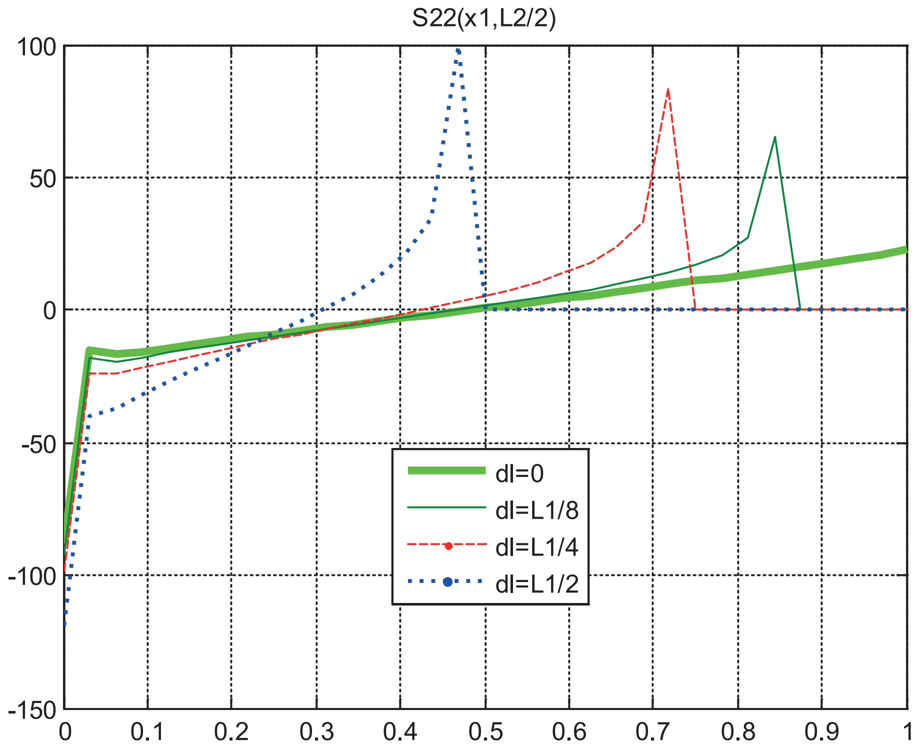

Figure 6. Stresses $\sigma 22$

2.2. The second example.
Let us consider the analysis of a deep beam (Figure 7), fixed along the lateral faces in both directions, under the action of equal external loads concentrated symmetrically relative to the point $x_2 = 0$, directed along the axis parallel to x_1. The loads are concentrated at points with coordinates $(0, x_2 f(1))$ and $(0, x_2 f(2))$ respectively.
Let us consider the following geometrical parameters of structure:

$$\ell_1 = 1\,\text{m}, \quad \ell_2 = 10\,\text{m}.$$

Let us consider $dl = \ell_1 / 2$ is the crack length.
Let us consider the following parameters of the material of the structure:

$$E = 2650 \cdot 10^4 \text{ kN/m2}$$

is the elasticity coefficient;

$$\nu = 0.15$$

is the Poisson's ratio.
Let us consider the following external load

$$P = 100\,\text{kN}.$$

Let us consider the dependence of the results of analysis on the distance between the points of application of loads ($x_2 f(2) - x_2 f(1)$).
Let us consider the initial problem in the form of joining two boundary problems (Figure 8).

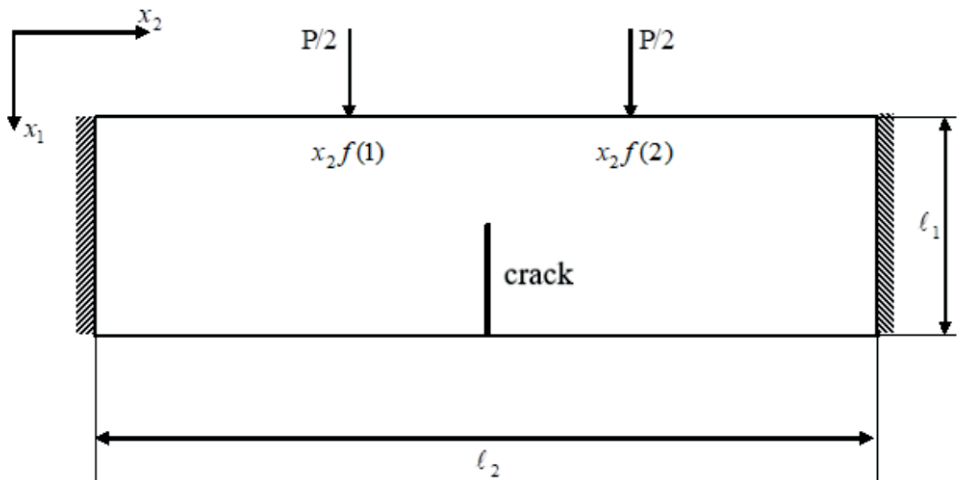

Figure 7. Example of analysis

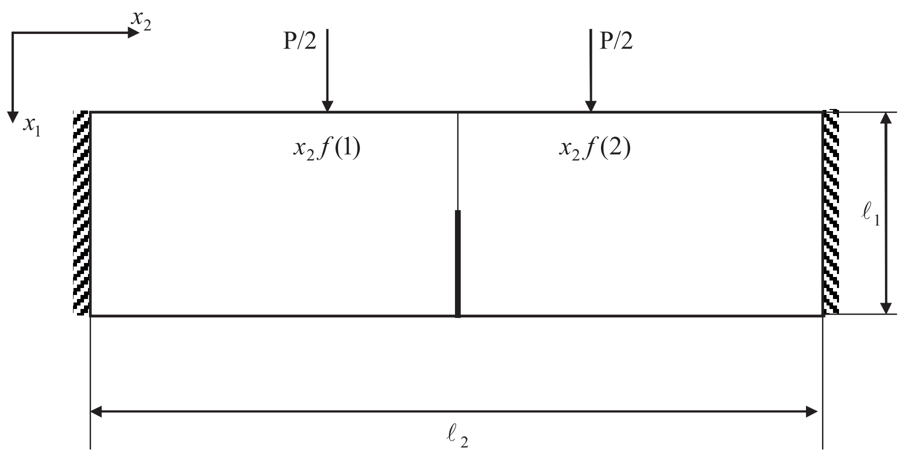

Figure 8. Design model

The joining boundary line passes along the axis $x_2 = 0$ (divides the structure in half). In this case, the joining boundary equations at the nodes outside the gap line represent the conditions of ideal contact. The equations at the nodes that fall within the gap zone represent the free edge conditions. The discrete component includes $n = 33$ nodes (32 finite elements).

A graphical comparison of the numerical solution at the joint nodes for different values of the distance $x_2 f(2) - x_2 f(1)$ between the points of application of loads is presented below.

In this case we have

$$x_2 f = (-3.75,\ 3.75)\,,\quad x_2 f = (-2.5,\ 2.5)\,,$$
$$x_2 f = (-1.25,\ 1.25)\,,\quad x_2 f = (-2\cdot 10^{-7},\ 2\cdot 10^{-7})\,.$$

From the graphical comparison it follows that the smaller the distance between the points of application of loads, the greater the gap opening (Figure 10), and the expansion stresses in the immediate vicinity of its beginning also increase (Figures 11, 12).

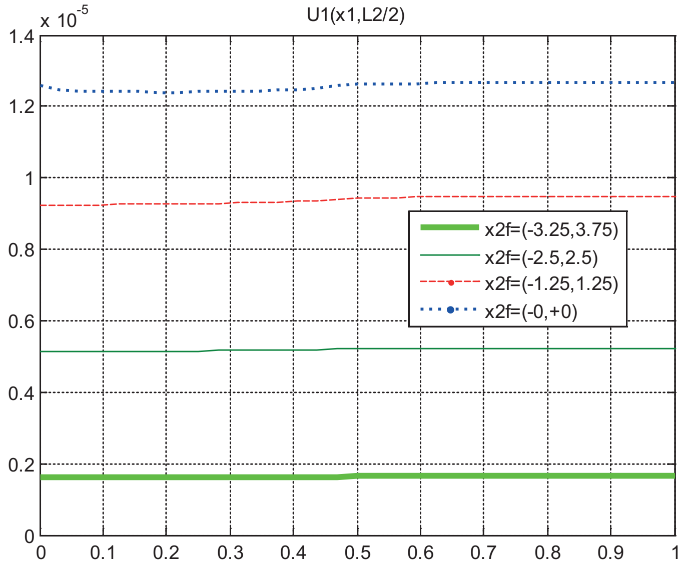

Figure 9. Displacements U1

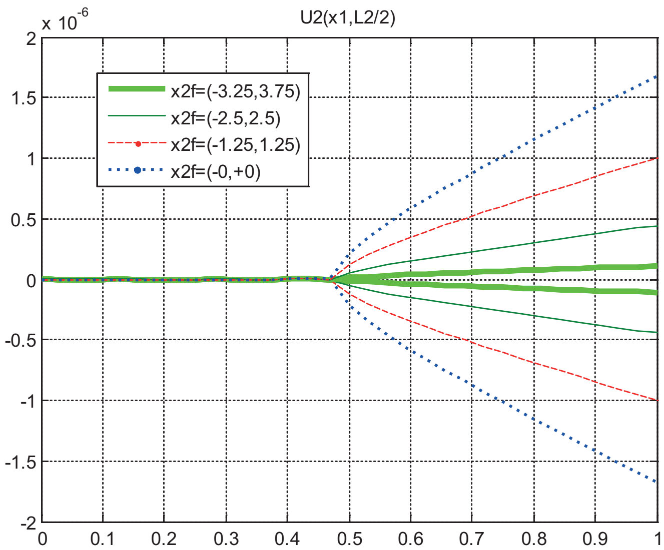

Figure 10. Displacements U2

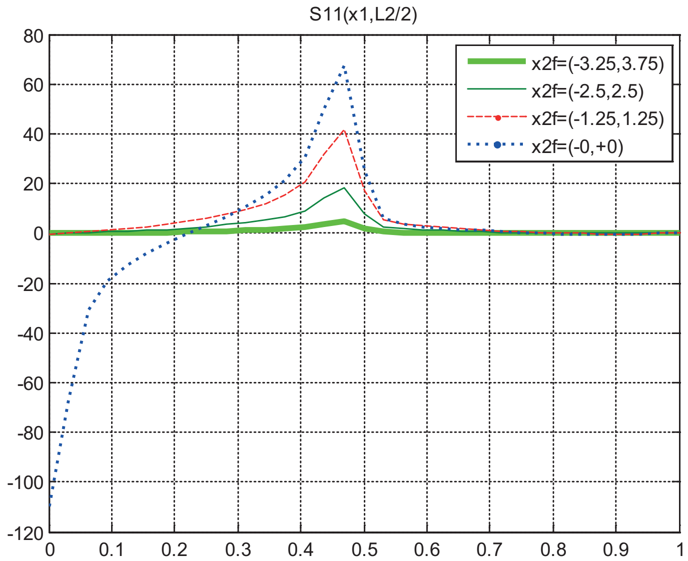

Figure 11. Stresses $\sigma 11$.

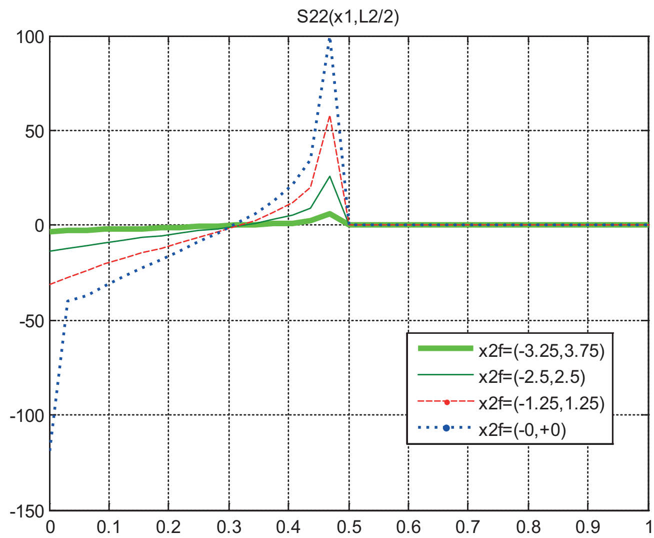

Figure 12. Stresses $\sigma 22$.

CONCLUSION

In the presented examples of analysis, there is a significant excess of the size of the structure along the axis x_2 compared to the size along the axis x_1. In this case, the numerical solution of the problem with the use of discrete-continual finite element method is the most effective, since the traditional finite element method would lead to a significant increase in the nodes of the mesh domain and, consequently, to an increase in the order of the resolving system of equations.

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Marina L. Mozgaleva, Senior Scientist Researcher, Dr.Sc.; Professor of Department of Applied Mathematics, National Research Moscow State University of Civil Engineering; 26, Yaroslavskoe Shosse, Moscow, 129337, Russia; Email: marina.mozgaleva@gmail.com.

Pavel A. Akimov, Full Member of the Russian Academy of Architecture and Construction Sciences, Professor, Dr.Sc.; Rector of National Research Moscow State University of Civil Engineering; 26, Yaroslavskoe Shosse, Moscow, 129337, Russia; E-mail: AkimovPA@mgsu.ru, rector@mgsu.ru, pavel.akimov@gmail.com.

Мозгалева Марина Леонидовна, старший научный сотрудник, доктор технических наук; профессор кафедры прикладной математики Национального исследовательского Московского государственного строительного университета; 129337, Россия, г. Москва, Ярославское шоссе, дом 26; E-mail: marina.mozgaleva@gmail.com.

Акимов Павел Алексеевич, академик РААСН, профессор, доктор технических наук; ректор Национального исследовательского Московского государственного строительного университета; 129337, Россия, г. Москва, Ярославское шоссе, дом 26; E-mail: AkimovPA@mgsu.ru, rector@mgsu.ru, pavel.akimov@gmail.com.