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ASSESSMENT OF THE DISTRIBUTION OF THE SOIL DEFORMATION MODULUS BASED ON CPT DATA

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Abstract: Consideration of soil deformation characteristics is a crucial factor in the design and construction of infrastructure facilities, as they can have a significant impact on the stability and serviceability of structures. Changes in the soil deformation modulus with depth reflect changes in soil density, layering, and stress levels. Accurate assessment of the soil deformation variability is important to ensure the safety and reliability of building foundations. This paper focuses on testing the statistical hypothesis about the type of the soil deformation modulus probability distribution when analyzing cone penetration testing (CPT) data. The soil deformation modulus probability distribution is assumed to be normal and this fact is not checked before modelling. In this paper it is shown that Weibull, Lognormal and Gamma distributions are more suitable than normal for CPT data taken from an open internet source.

Keywords: Cone penetration test (CPT), studies of soil deformation properties, statistical hypothesis testing, distribution type hypothesis

ОЦЕНКА РАСПРЕДЕЛЕНИЯ МОДУЛЯ ДЕФОРМАЦИИ ГРУНТА НА ОСНОВЕ ДАННЫХ CPT

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Аннотация: Учет деформационных характеристик грунта является решающим фактором при проектировании и строительстве объектов инфраструктуры, поскольку они может оказать существенное влияние на устойчивость и работоспособность сооружений. Изменения модуля деформации с глубиной отражают изменения плотности почвы, ее слоистости и уровней напряжения. Точная оценка изменчивости деформаций грунта имеет важное значение для обеспечения безопасности и надежности основания зданий. В работе особое внимание уделяется проверке статистической гипотезы о виде вероятностного распределения модуля деформации грунта при анализе данных статистического зондирования (CPT). Принято считать, что вероятностное распределение модуля деформации грунта является нормальным и этот факт не проверяют перед проведением моделирования. В работе показано, что распределения Вейбулла, Логнормальное и Гамма подходят больше, чем нормальное для данных статического зондирования грунта, взятых из открытого интернет – источника. Приведены примеры моделирования гауссовского и логнормального двумерных случайных полей модуля деформации грунта на фиксированной глубине (в плане).

Ключевые слова: Статическое зондирование, исследования деформационных свойств грунта, проверка статистических гипотез, гипотеза о виде распределения

INTRODUCTION

It is known that one of the main factors influencing the buildings settlements value and structures, as well as the possible settlements unevenness, is the soil deformation modulus values and its statistical heterogeneity. Cone penetration testing (CPT) data provide the most complete picture of the strain modulus values at various points in the soil mass [1 - 4]. The strain modulus distribution models could be useful in solving problems on systematization of static sensing results given in [5 - 10] to ensure safety and reliability. The probability distribution density of the soil deformation modulus plays an important role in building adequate models. According to [11], this distribution is assumed to be normal. Let us check this statement with the example data from [12]. These data represent the results of widely used field surveys that provide a continuous profile of soil physical properties through tip resistance, sleeve friction and pore water pressure during cone penetration. An initial statistical analysis of the data from [12] has already been performed by the authors in [13]. The purpose of such measurements is to assess the strength, stiffness and compressibility of the soil, which are key parameters for determining its deformation behavior [14]. The proper selection of the type of probability distribution can make the model of ground deformation behavior more accurate. We can consider the realization of the random field as a model of the distribution of the soil deformation modulus. The correct estimation of the variation of cone resistance q_c with depth is crucial in geotechnical engineering as it provides insight into the strength and stiffness of the soil. Hypothesis testing offers a statistical basis for assessing whether there is a statistically significant relationship between q_c and depth, indicating the presence of heterogeneity in the soil profile [15 - 16].

METHODS

There are several methods for testing the hypothesis about the type of distribution in mathematical statistics. Pearson's method χ^2 is the universal and most commonly used method. The maximum likelihood method is also widely adopted. It should be noted that the validity of the hypothesis about the type of probability distribution itself cannot be proved, only its admissibility is concluded [17]. Visualizing the data in different ways helps to make a correct hypothesis about the type of distribution. All calculations and graphs were performed in the MATLAB software environment.

The statistical heterogeneity of soil deformation modulus can be interpreted as a trajectory of a random process. For correct transition from the static sounding data to the construction of the ground deformation modulus model, it is possible, using the methods of the theory of random processes, to construct a continuous model using the available discrete statistical material. During construction of the model, the utilization of interpolation methods is not correct due to the peculiarities of the distribution of soil properties in space [16] and the influence of this distribution on the magnitude of non-uniform settlements.

Further we will use the model proposed in [18], when considering the statistical heterogeneity of the ground deformation modulus in plan view:

$$q_c(x, y) = m + \zeta\,(x, y),$$

where $q_c(x, y)$ is the realisation of a two-dimensional random process, $\zeta\,(x, y)$ is the realization of a two-dimensional stationary process with zero mean, and m is an average value of the ground deformation modulus at the considered depth.

Modelling of the Gaussian random field realizations is carried out by the method of canonical decompositions as a partial sum of Fourier series with random uncorrelated coefficients:

$$\zeta(x, y) = \sum_{k=-K}^{K} \sum_{n=-N}^{N} \psi_{k,n}\ exp\left\{\pi\left(\frac{k}{X_{max}}x + \frac{n}{Y_{max}}y\right)i\right\},$$

where X_{max} is the maximum value of the variable x, Y_{max} is the maximum value of the variable y, and $\psi_{k,n}$ are normal random variables with zero mean and variance defined through the covariance function.

The relationship between the lognormal and normal distribution was used to model the realization of a lognormal random field.

RESULTS AND DISCUSSION

Statistical analyses have been performed to quantify the variability of soil deformation parameters within each depth segment. Figures 1 - 4 show the histograms (purple color) of the soil deformation modulus with some theoretical probability densities, namely: Normal (red color), Weibull (blue color), Lognormal (brown color) and Gamma (grey color).

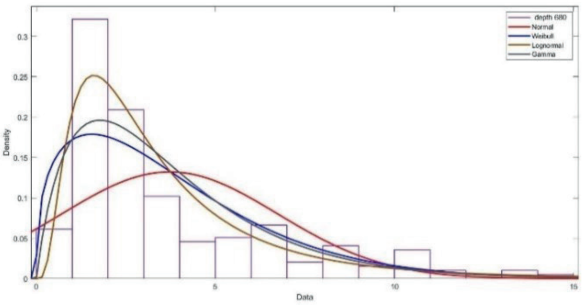

Figure 1. Histogram of the soil deformation modulus with some theoretical probability densities on 680 mm depth

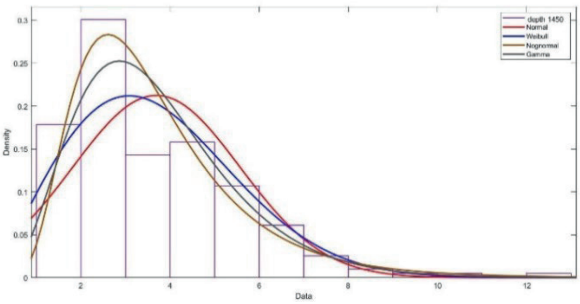

Figure 2. Histogram of the soil deformation modulus with some theoretical probability densities on 1450 mm depth

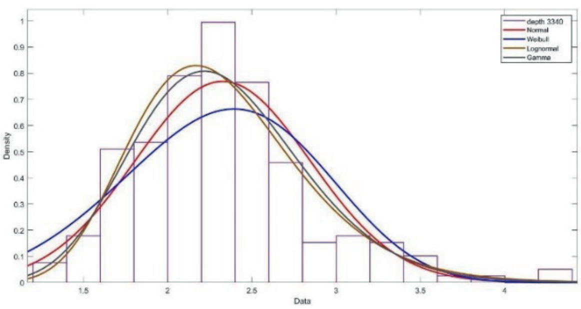

Figure 3. Histogram of the soil deformation modulus with some theoretical probability densities on 3340 mm depth

Table 1 summarizes the values of the logarithm of the likelihood function for the designated depths (mm). Note that the logarithm of the likelihood function is always negative. The greater its value, the more plausible the sample is, assuming that the hypothesis about the type of distribution is true. The lognormal or gamma distribution proved to be the most appropriate.

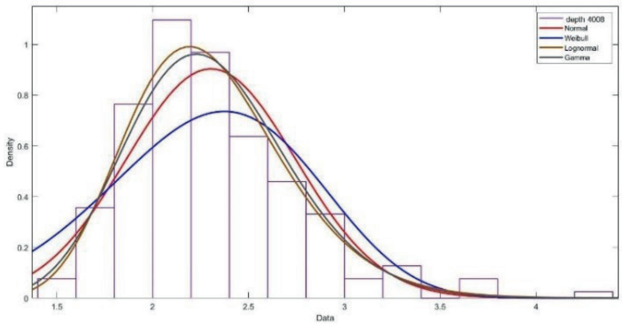

Figure 4. Histogram of the soil deformation modulus with some theoretical probability densities on 4005 mm depth

Table 1 The values of the logarithm of the likelihood function

	Log likelihood			
depth	Normal	Weibull	lognormal	Gamma
680	-494	-439	-422	-434
1450	-401	-383	-367	-372
1700	-363	-351	-336	-340
2300	-316	-306	-270	-280
2905	-235	-236	-212	-217
3340	-149	-160	-139	-140
3875	-125	-141	-109	-113
4005	-117	-136	-102	-106

Table 2 contains the probabilities for the designated depths (mm) for observing the test statistic χ^2 if the main hypothesis about the distribution type is true. Usually, the main hypothesis is rejected if this probability is less than 0.01. Here, too, the lognormal or gamma distribution proved to be the most appropriate.
Figures 5 - 8 show two different realizations each of Gaussian and lognormal random fields with the same numerical characteristics: mean and variance.

Table 2 The probabilities of the test statistic χ^2

depth	Normal	Weibull	lognormal	Gamma
680	8.6e-11	0.0008	0.0120	1.3e-04
1450	0.0026	0.0306	0.0023	0.0043
1700	7.9e-05	0.0056	0.2457	0.1410
2300	0.0225	0.0312	0.7060	0.3372
2905	3.5e-08	1.2e-08	0.0269	0.0012
3340	0.0071	1.8e-05	0.4525	0.1670
3877	0.0153	2.3e-05	0.3997	0.2031
4005	0.0119	5.3e-07	0.4558	0.2385

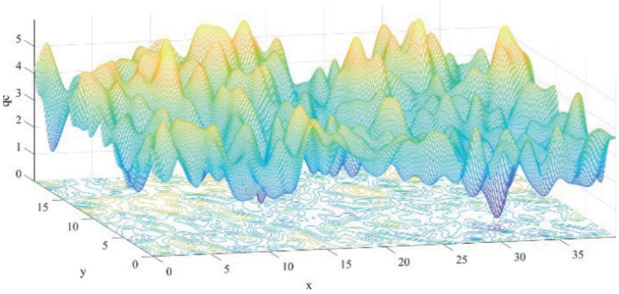

Figure 5. First realizations of the Gaussian random field on 1450 mm depth

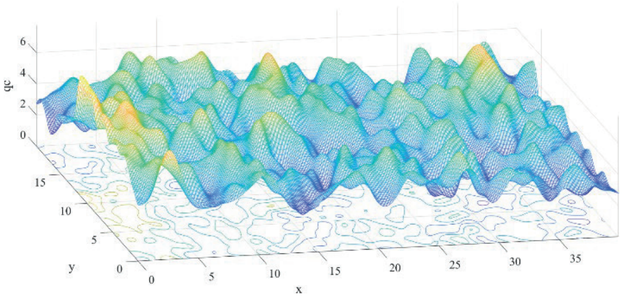

Figure 6. Second realizations of the Gaussian random field on 1450 mm depth

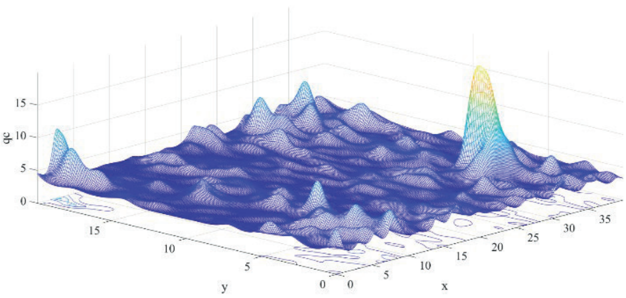

Figure 7. First realizations of the lognormal random field on 1450 mm depth

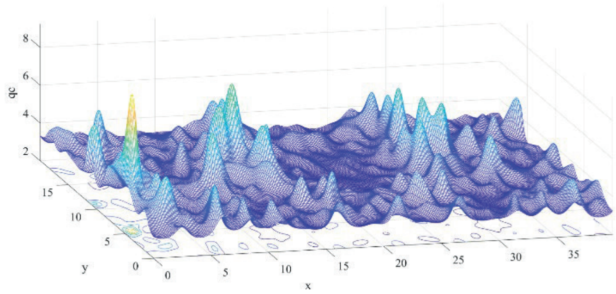

Figure 8. Second realizations of the lognormal random field on 1450 mm depth

It is noticeable that these fields have different distributions, which extends the possibility of using random field realizations in modeling the variability of the soil deformation modulus.

CONCLUSION

It is known (for example [19]) that the design of engineering geological objects that require ensuring the necessary reliability requires a deep understanding of the nature of soil variability, which could be modeled as a random field characterized by a probability density function with its basic statistical characteristics. When building random field models, the correct choice of the probability density function plays a crucial role. The studies carried out in this research have shown that it is incorrect to use only the normal distribution when modeling the behavior of the soil deformation modulus. Calculations show that in some cases the Lognormal distribution or the Gamma distribution are more appropriate than the Normal one. Therefore, before constructing a model of soil deformation modulus variability, it is necessary to test the hypothesis about the type of distribution of the studied quantity. The application of different types of random field distributions can improve the models used in the design of construction projects.

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