International Journal for Computational Civil and Structural Engineering, 21(1) 136–145 (2025)

DOI:10.22337/2587-9618-2025-21-1-136-145

TORSION IN THE ELEMENTS OF THE METAL DOME FRAME, SUPPORTED BY SPARSELY INSTALLED COLUMNS

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Annotation. The effect of torsion on the stress state of the main elements of a metal ribbed-annular dome was investigated. The dependence of the torsion effect on the increase in the distance between the columns supporting the dome was revealed. At the same time, the dependence of torsion on the type of nodal junctions of the frame elements among themselves was determined. The object of the study was a ribbed-ring dome, all elements of which are made of steel pipes. The dome had different support schemes on columns of steel pipes, installed not under each rib, but cyclically symmetrical along the contour. There were four such schemes. In addition, the type of nodal connections of the frame elements to each other has been changed for each scheme. There were five different types of conjugations. The research was carried out through calculations of various models. There were twenty models in total. During the calculations, the stresses in the main elements of the dome models were determined, which were compared with each other. In this case, comparative diagrams of the stress state dependences of the elements of the ribbed-ring dome are obtained. The effect of torsion on the stress state of the elements of the ribbed-ring dome of the considered models is estimated. The degree of change in the stress state of individual frame elements due to torsion has been established. According to the results of the study, significant stress changes due to torsion in the upper ring and noticeable in the meridional ribs were noted. The dependence of the nature of their changes on the type of nodal connections has been established. It is recommended to take into account the torsion effect when designing metal rib-ring domes.

Keywords: ribbed-ring dome, computer model, meridional ribs, upper and lower rings, columns, torsion, nodal connections, static calculation

КРУЧЕНИЕ В ЭЛЕМЕНТАХ КАРКАСА МЕТАЛЛИЧЕСКОГО КУПОЛА, ОПИРАЮЩЕГОСЯ НА РЕДКО УСТАНОВЛЕННЫЕ КОЛОННЫ

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Аннотация. Исследовалось влияния кручения на напряженное состояние основных элементов металлического ребристо-кольцевого купола. Выявлялась зависимость эффекта кручения от увеличения расстояния между поддерживающими купол колоннами. Одновременно с этим определялась зависимость кручения от вида узловых сопряжений элементов каркаса между собой. Объектом исследования был ребристо-кольцевой купол, все элементы которого приняты из стальных труб. Купол имел разные схемы опирания на колонны из стальных труб, установленные не под каждым ребром, но циклически симметрично по контуру. Таких схем было четыре. Кроме того, для каждой расчетной схемы изменялся вид узловых сопряжений элементов каркаса между собой. Разных видов сопряжений было пять. Исследования проводились посредством расчетов разных моделей. Всего моделей насчитывалось двадцать. В процессе расчетов определялись напряжения в основных элементах купольных моделей, которые сравнивались между собой. При этом получены сравнительные диаграммы зависимостей напряженного состояния элементов ребристо-кольцевого купола. Дана оценка влияния кручения на напряженное состояние элементов ребристо-кольцевого купола рассмотренных моделей. Установлена степень изменения напряженного состояния отдельных элементов каркаса из-за кручения. По результатам исследования были отмечены значительные изменения напряжений из-за кручения в верхнем кольце и заметные в меридиональных ребрах. Установлена зависимость характера их изменений от типа узловых соединений. Рекомендовано учитывать эффект кручения при проектировании металлических ребристо-кольцевых куполов.

Ключевые слова: ребристо-кольцевой купол, компьютерная модель, меридиональные ребра, верхнее и нижнее кольца, колонны, кручение, узловые сопряжения, статический расчет

INTRODUCTION

Recently, publications have begun to actively pay attention to the study of the operation of metal structural elements taking into account torsion. The vast majority of publications are usually devoted to open-profile beams. For example, the article [1] studies the behavior of a cantilever beam during torsion, and the article [2] compares the results of a theoretical and experimental study of the operation of an I-beam during bending. In addition to beams, frame rod systems are being considered. For example, in the article [3], an L-shaped and U-shaped frame with channel elements is considered, and the subject of the study is the bimoment diagrams. The article [4] considers a U-shaped rod systems of I-beams, and the subject of the study is the angle of rotation of the cross-section elements at the node.

There are no publications in the open press on the study of torsion in the elements of metal domes. Due to the spatial rigidity and cost-effectiveness of metal consumption, domes occupy a leading place as long-span coatings [5]. Metal domes are used as load-bearing frames for building coverings due to the reliability of such core systems [6, 7].

The geometric schemes of metal dome frames depend on the covered spans and the purpose of the building [8, 9]. Ribbed-ring domes are considered to be the simplest according to the geometric scheme. But even in ribbed-ring domes, various geometric schemes are possible, related to the number of sectors around the circumference and tiers in height. In addition, an important factor in the operation of the dome frame is the curvature and the number of columns supporting them. The static scheme of the entire building frame and the internal forces in the elements of the dome frames depend on this, but there are no studies of metal domes supported by sparsely installed columns in the open press.

Usually columns in ribbed-ring domes are placed under each meridional rib. However, with a large number of sectors or edges in the dome frame, such a design solution may be inconvenient for various reasons. In this case, fewer columns are used compared to the number of meridional ribs, which leads, as shown by the previous study of the author [10], to a change in the nature of the dome, which is manifested by a change in internal forces in the elements of the dome frame. The same study showed that, despite the similarity of the shape of the dome deformations, with a decrease in the number of columns under the dome, there is a significant increase in deflections of the dome frame.

Studies of dome-type rod systems in various computer programs have been carried out by many scientists. For example, the stress state of dome frames was analyzed when its geometric parameters changed [11], with different ratios of dome height to diameter for different spans [12]. As well as when the roof is included in the work in the cells of the frame between the steel ribs and rings [13], with different dome height-to-diameter ratios and different cross-sections of the elements [14], with different heights compared to the span of the dome frame with connections [15]. Previously, the author performed a comparative study of ribbed-ring domes with different numbers of connections [16] and different sizes of the upper ring [17].

In addition to torsion, there are no publications (except for the author's) on the study of the dependence of the stress state on the increase in the distance between the columns supporting the dome. In addition, it is possible to use various types of coupling elements of dome frames with each other, which also affects the torsion in the elements. The effect of the stiffness of the nodal joints on the stressed state of the dome is discussed in the publication [18]. This article presents a comparative analysis of a low steel dome of a ribbed-ring type with a diameter of 41 m and a height of 7 m. The dome consisted of 20 ribs and 10 rings, supported on foundations directly by each meridional rib. Purlins with glass cladding were used as rings. The models of this frame were considered, which differ in two approaches to calculation – linear and nonlinear, as well as the type of connection – with rings attached to the ribs and without attachment to them. There are no explanations about the specific interpretation of compounds in the computer model. In addition, in some models, cladding is included in the work, and in some– diagonal connections in all sectors. The publication analyzed dome deformations, internal forces, and stresses in the elements, but without torsion.

METHODS

In order to determine the effect of torsion on the stress state of the main elements of the metal rib-ring dome frame numerical studies using the finite element method using the SCAD software package were performed [19, 20]. Twenty variants of design models of a dome structure were considered for the study. They differed from each other, firstly, in the schemes of supporting the dome on the columns and, secondly, in the types of nodal connections of the frame elements to each other. Twenty different computer models of the dome structure frame were considered for the study. They differed from each other, firstly, by different schemes of supporting the dome on the columns and, secondly, by different types of nodal connections of the frame elements to each other.

The object of the study was the frame of a spherical ribbed-ring dome with a radius of curvature of 23 m, consisting of 36 ribs and 7 rings (Fig. 1). Thus, the dome is divided by ribs into 36 sectors. The diameter of the lower ring is 39.3 m, the diameter of the upper ring is 5.0 m, the height of the dome frame is 11 m. The parameters of the metal dome elements were adopted based on the results of a preliminary calculation for operational loads from electro-welded pipes: the meridional ribs are O 530×9, the upper ring is O 530×9, the lower ring is O 630×20, the remaining rings are O 273×7.

In the course of the study, four dome support schemes were considered, the distinguishing feature of which was the number of sectors between the columns. So, if the columns were installed under each riber of the dome, i.e. through one sector, then the scheme was designated as 1. If the columns were installed through two, four and six sectors, then the schemes were designated as 2, 4 and 6, respectively (Fig. 2). All support schemes are characterized by the cyclic symmetry of the arrangement of columns along the contour. The columns in all schemes have a height of 7.0 m and are made of electro-welded pipes O 402×10.

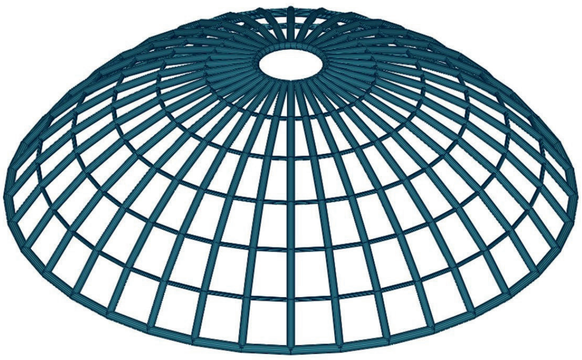

Figure 1. The ribbed-ring dome under study

The research was carried out on computational models of frameworks as spatial rod systems. In the calculated models, each structural element of the dome frame was represented by a single rod final element of the KE–10 with six degrees of freedom at the nodes. Calculations were performed for static impacts in a linear formulation. In the scheme 1 (Fig. 2, a), the number of elements 504, the number of nodes 288, total number of degrees of freedom 1728. In the scheme 2 (Fig. 2, b), the number of elements 486, the number of nodes 270, total number of degrees of freedom 1620. In the scheme 4 (Fig. 2, c), the number of elements 477, the number of nodes 261, total number of degrees of freedom 566. In the scheme 6 (Fig. 2, d), the number of elements 474, the number of nodes 258, total number of degrees of freedom 1548. In all schemes, restrictions on movement in the directions of all six degrees of freedom were imposed in the support nodes. In all schemes, the upper ring, the lower ring and the meridional ribs are integral, i.e. all 36 constituent rods of these rings and 6 rods of each rib are rigidly connected to each other. And each of the intermediate rings is formed from separate rods between the ribs.

Nodes at the junctions of the ribs with the upper and lower rings, the joints of the columns with the lower ring, and the joints of the rods of the intermediate rings with the meridional ribs in the computational models of each of the schemes (see Fig. 2) were assigned both hinged and rigid. In the course of the study, five types of combinations of nodal connections were considered, which are conventionally called types of connections. Their designation and description of the allowed rotations in the joints in the normal (UY) and tangential (UZ) planes are given in Table 1. For example, for the S3 type (see Table 1) these joints are hinged in the normal (UY) and tangential (UZ) planes (Fig. 3). Type S2 (see Table 1) differs from S3 by prohibiting UY rotation only at the junctions of the ribs with the upper ring (see Fig. 3). Type S1 (see Table 1) differs from S2 by prohibiting the rotation of UY only at the junctions of the intermediate rings with the ribs (see Fig. 3). Type R1 (see Table 1) differs from S1 by prohibiting UY rotation at the joints of the ribs and columns with the lower ring (see Fig. 3). And for the R2 type of connection, they are rigid in the normal (UY) and tangential (UZ) planes (see Table 1). The torsion of rods around their axes (UX) has always remained prohibited.

Note that the rotation designations UY, UZ here and in Table 1 correspond to the local axes of the finite elements in the SCAD program.

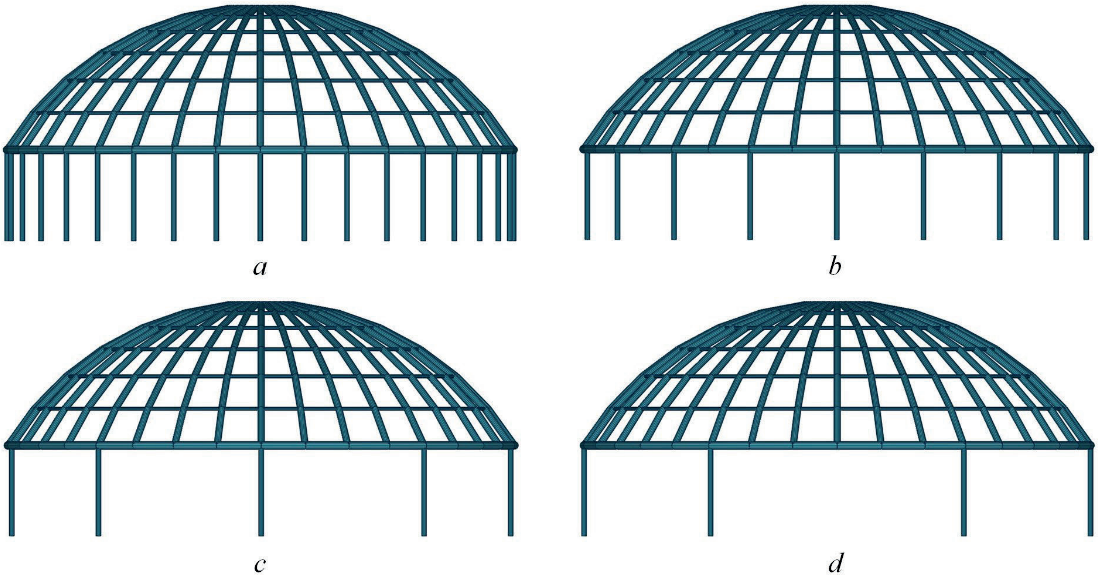

Figure 2. Schemes of frames with different numbers of sectors between the columns under the dome: a – 1 sector, b – 2 sectors, c – 4 sectors, d – 6 sectors.

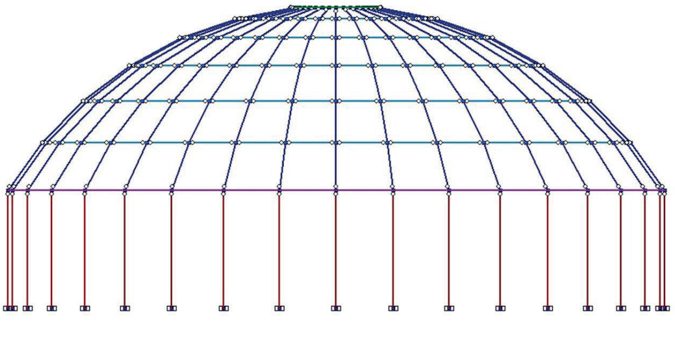

Figure 3. The initial calculation model of the frame with hinged nodal connections

Table 1. Resolution of rotations in nodes of various types of element connections of SCAD program

Types of nodes	Element connections							
	Ribs with upper ring		Ribs with lower ring		Columns with lower ring		Intermediate rings with ribs	
	UY	UZ	UY	UZ	UY	UZ	UY	UZ
S3	*	*	*	*	*	*	*	*
S2		*	*	*	*	*	*	*
S1		*	*	*	*	*		*
R1		*		*		*		*
R2								

The sign * means permission of rotation in node, and the absence of the sign means restrain of rotation.

The frames were calculated based on the combined effect of the load on the weight of the enclosing and load-bearing structures, as well as the asymmetric snow load acting on one side of the dome, as the most significant compared to the symmetrical one. All loads were applied at the nodes of the computational models of the framework.

In the course of the study internal forces N, M_x, M_y, M_t in the elements of the model frames were determined to calculate the stresses (in the SCAD program they were designated as N, M_y, M_z, M_k respectively), which were selected in the most stressed elements of the frame.

Since this study focuses on torsion, attention was focused on the torque M_t and the dependence of its magnitude on different frame schemes and types of connections elements in them.

Since the torsion of the elements causes tangential stresses in the sections, to assess the degree of influence of torsion on their stress state the reduced stresses were calculated using the formula

$$\sigma_t = \sqrt{\sigma^2 + 3\tau^2}\ . \tag{1}$$

Here are the normal stresses σ for the tubular section

$$\sigma = \frac{N}{A} \pm \frac{M}{W_x}, \tag{2}$$

where $M = \sqrt{M_x^2 + M_y^2}$ – this is the resultant of the moments M_x, M_y.

Tangential stresses τ in the presence of torque M_t in the pipes

$$\tau = \frac{M_t}{W_t}\ . \tag{3}$$

The effect of torsion on the stress state was determined by the difference between the reduced and normal stresses $\sigma_t - \sigma$.

RESULTS

The torques M_t manifests itself in all the elements of the framework, but in comparison with the moments M_x, M_y is characterized by relatively small values. In the columns and intermediate rings, the values of the torques M_t are insignificant. The meridional ribs of the dome the torques M_t are relatively small, but they show stable values by type of conjugation, with some increase with a decrease in the number of columns under the dome (Fig. 4).

In the upper ring of the dome the torque M_t values depend on the types of connections and increase with a decrease in the number of columns under the dome (Fig. 5). In the lower ring of the dome, the torque M_t values significantly depend on the types of connections and increase with a decrease in the number of columns under the dome by several times (Fig. 6). In schemes with the number of sectors between columns 4 and 6, the moments in the upper and lower rings of the dome reach significant values.

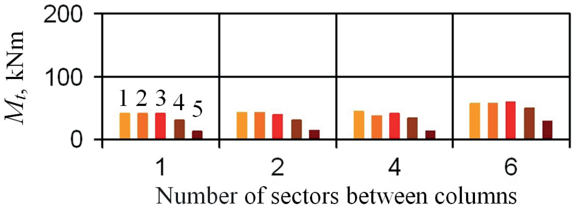

Figure 4. *Maximum moments M_t in the meridional ribs of the dome. Types of connections: 1 – S3, 2 – S2, 3 – S1, 4 – R1, 5 – R2*

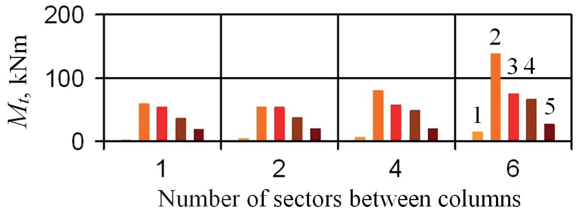

Figure 5. *Maximum moments M_t in the upper ring of the dome. Types of connections: 1 – S3, 2 – S2, 3 – S1, 4 – R1, 5 – R2*

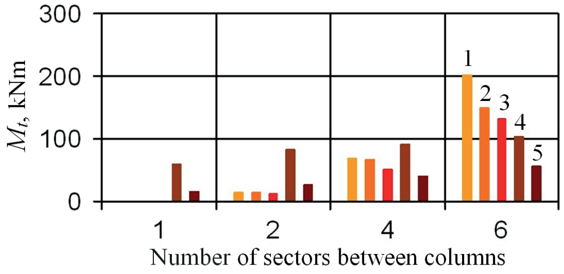

Figure 6. *Maximum moments M_t in the lower ring of the dome. Types of connections: 1 – S3, 2 – S2, 3 – S1, 4 – R1, 5 – R2*

Taking into account the identified torques M_t, a comparison of the reduced stresses σ_t calculated according to formula (1) was carried out. Note that these stresses are conditional, since their calculation uses the maximum values σ and the maximum values τ that occurred in different sections of the same type of elements. This is explained by the fact that in reality values are combined in a wide range of values – from maximum σ with minimum τ, to minimum σ with maximum τ.

In the meridional rings of the dome, the values of the reduced stresses σ_t depend on the types of connections and hardly change with a decrease in the number of columns under the dome (Fig. 7), with the exception of scheme 6. In the upper ring of the dome, the values of the reduced stresses σ_t also depend on the types of connections and clearly increase only when switching to scheme 6 with a reduced number of columns under the dome (Fig. 8).

The dependence of the reduced stresses σ_t on the number of columns under the dome is most reflected in the lower ring of the dome. The values σ_t increase several times when switching to schemes with a reduced number of columns (in schemes 2, 4 and 6) and, at the same time, depend on the types of connections (Fig. 9).

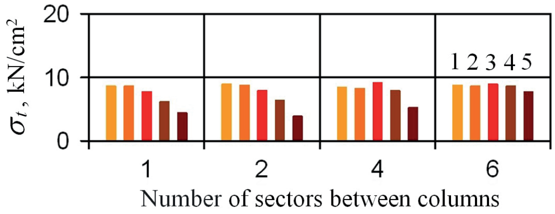

Figure 7. *Reduced stresses σ_t in the meridional ribs of the dome. Types of connections: 1 – S3, 2 – S2, 3 – S1, 4 – R1, 5 – R2*

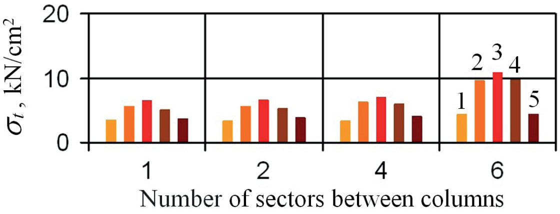

Figure 8. *Reduced stresses σ_t in the upper ring of the dome. Types of connections: 1 – S3, 2 – S2, 3 – S1, 4 – R1, 5 – R2*

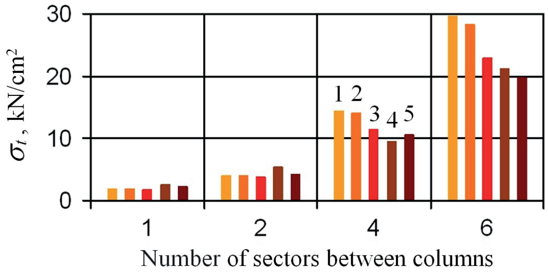

Figure 9. Reduced stresses σ_t in the lower ring of the dome. Types of connections: 1 – S3, 2 – S2, 3 – S1, 4 – R1, 5 – R2

To quantify the effect of torsion on the stress state of the dome elements, the ratio of the difference between reduced and normal stresses to reduced stresses, the so-called fractional effect of torsion, was calculated using the formula

$$\Delta_t = \frac{\sigma_t - \sigma}{\sigma_t}. \qquad (4)$$

In the meridional ribs of the dome the fractional effect of torsion Δ_t is relatively small and varies in the range from 1% to 5%. It depends on the type of node connections and increases slightly in scheme 6 with a reduced number of columns under the dome (Fig. 10).

In the upper ring of the dome, the fractional influence of torsion in one type of connections (S2), depending on the dome support scheme, is extremely high and varies from 11% to 26%, in the other (S1) – is not significant, since it does not exceed 1%. In other cases, the fractional effect of torsion Δ_t is relatively small and varies in the range from 3% to 8% (Fig. 11).

In the lower ring of the dome, the fractional effect of torsion Δ_t in only one type of connections (S1), depending on the dome support scheme, does not exceed 6%, and with the largest number of columns. In other types of connections, regardless of the support scheme, the torsion effect Δ_t does not reach even 1% (Fig. 12). This phenomenon, with a significant increase in torque M_t with a decrease in the number of dome support columns, is explained by the fact that the bending moments M_x, M_y increase much faster.

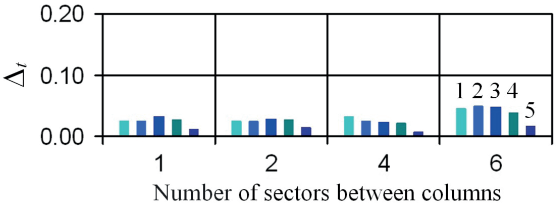

Figure 10. The fractional effect of torsion Δ_t on σ_t in the meridional ribs of the dome. Types of connections: 1 – S3, 2 – S2, 3 – S1, 4 – R1, 5 – R2

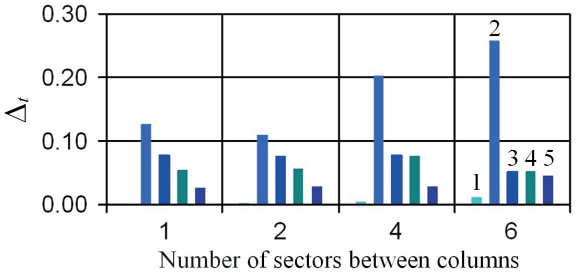

Figure 11. The fractional effect of torsion Δ_t on σ_t in the upper ring of the dome. Types of connections: 1 – S3, 2 – S2, 3 – S1, 4 – R1, 5 – R2

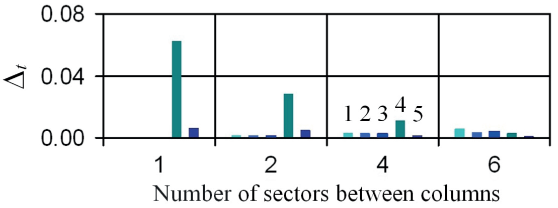

Figure 12. The fractional effect of torsion Δ_t on σ_t in the lower ring of the dome. Types of connections: 1 – S3, 2 – S2, 3 – S1, 4 – R1, 5 – R2

CONCLUSIONS

Based on the presented material, the following conclusions can be drawn:

1. With different types of nodal connections of the elements of the ribbed-ring dome, in combination with sparsely installed columns, torques

of different magnitudes arise in different elements.

2. In the meridional ribs of the dome, torsion has a noticeable (up to 5%) effect on their stress state in all dome support schemes and in various types of coupling elements, except for rigid ones.
3. Torsion has the most significant effect on the upper ring. When the nodes are swivel in the tangential and normal directions, with the exception of the junctions of the ribs with the upper ring, the voltage may increase by 11-26%, otherwise by no more than 1%. With other types of swivel connections, the torsion effect is limited to 8%, and with rigid connections - less than 5%.
4. In the lower ring of the dome, with hinged connections only in the tangential direction, torsion affects its stress state from 6% to 3% in schemes with columns under each rib and through the rib, respectively. In other types of nodes, regardless of the support scheme, the effect of torsion does not reach 1%.
5. Additional studies should be conducted to determine the possible effect of torsion on the stress state of metal ribbed-ring domes with non-tubular profile elements. It is also necessary to do this to evaluate the torsion in the intermediate rings of the dome, since it is necessary to take into account their work between the ribs along the beam circuit from a distributed load.
6. When designing metal ribbed-ring domes, it is recommended to take into account the effect of torsion on the bearing capacity of their elements, especially the upper ring.

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