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APPLICATION OF THE STEEL-RUBBER VIBRATION ISOLATORS WITH PERFORATION FOR VIBRATION ISOLATION OF BUILDINGS

Vladimir L. Mondrus [1]***, Dmitry K. Sizov*** [2]***, Timofei M. Kvasnikov*** [1]

[1] National Research Moscow State University of Civil Engineering, Moscow, RUSSIA
[2] OOO "Vibroseismozastchita", Moscow, RUSSIA

Abstract: The article presents the derivation of a formula taking into account the coefficient of convexity under load, which takes into account the bulging of rubber along the edges of a steel-rubber vibration isolator. The article also provides a comparison for a vibration isolator with a hole and a vibration isolator without holes. It also presents a formula that allows taking into account the ambient temperature when calculating vibration isolators. Graphs of the dependence of the elastic modulus on temperature for a rubber-metal isolator with a hole are given.

Keywords: vibration isolation, vibration isolator with holes, steel-rubber vibration isolator, vibration isolator without holes, effects of vibration isolation systems on human health

ПРИМЕНЕНИЕ ПЕРФОРИРОВАННЫХ РЕЗИНОМЕТАЛЛИЧЕСКИХ ВИБРОИЗОЛЯТОРОВ ДЛЯ ВИБРОИЗОЛЯЦИИ ЗДАНИЙ

В.Л. Мондрус [1]***, Д.К. Сизов*** [2]***, Т.М. Квасников*** [1]

[1] Национальный исследовательский Московский государственный строительный университет, г. Москва, РОССИЯ
[2] ООО «Вибросейсмозащита», г. Москва, РОССИЯ

Аннотация: В статье представлен вывод формулы с учетом коэффициента коэффициентом выпуклости под нагрузкой, который учитывает выпучиваение резины по краям резинометалличекого виброизолятора. Также в статье приведено сравнение для виброизолятора с отверстием и виброизолятора без отверстий. Также представлена формула, которая позволяет учитывать температуру окружающей среды при расчете виброизоляторов. Приведены графики зависимости модуля упругости от температуры для резинометаллического изолятора с отверстием.

Ключевые слова: виброизоляция, виброизолятор с отверстиями, резинометаллический виброизолятор, виброизолятор без отверстий, воздействия систем виброизоляции на здоровье людей

1. INTRODUCTION

Transportation routes are the circulatory system of any state. It is impossible to imagine trade, production, and everything that our world is based on without transportation routes. But lately it is not the quantity of roads, and even not their quality, but the quality of human life provided by transportation routes that comes to the fore.

Urban rail transport is an important part of the urban transportation system and one of the strategic vectors of development of megalopolises due to its advantages such as high speed, energy efficiency, environmental friendliness, as well as safety and punctuality. At the same time, the vibrations caused by the movement of the train on the rails propagate through the ground and affect the buildings located near the rail lines.

These vibrations have a negative impact on the health of peoples.

Thus, it is critical to study the vibrations caused by both rail and subway traffic and their impact on human health and the urban environment.

At the same time, as the need for people to improve their quality of life continues to grow, there are increasing demands for protection from vibration induced by external factors.

Although vibration isolation measures applied directly to the railroad track can somewhat reduce the impact on the building, they are still insufficient for full-fledged vibration isolation of the structure.

One of the ways to damp the vibration impact arising due to the movement of rail transport is the use of rubber-metal vibration isolators installed in a reinforced concrete deck under the lload-bearing structures of buildings, or in the walls and columns at different levels. With the rapid development of the subway, the requirements for the stability of movement and comfort of high-speed trains are constantly increasing, as well as the requirements for the comfort of buildings located near shallow subway lines [1-4]. Therefore, it is necessary to accurately define the stiffness of rubber vibration isolators in order to effectively design and predict the structure of buildings. The operating conditions and load capacity of vibration isolators are constantly changing. The change of static stiffness of vibration isolators depending on temperature and load should be expressed by the appropriate formula, which will be of great importance in the authors' further studies on the analysis of dynamic properties [5-7]. The calculation of mechanical properties of rubber vibration isolators has always been the focus of research, but previous studies have focused mainly on dynamic stiffness. Most models consider different combinations of elements such as springs, isolators; and the main impact factors are frequency and amplitude [8-10]. Currently, there are some studies on the static stiffness of rubber vibration isolators under deformation conditions at room temperature. For rubber vibration isolators with different shapes, there are several theories and empirical formulas for calculation. Most of these formulas assume linear stiffness or consider a certain geometric nonlinear stiffness. In addition, the finite element method is widely used in the study of rubber behavior. This method can accurately simulate small static deformation in regard with experiments. These deformations are calculated using strain energy and tensor function [11, 12], but the parameters calculated by computer simulation still depend on the experimental data.

Ogden [13] has already pointed out that the procedure for modeling vibration isolators at different temperatures is a very difficult task, and Destrade [14] proposed mathematical equations to describe rubber vibration isolators based on commonly available data.

Cheremisinoff [15] studied the effect of humidity, temperature on the load carrying capacity of rubber isolators.

Dickens [16] examined the effect of temperature on the stiffness of insulators at temperatures ranging from 10°C to 60°C.

Hwang [17] conducted field experiments of isolators at temperatures ranging from 0°C to 28°C to validate his fractional derivative model for calculation of isolators.

The insulator developed by Hou [18] was tested in the extreme range from -70°C to 300°C and confirmed its intended characteristics.

Gajewski [19] presented the theory of finite element modeling for large deformations and analyzed the effectiveness of different models. In addition, Gajewski [20] proposed a method to estimate the magnitude of energy dissipation of elastic materials at large deformations. These results are important in the study of large deformations.

Cardon et al. [21] studied the relationship between rubber properties and temperature. They focused mainly on the shear modulus, and the temperature ranged from 20°C to 60°C. But at present there are very few studies on the behavior of rubber vibration isolator at low temperatures.

Stevenson [22] studied the relationship between low-temperature crystallization and the strain modulus of rubber vibration isolator. The results showed that the strain modulus increases significantly with low-temperature crystallization. In addition, the relationship between low-temperature deformation and crystallization was also studied. Authors of the papers [23] and [24] analyzed and studied the performance of rubber insulators at low temperature. The results showed that low temperature has a significant effect on the properties of rubber.

Kari [25] proposed a nonlinear temperature model of rubber vibration isolator based on shape factor and investigated the effect of temperature on geometric parameters under load. To calculate the shape factor, they used the equivalent cylinder approximation in the equation to calculate the shape factor.

2. RESEARCH METHODS

For an ideal rubber material, the entropy of the molecular chain can be utilized to analyze the elasticity. In microscopic theory, the complex structure is simplified using some assumptions and the ideal structure is considered to facilitate calculation and analysis.

Let us summarize the static theory and derive the modulus of elasticity E. According to the theory of elasticity, the following equation is satisfied for the rubber layer:

$$E = \frac{\sigma}{\varepsilon} = \frac{3\rho RairT}{Mc}, \tag{1}$$

where E is the rubber modulus of elasticity; σ is the stress; ε is the strain; M_c is the mean molecular weight between cross-linked nodes; ρ is the rubber density; R_{air} is the Universal gas constant; T is the temperature.

Due to the complexity of the composition and inner structure of rubber materials, Mc is difficult to measure directly. However, Mc can be obtained from the modulus of elasticity at a certain temperature and small strain. This method can be used to derive an expression for Mc. The formula for Mc can be used to further derive the modulus of elasticity of rubber at different temperatures.

$$Mc = \frac{3\rho k\mathrm{RTk}}{Ek},$$
$$E = \frac{3\rho \mathrm{RT}}{Mc} = \frac{\rho \mathrm{TEk}}{\rho kTk}, \tag{2}$$

where T_k is the room temperature adopted as 25±2°C;

ρ_k is the rubber density at room temperature; E_k is the modulus of elasticity at room temperature.

The density of rubber varies with temperature since its volume varies with temperature. The linear coefficient of thermal expansion of rubber is denoted as α. Since α is extremely small, the coefficient of volume expansion can be expressed as (1 + α)3 − 1 ≈ 3α. Thus the density of rubber is as follows:

$$[1 + 3\alpha(T - Tk)]\rho = \rho k \quad for\ T > Tk,$$
$$\rho = [1 + 3\alpha(Tk - T)]\rho k \quad for\ T < Tk,$$
$$\rho = \rho k \quad for\ T = Tk, \tag{3}$$

In addition, the following formula can be derived:

$$\frac{\rho}{\rho k} = \frac{1}{[1 + 3\alpha(Tk - T)]} \quad for\ T > Tk,$$
$$\frac{\rho}{\rho k} = [1 + 3\alpha(Tk - T)] \quad for\ T < Tk, \tag{4}$$
$$\frac{\rho}{\rho k} = 1\ for\ T = Tk,$$

If one accepts $\Delta\mathrm{T} = T - Tc \quad for\ T < Tc$, and multiply the numerator and denominator by $1 + 3\alpha\Delta\mathrm{T}$, then it follows:

$$1 + 3\alpha(Tk - T) = 1 + 3\alpha\Delta T \ = \frac{(1 - 3\alpha\Delta T)(1 + 3\alpha\Delta T)}{1 + 3\alpha\Delta T} = \\ = \frac{(1 - 9\alpha^2\Delta T^2)}{1 + 3\alpha\Delta T} = \frac{1}{1 + 3\alpha\Delta T}, \quad (5)$$

If α is extremely small, we can use an approximation and simplify equation (2) as follows by substituting $\frac{\rho}{\rho k}$:

$$E = \frac{T_k}{(1 + 3\alpha\Delta \mathrm{T})T_k}, \quad (6)$$

The formula shows that for a rubber vibration isolator, the modulus of elasticity increases when the temperature increases in a certain range. Other relationships are also available, such as formula (33).

In addition, due to the nonlinearity of rubber, the stiffness will gradually decrease with increasing temperature when the load is relatively high. Changes in modulus of elasticity and stiffness do not always correspond to each other. This will be discussed in detail: see conclusions, Fig. 5b formula (31).

3. CALCULATION OF VIBRATION ISOLATOR WITH A HOLE

Based on extensive experience, researchers have developed a simplified method for calculating the stiffness of vibration isolators with a certain accuracy. For a rubber vibration isolator with a round shape, at small deformations, the empirical formula for the static stiffness K is as follows:

$$K = \frac{A_c \mu E}{H}, \quad (7)$$

where A_c is the support area, E is the elasticity modulus, H is the height of the vibration isolator, μ is the form factor.

$$\mu = 1 + 2S^2. \quad (7.1)$$

This factor accounts for the ratio between the height and width of the vibration isolator, which directly affects its stiffness and deformation properties [26].

Let us assume the area factor S, which can be obtained from the following formula:

$$S = \frac{A_c}{A_f} = \frac{\pi(R^2 - r^2)}{2\pi(\mathrm{R} + \mathrm{r})\mathrm{H}} = \frac{R - r}{2\mathrm{H}}, \quad (8)$$

where Af is the free area, which is a sum of inner and external areas of vibration isolator with a hole, R is the outer radius of the vibration isolator, r is the inner radius of the vibration isolator.

Based on formulas (7) and (8), the empirical expression for the stiffness of the vibration isolator is as follows:

$$K = \frac{\pi r^2 E(H^2 + (\frac{1}{2})r^2)}{H^3} = \frac{\pi r^2 E}{\mathrm{H}} + \frac{\pi r^4 E}{2H^3}, \quad (9)$$

As a vibration isolator is compressing, the size of the rubber gasket is continuously changing. The stiffness K is related to the deformation and is not a fixed value. Thus, the static stiffness of the vibration isolator is nonlinear. In the calculation of the free area, we assume that the vibration isolator is compressed uniformly, but it has convex edges after deformation. Assuming that the change of the inner radius is the same as the outer radius, we obtain,

$$R_1 - R = r - r_1 = d, \quad (9.1)$$

where R_1 is the outer radius of the vibration isolator, r_1 is the inner radius after compression, R is the outer radius of the vibration isolator before compression, r is the inner radius of the vibration isolator before compression, d is the change in the radius. Since the volume remains constant, we get:

$$\pi({R_1}^2 - {r_1}^2)(\mathrm{H} - h_{def}) = \pi(R^2 - r^2)H,$$

$$d = \frac{1}{2}\frac{(R-r)h_{def}}{\mathrm{H}-h_{def}},$$

$$R_1 = R + \frac{1}{2}\frac{(R-r)h_{def}}{\mathrm{H}-h_{def}}, \quad (10)$$

$$r_1 = r - \frac{1}{2}\frac{(R-r)h_{def}}{\mathrm{H}-h_{def}},$$

Where H is the initial height of the vibration isolator, and h_{def} is the deformation. Formulas (8) and (9) are modified as follows:

$$S_1 = \frac{A_{c1}}{A_{f1}} = \frac{\pi({R_1}^2 - {r_1}^2)}{2\pi(R_1 + r_1)(\mathrm{H}-h_{def})} = \frac{R_1 - r_1}{2(\mathrm{H}-h_{def})}, \quad (11)$$

$$K_1 = \frac{\mu_1 A_{c1} E}{\mathrm{H}-h_{def}} = \frac{A_{c1}(1+2{S_1}^2)E}{\mathrm{H}-h_{def}} = \frac{\pi({R_1}^2 - {r_1}^2)\mathrm{E}}{\mathrm{H}-h_{def}} \cdot \left[1 + \frac{({R_1}^2 - {r_1}^2)}{2(\mathrm{H}-h_{def})^2}\right], \quad (11)$$

where S_1 is the ratio of areas, A_{f1} is the free area, K_1 is the stiffness, A_{c1} is the support area, μ_1 is the form factor.
Static force F_1 is defined as follows:

$$F_1 = \int_0^{h_{def}} K_{c1} dz = \int_0^{h_{def}} \frac{\pi({R_1}^2 - {r_1}^2)\mathrm{E}}{\mathrm{H}-h_{def}} \cdot \left[1 + \frac{({R_1}^2 - {r_1}^2)}{2(\mathrm{H}-h_{def})^2}\right] dz, \quad (12)$$

Usually, the end friction causes the area change of the vibration isolator after deformation to be smaller than the calculated value, but the free area change is larger in the presence of the supporting steel plate.

$$A_{c1} = A_c = \pi(R^2 - r^2), \quad (13)$$

As for rubber, the compression behavior of the sample between bonded surfaces can be estimated quantitatively by means of the elastic modulus. This parameter is related to the geometry of the rubber sample. In the case of compression without bonded surfaces, the rubber block exhibits axial and uniform lateral deformation as shown in Figure 1(a).
The deformation of the vibration isolator is non-uniform in compression. Therefore, the assumption of cylindrical deformation of the rubber gasket is unfeasible. It should be close to elliptical deformation because the lateral surface will take the shape of an ellipse. Figure 1(a) shows the initial state of the vibration isolator, and Figure 1(b) shows the hypothetical half-elliptical state, which is obtained in practice.
During deformation, the steel plate is treated as a rigid body, and hence the bearing area of the vibration isolator is not changed. However, the free area of the rubber gasket is changed. When the rubber pad is deformed, the side surface takes the shape of a semi-ellipse. It is assumed that the convex part is a semi-ellipse in the vertical section, and that the semi-axes of the inner and outer semi-ellipses are equal, as shown in Figure 1(b). By taking the center of the vibration isolator as the origin of the rectangular coordinate system, we denote the axis of the convex part as a and the axis to the steel plate as b.

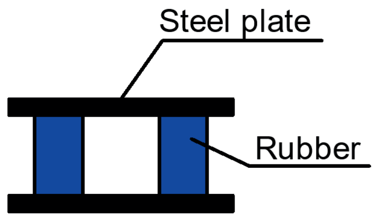


Figure 1a. Vibration isolator with a hole before loading

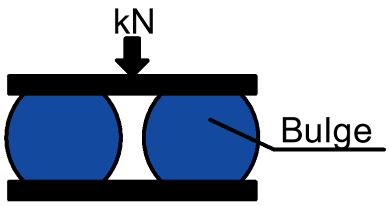
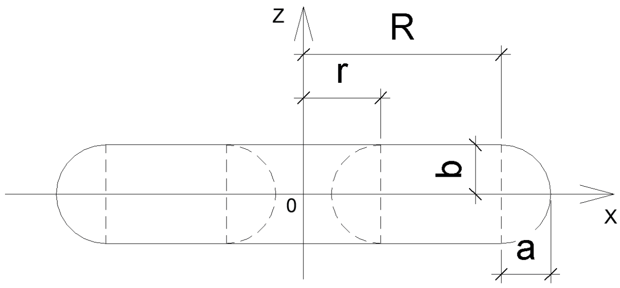


Figure 1b. Vibration isolator with a hole after loading



<u>Figure 2.</u> Diagram of geometrical characteristics of the vibration isolator

$$\frac{(x-R)^2}{a^2}+\frac{z^2}{b^2}=1 \quad for\; R \le x \le R+a,$$
$$z=\pm b \quad for -R < x < -rUr < x < r,$$
$$\frac{(x-r)^2}{a^2}+\frac{z^2}{b^2}=1 \quad for\; r-a \le x \le r, \quad (14)$$
$$\frac{(x+r)^2}{a^2}+\frac{z^2}{b^2}=1 \quad for -r \le x \le -r+a,$$
$$\frac{(x+R)^2}{a^2}+\frac{z^2}{b^2}=1 \quad for -R-a \le x \le -R,$$

In the present paper, a vibration isolator with one central hole is considered. For further studies, the authors aim to derive formulas for a rectangular isolator with multiple holes.

The volume enclosed in the outer elliptic curve is represented as V_{out}, and the volume inside the elliptic curve is represented as V_{in}. Considering the right half, the following can be derived:

$$V=\pi(R^2-r^2)H = Vout - Vin =$$
$$=2\int_0^b \pi({x_1}^2-{x_2}^2)dz =$$
$$=2\pi\int_0^b \left[R+\frac{a}{b}\sqrt{b^2-z^2}\right]^2 -$$
$$-\left[r-\frac{a}{b}\sqrt{b^2-z^2}\right]^2 dz = \quad (15)$$
$$=2\pi(R^2b+\frac{2}{3}a^2b+\frac{1}{2}ab\pi R)-2\pi(r^2b+$$
$$+\frac{2}{3}a^2b-\frac{1}{2}ab\pi r)=$$
$$=2\pi b(R^2-r^2)+a\pi b(R+r),$$

Thus, we have a:

$$a=\frac{(R-r)(H-2b)}{b}, \quad (16)$$

Since b satisfies

$$b=\frac{H-h_{def}}{2}, \quad (17)$$

then

$$a=\frac{(R-r)h_{def}}{H-h_{def}}, \quad (18)$$

The free area of the vibration isolator, denoted as A_{f2}, can be calculated as follows:

$$A_{f2}=4\pi\int_0^b \pi(x_1-x_2)dz = 4\pi\int_0^b \left[R+\frac{a}{b}\sqrt{b^2-z^2}\right]-\left[r-\frac{a}{b}\sqrt{b^2-z^2}\right]dz =$$
$$=4b\pi R+ab\pi^2+(4b\pi r-ab\pi^2)=$$
$$4b\pi(R+r)=2\pi(R+r)\big(H-h_{def}\big), \quad (19)$$

Considering the free area in the cases of the elliptic deformation hypothesis, it can be seen that:

$$A_{f2}=A_{f1}=2\pi(R+r)\big(H-h_{def}\big), \quad (20)$$

Formula (20) is based on the assumption that the internal and external deformations are of the same elliptic shape. If this assumption is not fulfilled, then $A_{f2} \neq A_{f1}$. However, the difference between A_{f2} and A_{f1} is always extremely small.

In practice, the stiffness of rubber increases as the load increases. This is because an increase in load leads to a change in both the geometric shape and nonlinearity of the material. When the rubber element is compressed, the sample becomes anisotropic since the molecular chain is more oriented in the transverse direction.

As the compression ratio of the vibration isolator increases, the greater the degree of rubber bulging and the greater the change in rubber properties. This change can be expressed by the convexity factor μ_{bul}, which can be defined as follows:

$$\mu_{bul} = 1 + \frac{a}{b}\ , \qquad (21)$$

where a and b are accepted in accordance with (16) and (17).

Then the stiffness of the vibration isolator, K_2, taking into account the convexity coefficient, is defined as follows:

$$K_1 = \frac{A_{c1}\left(1 + 2{S_2}^2\right)E}{\mathrm{H} - h_{def}}\mu_2 = \\ = \frac{\pi(\mathrm{R}^2 - \mathrm{r}^2)\mathrm{E}}{\mathrm{H} - h_{def}} \cdot \left[1 + \frac{(\mathrm{R}^2 - \mathrm{r}^2)}{8\mathrm{b}^2(\mathrm{R} + \mathrm{r})^2}\right]\left(1 + \frac{a}{b}\right) = \\ = \frac{\pi(\mathrm{R}^2 - \mathrm{r}^2)\mathrm{E}}{\mathrm{H} - h_{def}} \cdot \\ \cdot \left[1 + \frac{(\mathrm{R}^2 - \mathrm{r}^2)}{2\left(\mathrm{H} - h_{def}\right)^2}\right]\left(1 + \frac{a}{b}\right), \qquad (22)$$

The applied force F_2 is determined by the formula:

$$F_{c2} = \int_0^{h_{def}} K_c dz = \int_0^{h_{def}} \frac{\pi(\mathrm{R}^2 - \mathrm{r}^2)\mathrm{E}}{\mathrm{H} - \mathrm{z}} \cdot \\ \cdot \left[1 + \frac{(\mathrm{R}^2 - \mathrm{r}^2)}{2(\mathrm{H} - \mathrm{z})^2}\right]\left(1 + \frac{a}{b}\right) dz, \qquad (23)$$

where $h_{def} = z$

During operation, the temperature of the vibration isolators may change, usually between -10°C and +10°C. If the temperature changes, the volume of the rubber will change due to thermal expansion and contraction.

Considering the room temperature TK as the standard condition, when the temperature change is ΔT, the volume of the round rubber part expands to

$$(1 + \alpha\Delta\mathrm{T})^3 3\pi(\mathrm{R}^2 - \mathrm{r}^2), \qquad (24)$$

and the height is as follows

$$(1 + \alpha\Delta\mathrm{T})H, \qquad (25)$$

However, because of the restraint imposed by the steel plate, the bearing surface is assumed to remain unchanged since the thermal expansion of steel is much lower than that of rubber. The lateral changes are limited to a certain extent. If the temperature increases, the rubber will protrude outward. Otherwise it will become concave inward.

Then equation (15) can be modified as follows:

$$(1 + \alpha\Delta T)^3 V = (1 + \alpha\Delta T)^3 3\pi(R^2 - r^2)H = \\ = Vout - Vin = 2\int_0^{\mathrm{b}} \pi({x_1}^2 - {x_2}^2)dz = \\ = 2\pi b(\mathrm{R}^2 - \mathrm{r}^2) + \mathrm{a}\pi b(R + r), \qquad (26)$$

From the equation (26) we can conclude that

$$a = \frac{(R - r)[(1 + \alpha\Delta\mathrm{T})^3 H - 2b)]}{b}, \qquad (27)$$

where

$$b = \frac{(1 + \alpha\Delta\mathrm{T})\mathrm{H} - h_{def}}{2}, \qquad (28)$$

then

$$A_{f2} = 2\pi(R + r)\left[(1 + \alpha\Delta\mathrm{T})\mathrm{H} - h_{def}\right], \quad (29)$$

Form factor μ_2 is defined as follows

$$\mu_2 = 1 + 2{S_2}^2 = 1 + 2\left(\frac{A_c}{A_{f2}}\right)^2 = \\ 1 + \frac{(\mathrm{R} - \mathrm{r})^2}{2\left((1 + \alpha\Delta\mathrm{T})\mathrm{H} - h_{def}\right)^2}, \qquad (30)$$

As the temperature changes, equations (22) and (23) can be further modified as follows:

$$F_2 = \int_0^{h_{def}} K_{c2} dz = \\ \int_0^{h_{def}} \frac{\pi(\mathrm{R}^2 - \mathrm{r}^2)\mathrm{E}}{(1 + \alpha\Delta\mathrm{T})\mathrm{H} - \mathrm{z}} \cdot \\ \cdot \left[1 + \frac{(\mathrm{R}^2 - \mathrm{r}^2)}{2\left((1 + \alpha\Delta\mathrm{T})\mathrm{H} - \mathrm{z}\right)^2}\right]\left(1 + \frac{a}{b}\right) dz$$

$$K_2 = \frac{A_c\left(1+2{S_2}^2\right)E}{(1+\alpha\Delta\mathrm{T})\mathrm{H}-h_{def}}\mu_2 = \\ = \frac{\pi(\mathrm{R}^2-\mathrm{r}^2)\mathrm{E}}{(1+\alpha\Delta\mathrm{T})\mathrm{H}-h_{def}} \cdot \\ \left[1+\frac{(\mathrm{R}^2-\mathrm{r}^2)}{2\left((1+\alpha\Delta\mathrm{T})\mathrm{H}-h_{def}\right)^2}\right]\left(1+\frac{a}{b}\right), \quad (31)$$

4. RESULTS AND DISCUSSION

At high temperatures, the degree of rubber crosslinking gradually increases with temperature, resulting in an increase in the hardness and elastic modulus of the rubber. If the temperature increases even further, the polymer molecular chain will fracture. There is no need to consider the role of material degradation, as the range of engineering applications is usually less than 60 °C. When the rubber temperature is lower, the molecular activity is weakened and the rubber crystallizes. This condition leads to an increase in the elastic modulus of the rubber at low temperatures.

When the rubber is above 0 °C, the crystallization effect is significantly weaker.

The elastic modulus and stiffness of the rubber increase significantly with decreasing temperature when the temperature is below 0 °C. Therefore, in some theoretical studies, 0 °C is chosen as the transition point. In this study, it is assumed that at temperatures above 0 °C, the modulus of elasticity changes according to equation (6), and at temperatures below 0 °C, the modulus of elasticity has a certain dependence on the ambient temperature.

To account for temperature as a variable, a coefficient φ is adopted. This coefficient is the ratio of the modulus of elasticity of rubber at low temperature to the modulus of elasticity at 0°C , which can be expressed as follows:

$$\varphi = \frac{E}{E_0}, \quad (32)$$

$$\Delta T_1 = T - T_0, \quad (32.1)$$

where E is the elasticity modulus at 0°C, and T0 is the temperature at 0°C.

Thus, we get:

$$E = \frac{TE_k}{(1+3\alpha\Delta\mathrm{T})T_k} \quad for\ T \geq 273\ K, \\ E = \frac{T_0 E_k}{(1+75\alpha)T_k} \quad for\ T < 273\ K, \quad (33)$$

where $(1+75\alpha)$ have been previously derived in the study [27].

Consider the vibration isolator of the following sizes with and without hole.

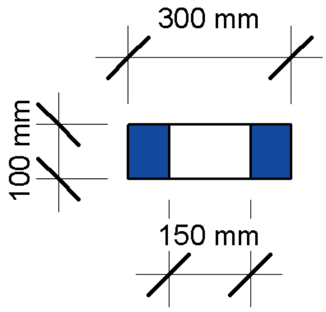


Figure 3a. Adopted vibration isolator with an aperture

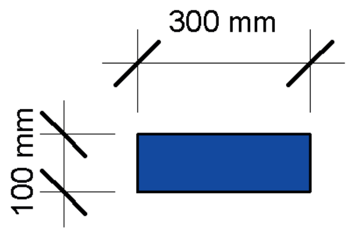


Figure 3b. Adopted vibration isolator without hole

The modulus of elasticity E is 2.28 MPa at room temperature [28].

For the isolator under consideration, the coefficient of thermal expansion is as follows

$$\alpha = 6.6 \text{ x } 10^{-4} K^{-1}, \quad (34)$$

Using equation (33), the modulus of elasticity E can be calculated at different temperatures, which is shown in the diagrams below.

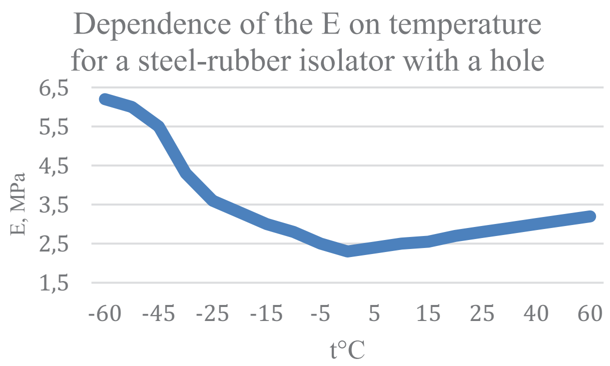


Figure 4a. Plot of elastic modulus versus temperature for a rubber-metal isolator

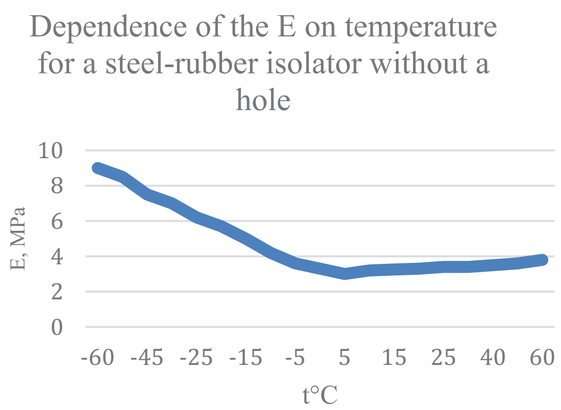


Figure 4b. Plot of elastic modulus versus temperature for a rubber-metal isolator without a hole

These figures show that at temperatures above 0°C, the modulus of elasticity E increases slowly with increasing temperature, and when the temperature drops below 0°C, the modulus of elasticity E increases rapidly with decreasing temperature. Although there are some uncertainties, the main trend of this curve reflects the theoretical analysis and the results of tests conducted by other authors [29].
These plots show the variation of stiffness obtained from formula (31) with convexity factor, as well as formulas (11) and (12) without convexity factor at T=25°C under different load.

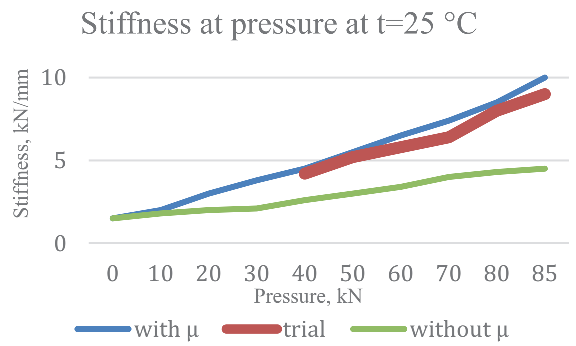


Figure 5a. Stiffness of the vibration isolator with a hole under different load at t=25C

Figure 5a shows that the difference between the original formula (11) and the formula with convexity factor (31) becomes more significant with increasing pressure. The maximum error is about 10.86% and the RMS error is 7.71%. The error increases with increase in pressure, but the accuracy is higher at low pressure. From the above analysis, it can be noted that the formula with convexity coefficient is effective and accurate in reflecting the change in stiffness caused by load variation. The experimental data shown in Fig.5 and Fig.5b are taken from [27] and [29].

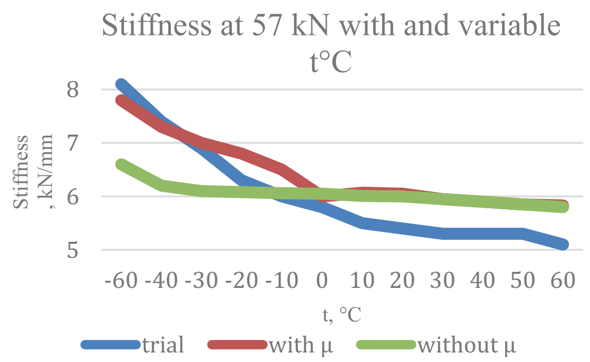


Figure 5b. Stiffness of vibration isolator with hole at different t under 57kN load

Figure 6 shows that formula (31) with the convexity factor is more accurate between -60°C and +60°C. If the temperature is above 0°C, the calculation result obtained using the modified formula (31) is close to the result obtained using the original (12) empirical formula. The maximum error with the modified formula (31) is

12% and the RMS error is 9%. Although the error gradually increases when the temperature is above 60°C, it is usually ignored. If the temperature is below 0°C, the maximum error with formula (31) is 5% and the RMS error is 3%. The maximum error with empirical formula (12) is 16% and the RMS error is 10%. From the above analysis, it can be seen that formula (31) can reflect the change in stiffness depending on temperature [28].

The stiffness of the vibration isolator changes as the pressure or compression amplitude changes. This is due to the nonlinearity of the stiffness. In particular, the stiffness decreases slightly with increasing temperature when the temperature is above 0°C and the preload is below 45 kN. The stiffness increases slightly with increasing temperature when the temperature is above 0°C and the preload is above 45 kN. This is mainly because the temperature affects not only the elastic modulus but also the geometric nonlinearity of the vibration isolator. When the temperature is below 0°C, changes in pressure have little effect on the stiffness trend. The stiffness hardly changes with temperature at the same load when the temperature is above 0°C. If the temperature is below 0°C, the greater the load, the more dramatic the change of stiffness with temperature [30, 31].

5. CONCLUSIONS

This study has examined the relationship between modulus of elasticity and rubber temperature. At low temperature, a more efficient formula for determining the modulus of elasticity (33) is proposed. The formula (31) with the convexity coefficient μ_{con} for more accurate calculation of the stiffness and modulus of elasticity of rubber vibration isolator is obtained.

There is a strong relationship between the modulus of elasticity and the temperature of the rubber. If the temperature is above 0°C, the modulus of elasticity gradually increases with increasing temperature. While the temperature is below 0°C, the modulus of elasticity increases significantly with decreasing temperature due to the effect of partial crystallization. Therefore, it is effective to use variable modulus of elasticity in calculations.

Thus, formula (31) with the correction by the convexity coefficient μ_{bul} has a good applied value. Formula (31) can also be applied to rubber vibration isolators of other sizes and types.

The modified formula (31) with convexity coefficient μ_{con} takes into account the effect of temperature and load simultaneously. The calculated values are close to the actual ones, and the calculation error under normal operating conditions is less than 10%. Therefore, it can be considered that formula (31) with convexity coefficient μ_{con} is effective for stiffness calculation. Consideration of the influence of temperature in formula (31) significantly increased the accuracy of the results compared to the original formula (11).

Different loads have the same effect on the stiffness trend at low temperature, and the stiffness increases significantly with decreasing temperature. However, the stiffness is less temperature dependent when the temperature is above 0°C, but still increases slightly with increasing temperature.

Theoretically, the elastic modulus increases with increasing temperature as can be seen from (6) and Fig. 4a. However, in most cases, the measured stiffness decreases with increasing temperature Fig. 6. This seemingly contradictory phenomenon is explained in the derived equation (31).

The theory of the static stiffness of rubber vibration isolator is of great importance in the study of the dynamic performance of rubber and provides a method for further derivation of the dynamic stiffness formula. The dynamic properties of rubber under temperature and pre-pressure will be given more attention in future research.

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Mondrus Vladimir Lvovich – Doctor of Technical Sciences, Professor, Corresponding Member. RAASN, Professor of the Department of Structural and Theoretical Mechanics, Institute of Industrial and Civil Engineering, National Research Moscow State University of Civil Engineering (NRU MGSU). 405, 26 Yaroslavskoe highway, Moscow, 129337, Russia. E-mail: mondeusvl@mgsu.ru

Sizov Dmitry Konstantinovich – Ph.D., Vibroseysmozashchita LLC,6 st. Bratislavskaya, Moscow, 109341, Russia. E-mail: newfff@mail.ru

Kvasnikov Timofei Mikhailovich - 3rd year graduate student of the Department of Structural and Theoretical Mechanics, Institute of Industrial and Civil Engineering, National Research Moscow State University of Civil Engi-neering (NRU MGSU). 405, 26 Yaroslavskoe shosse, Moscow, 129337, Russia. E-mail: ti-ma.kvasnikov@gmail.com

Мондрус Владимир Львович – д.т.н., профессор, член корр. РААСН, профессор кафедры строительной и теоретической механики, институт Промышленного и гражданского строительства, Национальный Исследовательский Московский Государственный Строительный Университет (НИУ МГСУ). 405, 26 Ярославское шоссе, Москва, 129337, Россия. E-mail: mondeusvl@mgsu.ru

Сизов Дмитрий Константинович – к.т.н., ООО «Вибросейсмозащита», ул. Братиславская, 6, Москва, 109341,Россия. E-mail: newfff@mail.ru,

Квасников Тимофей Михайлович -аспирант 3 курса кафедры строительной и теоретической механики, институт Промышленного и гражданского строительства, Национальный Исследовательский Московский Государственный Стро-ительный Университет (НИУ МГСУ). 405, 26 Ярославское шоссе, Москва, 129337, Россия. E-mail: tima.kvasnikov@gmail.com, https://orcid.org/0000-0002-7836-4918