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A.A. ILYUSHIN'S FINAL RELATION, ALTERNATIVE EQUIVALENT RELATIONS AND VERSIONS OF ITS APPROXIMATION IN PROBLEMS OF ELASTIC DEFORMATION OF PLATES AND SHELLS PART 2: ALTERNATIVE EQUIVALENT RELATIONS OF A.A. ILYUSHIN

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Abstract: The finite relationship between the forces and moments of plates and shells in the parametric form of the theory of small elastoplastic deformations is investigated of A.A. Ilyushin, to determine the load-bearing capacity of structures from a material without hardening. A geometric image of the exact yield surface in the space of generalized stresses is obtained. In the first part of the article the conclusion of the final relation is given. In the second and third parts, by introducing other parameters, alternative equivalent dependences of the final relationship have been developed and variants of its approximation for application in computational practice are considered. In the fourth part, additional properties of the final relationship are considered, the possibility and necessity of its use in problems of plastic deformation of plates and shells is shown.

Keywords: the plasticity theory, plastic deformation of plates and shells, a surface of fluidity, a plasticity condition.

КОНЕЧНОЕ СООТНОШЕНИЕ А.А. ИЛЬЮШИНА, АЛЬТЕРНАТИВНЫЕ ЭКВИВАЛЕНТНЫЕ ЗАВИСИМОСТИ И ВАРИАНТЫ ЕГО АППРОКСИМАЦИИ В ЗАДАЧАХ ПЛАСТИЧЕСКОГО ДЕФОРМИРОВАНИЯ ПЛАСТИН И ОБОЛОЧЕК ЧАСТЬ 2: АЛЬТЕРНАТИВНЫЕ ЭКВИВАЛЕНТНЫЕ ЗАВИСИМОСТИ КОНЕЧНОГО СООТНОШЕНИЯ А.А. ИЛЬЮШИНА

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Аннотация: Выполнено исследование конечного соотношения между силами и моментами пластин и оболочек в параметрическом виде теории малых упругопластических деформаций А.А. Ильюшина, для определения несущей способности конструкций из материала без упрочнения. Получен геометрический образ точной поверхности текучести в пространстве обобщенных напряжений. В первой части статьи приводится вывод конечного соотношения. Во второй и третьей частях введением других параметров разработаны альтернативные эквивалентные зависимости конечного соотношения и рассмотрены варианты его аппроксимации для применения в расчетной практике. В четвертой части рассмотрены дополнительные свойства конечного соотношения, показана возможность и необходимость его использования в задачах пластического деформирования пластин и оболочек.

Ключевые слова: теория пластичности, пластическое деформирование пластин и оболочек, поверхность текучести, условия пластичности.

2.1. Alternative equivalent relations of a final relation

In the work [9], in integrating the integrals (4.25), integration over the intensity of the deformations e_i is performed instead of integrating over the coordinate z. Let us show that we can obtain an alternative finite relation by calculating the integrals (4.25) with respect to the coordinate z, and compare the results of the calculations.

Intensity of deformations, according to (4.7) [9]:

$$e_i = \frac{2}{\sqrt{3}}\sqrt{P_\varepsilon - 2zP_{\varepsilon\chi} + z^2 P_\chi},$$
$$P_\varepsilon = \varepsilon_1^2 + \varepsilon_1\varepsilon_2 + \varepsilon_2^2 + \varepsilon_{12}^2,\ P_\chi = \chi_1^2 + \chi_1\chi_2 + \chi_2^2 + \chi_{12}^2,$$
$$P_{\varepsilon\chi} = \varepsilon_1\chi_1 + \varepsilon_2\chi_2 + \frac{1}{2}\varepsilon_1\chi_2 + \frac{1}{2}\varepsilon_2\chi_1 + \varepsilon_{11}\chi_{12}. \qquad (2.1)$$

Let's consider values of intensity of deformations in three points disposed on an axis z $z = -\frac{h}{2},\ z = +\frac{h}{2},\ z = 0.$ Let's designate them accordingly:

$$e_{i1} = \frac{2}{\sqrt{3}}\sqrt{P_\varepsilon + hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} \quad \left(z = -\frac{h}{2}\right),$$
$$e_{i2} = \frac{2}{\sqrt{3}}\sqrt{P_\varepsilon - hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} \quad \left(z = +\frac{h}{2}\right), \qquad (2.2)$$
$$e_{i0} = \frac{2}{\sqrt{3}}\sqrt{P_\varepsilon} \quad (z = 0).$$

Considering the last as the equations concerning three quadratic forms P_χ, $P_{\varepsilon\chi}$, P_ε, we copy them in a kind:

$$P_\varepsilon + hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi = \frac{3}{4}e_{i1}^2,$$
$$P_\varepsilon - hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi = \frac{3}{4}e_{i2}^2,\ P_\varepsilon = \frac{3}{4}e_{i0}^2. \qquad (2.3)$$

Solving them with respect to quadratic forms leads to the following results:

$$P_\varepsilon = \frac{3}{4}e_{i0}^2,\ hP_{\varepsilon\chi} = \frac{3}{8}\left(e_{i1}^2 - e_{i2}^2\right),$$
$$\frac{h^2}{4}P_\chi = \frac{3}{16}\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right). \qquad (2.4)$$

We introduce two basic parameters λ and μ:

$$\lambda = \frac{e_{i2}}{e_{i1}},\ \mu = \frac{e_{i0}}{e_{i1}}. \qquad (2.5)$$

These parametres satisfy to conditions: $0 \le \lambda \le 1$, $0 \le \mu \le 1$ as e_{i1} – there is a maximum value of intensity of deformations, if $P_{\varepsilon\chi} < 0$. Then formulas (2.3) can be copied in a kind:

$$P_\varepsilon = \frac{3}{4}\mu^2 e_{i1}^2,\ hP_{\varepsilon\chi} = \frac{3}{8}\left(1-\lambda^2\right)e_{i1}^2,$$
$$\frac{h^2}{4}P_\chi = \frac{3}{16}\left(2 + 2\lambda^2 - 4\mu^2\right)e_{i1}^2. \qquad (2.6)$$

In formulas (4.23')-(4.24') [9], there are three types of integrals that are common in shell thickness:

$$J_1 = \frac{\sqrt{3}}{2}\sigma_s \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{dz}{X^{\frac{1}{2}}},\ J_2 = \frac{\sqrt{3}}{2}\sigma_s \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{zdz}{X^{\frac{1}{2}}},$$
$$J_3 = \frac{\sqrt{3}}{2}\sigma_s \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{z^2dz}{X^{\frac{1}{2}}},\ X^{\frac{1}{2}} = \sqrt{c + bz + az^2}, \qquad (2.7)$$
$$c = P_\varepsilon,\ b = -2P_{\varepsilon\chi},\ a = P_\chi.$$

These integrals tabular. According to formulas (380.001, 380.011, 380.021) [42]

$$J_1 = \frac{\sqrt{3}}{2}\sigma_s \left[\frac{1}{\sqrt{a}}\ln\left|2\sqrt{a}X^{\frac{1}{2}} + 2az + b\right|\right]_{-\frac{h}{2}}^{\frac{h}{2}},$$
$$J_2 = \frac{\sqrt{3}}{2}\sigma_s \left[\frac{X^{\frac{1}{2}}}{a} - \frac{b}{2a}\int\frac{dz}{X^{\frac{1}{2}}}\right]_{-\frac{h}{2}}^{\frac{h}{2}},$$
$$J_3 = \frac{\sqrt{3}}{2}\sigma_s \left[\begin{array}{l}\left(\frac{z}{2a} - \frac{3b}{4a^2}\right)X^{\frac{1}{2}} + \\ +\left(\frac{3b^2 - 4ac}{8a^2}\right)\int\frac{dz}{X^{\frac{1}{2}}}\end{array}\right]_{-\frac{h}{2}}^{\frac{h}{2}}, \qquad (2.8)$$
$$X^{\frac{1}{2}} = \sqrt{c + bz + az^2},\ c = P_\varepsilon,\ b = -2P_{\varepsilon\chi},\ a = P_\chi.$$

As well as in [9], we will consider that tensile deformation and shift of a middle surface ε_1, ε_2, ε_{12} are commensurable or small compared with bending strains of a shell $\pm\frac{h}{2}\chi_1,\ \pm\frac{h}{2}\chi_2,\ \pm\frac{h}{2}\chi_{12}$ or that the last are dominating if the point z_0 (minimum) e_i does not fall outside the limits a thickness of a shell, i.e. if $-\frac{h}{2} \le z_0 = \frac{P_{\varepsilon\chi}}{P_\chi} \le \frac{h}{2}$.

Deformations of a middle surface we will name large or dominating compared with bending strains if the point z_0 is disposed out of a thickness of a shell i.e. if one of inequalities takes place $z_0 = \frac{P_{\varepsilon\chi}}{P_\chi} > \frac{h}{2}$, $z_0 = \frac{P_{\varepsilon\chi}}{P_\chi} < -\frac{h}{2}$.

Taking into account (2.8) also it is possible to express an integral J_3 through integrals J_2 and J_1:

$$J_3 = \frac{\sqrt{3}}{2}\sigma_s \left[\begin{array}{l} \frac{z}{2a}X^{\frac{1}{2}} - \frac{3b}{4a}\left(\frac{X^{\frac{1}{2}}}{a} - \frac{b}{2a}\int \frac{dz}{X^{\frac{1}{2}}} \right) - \\ -\frac{c}{2a}\int \frac{dz}{X^{\frac{1}{2}}} \end{array} \right]_{-\frac{h}{2}}^{\frac{h}{2}},$$

$$J_3 = \frac{\sqrt{3}}{2}\sigma_s \left[\frac{z}{2a}X^{\frac{1}{2}} \right]_{-\frac{h}{2}}^{\frac{h}{2}} - \frac{3b}{4a}J_2 - \frac{c}{2a}J_1, \tag{2.9}$$

$$X^{\frac{1}{2}} = \sqrt{c + bz + az^2},\ c = P_\varepsilon,\ b = -2P_{\varepsilon\chi},\ a = P_\chi.$$

Corresponding integrals according to (2.8)-(2.9):

$$J_1 = \frac{\sqrt{3}\sigma_s}{2} \cdot \frac{1}{\sqrt{P_\chi}} \times \\ \times \ln \frac{2\sqrt{P_\chi} \cdot \sqrt{P_\varepsilon - hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} + hP_\chi - 2P_{\varepsilon\chi}}{2\sqrt{P_\chi} \cdot \sqrt{P_\varepsilon + hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} - hP_\chi - 2P_{\varepsilon\chi}}, \tag{2.10}$$

$$J_2 = \frac{\sqrt{3}\sigma_s}{2} \cdot \frac{1}{P_\chi} \cdot \left(\begin{array}{l} \sqrt{P_\varepsilon - hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} - \\ -\sqrt{P_\varepsilon + hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} \end{array} \right) + \frac{P_{\varepsilon\chi}}{P_\chi}J_1, \tag{2.11}$$

$$J_3 = \frac{\sqrt{3}\sigma_s}{2} \cdot \frac{1}{4P_\chi^2} \times \\ \times \left(\begin{array}{l} \left(hP_\chi + 6P_{\varepsilon\chi}\right)\sqrt{P_\varepsilon - hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} + \\ +\left(hP_\chi - 6P_{\varepsilon\chi}\right)\sqrt{P_\varepsilon + hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} \end{array} \right) + \frac{3P_{\varepsilon\chi}^2 - P_\varepsilon P_\chi}{P_\chi^2}J_1. \tag{2.12}$$

Taking into account (2.9) also it is possible to present an integral J_3 in a kind

$$J_3 = \frac{\sqrt{3}\sigma_s h}{2} \cdot \frac{1}{4P_\chi} \cdot \left(\begin{array}{l} \sqrt{P_\varepsilon - hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} + \\ +\sqrt{P_\varepsilon + hP_{\varepsilon\chi} + \frac{h^2}{4}P_\chi} \end{array} \right) + \\ + \frac{3}{2}\frac{P_{\varepsilon\chi}}{P_\chi}J_2 - \frac{1}{2}\frac{P_\varepsilon}{P_\chi}J_1. \tag{2.13}$$

At change of a sign $P_{\varepsilon\chi}$ integrals according to (2.10)–(2.13) $J_1 = J_1$, $J_2 = -J_2$, $J_3 = J_3$. If $P_\varepsilon \to 0$ $J_1 \to \infty$, $J_2 \to 0$, $J_3 \to \frac{\sqrt{3}\sigma_s}{2} \cdot \frac{h^2}{4\sqrt{P_\chi}}$.

Intensity of deformations (2.1) taking into account (2.4) becomes

$$e_i = \sqrt{e_{i0}^2 - \frac{z}{h}\left(e_{i1}^2 - e_{i2}^2\right) + \frac{z^2}{h^2}\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)}. \tag{2.14}$$

According to (2.14) integrals in formulas (4.23')-(4.24') [9]:

$$J_1 = \sigma_s \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{dz}{X^{\frac{1}{2}}},\ J_2 = \sigma_s \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{zdz}{X^{\frac{1}{2}}},\ J_3 = \sigma_s \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{z^2dz}{X^{\frac{1}{2}}},$$

$$X^{\frac{1}{2}} = \sqrt{c + bz + az^2},\ c = e_{i0}^2,\ b = -\frac{1}{h}\left(e_{i1}^2 - e_{i2}^2\right),$$

$$a = \frac{1}{h^2}\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right). \tag{2.15}$$

Corresponding integrals according to (2.8)–(2.9) which can be received also substitution (2.4) in (2.10)–(2.13):

$$J_1 = \frac{\sigma_s h}{\sqrt{2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2}} \times \\ \times \ln \frac{2e_{i2}\sqrt{2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2} + \left(e_{i1}^2 + 3e_{i2}^2 - 4e_{i0}^2\right)}{2e_{i1}\sqrt{2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2} - \left(3e_{i1}^2 + e_{i2}^2 - 4e_{i0}^2\right)}, \tag{2.16}$$

$$J_2 = \frac{\sigma_s h^2\left(e_{i2} - e_{i1}\right)}{\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)} + \frac{h\left(e_{i1}^2 - e_{i2}^2\right)}{2\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)}J_1, \tag{2.17}$$

$$J_3 = \frac{\sigma_s h^3\left(e_{i1} + e_{i2}\right)}{4\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)} - \\ - \frac{3\sigma_s h^3\left(e_{i1}^2 - e_{i2}^2\right)\left(e_{i1} - e_{i2}\right)}{4\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)^2} + \\ + \frac{h^2\left[3\left(e_{i1}^2 + e_{i2}^2\right)^2 - 4e_{i0}^2\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)\right]}{8\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)^2}J_1. \tag{2.18}$$

Taking into account (2.9) also it is possible to present an integral J_3 in a kind

$$J_3 = \frac{\sigma_s h^3 \left(e_{i1} + e_{i2}\right)}{4\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)} + \frac{3h\left(e_{i1}^2 - e_{i2}^2\right) J_2}{4\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)} - \frac{h^2 e_{i0}^2}{2\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)} J_1. \tag{2.19}$$

Taking into account (2.5) formulas (2.16)-(2.19) become:

$$J_1 = \frac{\sigma_s h}{e_{i1}\sqrt{2+2\lambda^2-4\mu^2}} \times \ln\frac{2\lambda\sqrt{2+2\lambda^2-4\mu^2} + \left(1+3\lambda^2-4\mu^2\right)}{2\sqrt{2+2\lambda^2-4\mu^2} - \left(3+\lambda^2-4\mu^2\right)}, \tag{2.20}$$

$$J_2 = \frac{\sigma_s h^2 (\lambda - 1)}{e_{i1}\left(2+2\lambda^2-4\mu^2\right)} + \frac{h\left(1-\lambda^2\right)}{2\left(2+2\lambda^2-4\mu^2\right)} J_1, \tag{2.21}$$

$$J_3 = \frac{\sigma_s h^3 (1+\lambda)}{4e_{i1}\left(2+2\lambda^2-4\mu^2\right)} - \frac{3\sigma_s h^3\left(1-\lambda^2\right)}{4e_{i1}\left(2+2\lambda^2-4\mu^2\right)^2} + \frac{h^2\left[3\left(1+\lambda^2\right)^2 - 4\mu^2\left(2+2\lambda^2-4\mu^2\right)\right]}{8\left(2+2\lambda^2-4\mu^2\right)^2} J_1, \tag{2.22}$$

$$J_3 = \frac{\sigma_s h^3 (1+\lambda)}{4e_{i1}\left(2+2\lambda^2-4\mu^2\right)} + \frac{3h\left(1-\lambda^2\right)}{4\left(2+2\lambda^2-4\mu^2\right)} J_2 - \frac{h^2\mu^2}{2\left(2+2\lambda^2-4\mu^2\right)} J_1. \tag{2.23}$$

Formulas (4.44) taking into account (4.66)-(4.68) [9]

$$P_S = \frac{3}{4}\cdot\frac{\sigma_s^2 h^2}{1}\left(n_1^2 - n_1 n_2 + n_2^2 + 3n_{12}^2\right) = \frac{3}{4}\cdot\frac{\sigma_s^2 h^2}{1} Q_n,$$
$$P_H = \frac{3}{4}\cdot\frac{\sigma_s^2 h^4}{16}\left(m_1^2 - m_1 m_2 + m_2^2 + 3m_{12}^2\right) = \frac{3}{4}\cdot\frac{\sigma_s^2 h^4}{16} Q_m, \tag{2.24}$$
$$P_{SH} = \frac{3}{4}\cdot\frac{\sigma_s^2 h^3}{4}\begin{pmatrix} n_1 m_1 + n_2 m_2 - \frac{1}{2} n_1 m_2 - \\ -\frac{1}{2} n_2 m_1 + 3n_{12} m_{12} \end{pmatrix} = \frac{3}{4}\cdot\frac{\sigma_s^2 h^3}{4} Q_{nm},$$

where

$$Q_n == \frac{4}{3}\cdot\frac{1}{\sigma_s^2 h^2} P_S, \quad Q_m = \frac{4}{3}\cdot\frac{16}{\sigma_s^2 h^4} P_H, \quad Q_{nm} = \frac{4}{3}\cdot\frac{4}{\sigma_s^2 h^3} P_{SH}. \tag{2.25}$$

From here with the account (4.45'), (4.45''), (4.45''') [9] we receive a required final relation:

$$Q_n = \frac{4}{3}\cdot\frac{1}{\sigma_s^2 h^2} P_S = \frac{4}{3}\cdot\frac{1}{\sigma_s^2 h^2}\left[J_1^2 P_\varepsilon - 2J_1 J_2 P_{\varepsilon\chi} + J_2^2 P_\chi\right],$$
$$Q_{nm} = \frac{4}{3}\cdot\frac{4}{\sigma_s^2 h^3} P_{SH} = \frac{4}{3}\cdot\frac{4}{\sigma_s^2 h^3}\left[J_1 J_2 P_\varepsilon - \left(J_1 J_3 + J_2^2\right) P_{\varepsilon\chi} + J_2 J_3 P_\chi\right],$$
$$Q_m = \frac{4}{3}\cdot\frac{16}{\sigma_s^2 h^4} P_H = \frac{4}{3}\cdot\frac{16}{\sigma_s^2 h^4}\left[J_2^2 P_\varepsilon - 2J_2 J_3 P_{\varepsilon\chi} + J_3^2 P_\chi\right]. \tag{2.26}$$

As in A.A. Ilyushin's theory e_{i0} – the minimum value of intensity of deformations e_i at $z = z_0$, and in offered model e_{i0} – value of intensity of deformations e_i at $z = 0$ also have different physical sense, we will designate these parametres as follows:

$$e_i\big|_{z=z_0} = e_{i0,\min}, \quad \mu_{\min} = \frac{e_{i0,\min}}{e_{i1}}, \quad e_i\big|_{z=0} = e_{i0}, \quad \mu = \frac{e_{i0}}{e_{i1}}.$$

The relationship between these parameters is obtained from (4.34) [9]

$$e_{i0,\min}^2 = \frac{2}{\sqrt{3}}\sqrt{P_\varepsilon - \frac{P_{\varepsilon\chi}^2}{P_\chi}}, \tag{2.27}$$

Where P_ε, $P_{\varepsilon\chi}$, P_χ according to (2.4):

$$e_{i0,\min}^2 = e_{i0}^2 - \frac{\left(e_{i1}^2 - e_{i2}^2\right)^2}{4\left(2e_{i1}^2 + 2e_{i2}^2 - 4e_{i0}^2\right)}, \tag{2.28}$$

$$\mu_{\min}^2 = \mu^2 - \frac{\left(1-\lambda^2\right)^2}{4\left(2+2\lambda^2-4\mu^2\right)}. \tag{2.29}$$

Deciding biquadratic the equations (2.28)-(2.29), we find

$$e_{i0}^2 = \frac{1}{4}\begin{pmatrix} e_{i1}^2 + e_{i2}^2 + 2e_{i0,\min}^2 \mp \\ \mp 2\sqrt{e_{i1}^2 - e_{i0,\min}^2\left(e_{i1}^2 + e_{i2}^2\right) + e_{i0,\min}^4} \end{pmatrix}, \tag{2.30}$$

$$\mu^2 = \frac{1}{4}\left(\begin{array}{l} 1+\lambda^2+2\mu_{\min}^2 \mp \\ \mp 2\sqrt{\lambda^2-\mu_{\min}^2\left(1+\lambda^2\right)+\mu_{\min}^4} \end{array}\right). \quad (2.31)$$

In formulas (2.30)-(2.31) upper sign (–) concerns to a case of a dominating bending of a shell, and lower (+) to a case of a dominating stretching – compression. Analysing (2.29), (2.31), we find limits of change of parametres λ, μ_{min}, μ:

For a dominating bending of a shell:

$$\lambda = 1,\ 0 \le \mu_{\min} \le \lambda,\quad \mu = \mu_{\min};$$

$$\lambda < 1,\ \mu_{\min} = 0,\ \mu = \frac{1-\lambda}{2},\ 0 \le \mu \le \frac{1}{2};$$

$$\lambda < 1,\ \mu_{\min} = \lambda,\ \mu = \frac{\sqrt{1+3\lambda^2}}{2},\ \frac{1}{2} \le \mu < 1;$$

$$\lambda = 0,\ \mu_{\min} = 0,\ \mu = \frac{1}{2}. \quad (2.32)$$

For the dominant extension – compression of the shell:

$$\lambda = 1,\ 0 \le \mu_{\min} \le \lambda,\ \ \mu = 1;$$

$$\lambda < 1,\ \mu_{\min} = 0,\ \mu = \frac{1+\lambda}{2},\ \frac{1}{2} \le \mu \le 1;$$

$$\lambda < 1,\ \mu_{\min} = \lambda,\ \mu = \frac{\sqrt{1+3\lambda^2}}{2},\ \frac{1}{2} \le \mu < 1;$$

$$\lambda = 0,\ \mu_{\min} = 0,\ \mu = \frac{1}{2}. \quad (2.33)$$

Another variant of the relation between the parameters $e_{i0,\min}$, e_{i0}, $\mu_{\min}$, μ is obtained from (4.60) [9] and (2.4)

$$e_{i0}^2 = \frac{1}{4}\left(\begin{array}{l} e_{i1}^2 + e_{i2}^2 + 2e_{i0,\min}^2 \mp \\ \mp 2\sqrt{e_{i1}^2 - e_{i0,\min}^2}\cdot\sqrt{e_{i2}^2 - e_{i0,\min}^2} \end{array}\right), \quad (2.34)$$

$$\mu^2 = \frac{1}{4}\left(1+\lambda^2+2\mu_{\min}^2 \mp 2\sqrt{1-\mu_{\min}^2}\cdot\sqrt{\lambda^2-\mu_{\min}^2}\right). \quad (2.35)$$

In formulas (2.34)-(2.35) upper sign (–) concerns to a case of a dominating bending of a shell, and lower (+) to a case of a dominating stretching – compression. Formulas (2.30), (2.34), (2.31), (2.35) are equivalent. Product of radicals in (2.34)–(2.35) is equal to a radical in (2.30)–(2.31). Limits of change of parametres are naturally identical. Deciding (2.34) and (2.35) rather $e_{i0,\min}$, $\mu_{\min}$, we receive (2.28) and (2.29).

The right parts of system of the equations (2.26) are functions only two parametres λ, μ, in three-dimensional space with variables Q_n, Q_m, Q_{nm} they represent a surface $F(Q_n, Q_m, Q_{nm}) = 0$, and (2.26) is the parametric equation of this surface and coincides with (4.70') [9].

If to enter new functions by analogy with (4.62)-(4.65) [9] after enough bulky transformations of the right parts of the equations (2.26), relation (2.26) can be resulted in a kind (4.70') [9]

$$Q_n = Q_n\left[\begin{array}{l}\Delta_1(\lambda,\ \mu),\ \Delta(\lambda,\ \mu),\ \psi(\lambda,\ \mu), \\ \varphi(\lambda),\ \mu\end{array}\right],$$

$$Q_{nm} = Q_{nm}\left[\begin{array}{l}\Delta_1(\lambda,\ \mu),\ \Delta(\lambda,\ \mu),\ \psi(\lambda,\ \mu), \\ \varphi(\lambda),\ \mu\end{array}\right],$$

$$Q_m = Q_m\left[\begin{array}{l}\Delta_1(\lambda,\ \mu),\ \Delta(\lambda,\ \mu),\ \psi(\lambda,\ \mu), \\ \varphi(\lambda),\ \mu\end{array}\right],$$

$$\Delta_1^2 = 2+2\lambda^2-4\mu^2,\ \Delta = \frac{1-\lambda^2}{\Delta_1},$$

$$\psi = J_1\cdot\Delta_1,\ \varphi = \lambda - 1. \quad (2.36)$$

It is possible to notice that function χ here does not enter, as and in (4.70') [9] it is not independent and is equal $\chi = \frac{(\lambda+1)\Delta_1}{2} - \frac{(\lambda-1)\Delta}{2}$.

Similar transformations are necessary in the absence of high-power computer facilities. Now in it there are no necessities and the right parts of the equations (2.26) are more convenient for calculating directly. Ratio (2.26) and (4.70') [9] are equivalent.

As well as in the work [9] we consider three special cases of a final relation.

1. The momentless tension state occurs if the deformations of the fibers along the thickness of the shell are the same:

$$e_{i1} = e_{i2} = e_{i0} = e_{i0,\min},\ \lambda = \mu = \mu_{\min} = 1.$$

In the formulas (2.31)-(2.35) it is necessary to take the lower sign (+). Expanding the uncertainties in the formulas (2.20)-(2.23) and (2.26), we obtain the Mizes condition (4.71)-(4.71') [9]

In formulas (2.31)-(2.35) it is necessary to take the lower sign (+). Opening uncertainty of formulas (2.20)-(2.23) and (2.26), we receive a condition of Mizes (4.71)-(4.71') [9]

$$Q_m = Q_{nm} = 0,\ Q_n = n_1^2 - n_1 n_2 + n_2^2 + 3n_{12}^2 = 1. \quad (2.37)$$

2. Purely moments the tension takes place in the absence of lengthening of a middle surface. Quadratic forms $P_\varepsilon = P_{\varepsilon\chi} = 0$.

As appears from (4.19) [9], intensity of deformations e_i is even function z and, according to (4.34) [9], (2.2) is had: $e_{i1} = e_{i2}$, $e_{i0} = e_{i0,\min} = 0$, $\lambda = 1$, $\mu = \mu_{\min} = 0$.

In formulas (2.31)-(2.35) it is necessary to take the upper sign (–). Opening uncertainty of formulas (2.20)-(2.23) and (2.26), we receive a condition (4.72)-(4.72') [9]. The final relation (4.70') [9] becomes:

$$Q_n = Q_{nm} = 0,\ Q_m = m_1^2 - m_1 m_2 + m_2^2 + 3m_{12}^2 = 1. \quad (2.38)$$

3. The elementary difficult tension of shells at $P_\chi \neq 0$, $P_\varepsilon \neq 0$ takes place, if the bilinear form

$$P_{\varepsilon\chi} = \varepsilon_1\chi_1 + \varepsilon_2\chi_2 + \frac{1}{2}\varepsilon_2\chi_1 + + \frac{1}{2}\varepsilon_1\chi_2 + \chi_{12}\varepsilon_{12} = 0.$$

Possible versions:

$$P_{\varepsilon\chi} = \chi_1\left(\varepsilon_1 + \frac{1}{2}\varepsilon_2\right) + \chi_2\left(\varepsilon_2 + \frac{1}{2}\varepsilon_1\right) + \chi_{12}\varepsilon_{12} = 0,$$

$$P_{\varepsilon\chi} = \varepsilon_1\left(\chi_1 + \frac{1}{2}\chi_2\right) + \varepsilon_2\left(\chi_2 + \frac{1}{2}\chi_1\right) + \chi_{12}\varepsilon_{12} = 0.$$

It can take place in cases (4.74) [9] and in addition:

$a)\ \chi_1 \neq 0,\ \chi_{12} = \chi_2 = 0,\ \varepsilon_1 + \frac{1}{2}\varepsilon_2 = 0,$

$b)\ \chi_2 \neq 0,\ \chi_{12} = \chi_1 = 0,\ \varepsilon_2 + \frac{1}{2}\varepsilon_1 = 0,$

$c)\ \varepsilon_1 \neq 0,\ \varepsilon_{12} = \varepsilon_2 = 0,\ \chi_1 + \frac{1}{2}\chi_2 = 0,$

$d)\ \varepsilon_2 \neq 0,\ \varepsilon_{12} = \varepsilon_1 = 0,\ \chi_2 + \frac{1}{2}\chi_1 = 0,$

$e)\ \chi_1 = \chi_2,\ \varepsilon_1 = -\varepsilon_2,\ f)\ \chi_1 = -\chi_2,\ \varepsilon_1 = \varepsilon_2.$

From (4.60) [9] – (2.4) it is had: $e_{i1} = e_{i2} > e_{i0} = e_{i0,\min}$, $\lambda = 1$, $\mu = \mu_{\min} < 1$, i.e. dominating bending strain. According to (2.6)

$$P_\varepsilon = \frac{3}{4}\mu^2 e_{i1}^2,\ hP_{\varepsilon\chi} = 0,\ \frac{h^2}{4}P_\chi = \frac{3}{4}\left(1-\mu^2\right)e_{i1}^2. \quad (2.39)$$

Corresponding integrals according to (2.20)-(2.23):

$$J_1 = \frac{\sigma_s h}{2e_{i1}\left(1-\mu^2\right)}\ln\frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}},\ J_2 = 0,$$

$$J_3 = \frac{\sigma_s h^2}{8e_{i1}\left(1-\mu^2\right)}\left(1-\mu^2 J_1\right). \quad (2.40)$$

The final relation (2.26) becomes:

$$Q_n = \frac{\mu^2}{4\left(1-\mu^2\right)}\ln^2\frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}},\ Q_{nm} = 0,$$

$$Q_m = \left(\frac{1}{\sqrt{1-\mu^2}} - \frac{\mu^2}{2\left(1-\mu^2\right)}\ln\frac{1+\sqrt{1-\mu^2}}{1-\sqrt{1-\mu^2}}\right)^2. \quad (2.41)$$

Considering identity (341.01 [42]

$$\frac{1}{a}\ln\frac{a+\sqrt{a^2-x^2}}{x} = \frac{1}{2a}\ln\frac{a+\sqrt{a^2-x^2}}{a-\sqrt{a^2-x^2}},$$

the final relation (2.26) becomes (4.74) [9]:

$$Q_n = \frac{\mu^2}{1-\mu^2}\ln^2\frac{1+\sqrt{1-\mu^2}}{\mu},\ Q_{nm} = 0,$$

$$Q_m = \left(\frac{\mu^2}{1-\mu^2}\ln\frac{1+\sqrt{1-\mu^2}}{\mu} - \frac{1}{\sqrt{1-\mu^2}}\right)^2. \quad (2.41')$$

In table 2.1 shows the coordinates of points of a curve (2.42) and (4.74) [9] for the elementary difficult tension of a shell are presented.

4. A difficult tension of shells if the bilinear form $P_{\varepsilon\chi}$ submits to a relation $P_{\varepsilon\chi}^2 = P_\varepsilon \cdot P_\chi$.

In case of a dominating stretching of a shell at the lower sign (+) in (2.31) it is had:

$\lambda < 1$, $\mu_{\min} = 0$, $\mu = \frac{1+\lambda}{2}$. Substituting corresponding integrals in (2.26), we receive $Q_n = 1$, $Q_{nm} = Q_m = 0$, i.e. the line $\mu = 0$ degenerates in a point.

In case of a dominating bending the upper sign (–) in (2.31) it is received:

$$\lambda < 1,\ \mu_{\min} = 0,\ \mu = \frac{1-\lambda}{2}.$$

Substituting corresponding integrals in (2.26), we receive (4.79') [9], and excepting parametre λ also (4.77), (4.79), (4.80) [9]

$$Q_n = \left(\frac{1-\lambda}{1+\lambda}\right)^2,\ Q_{nm} = -\frac{4\lambda(1-\lambda)}{(1+\lambda)^3},\ Q_m = \frac{16\lambda^2}{(1+\lambda)^4},$$

$$Q_{nm}^2 = Q_n \cdot Q_m,\ Q_m = \left(1-Q_n\right)^2,\ \left|Q_{nm}\right| = \left(1-Q_n\right)\sqrt{Q_n} \quad (2.42)$$

In table 2.2 coordinates of points of a surface (2.26) and (4.70) [9] on lines $\lambda = const$ for a dominating bending of a shell are presented $\lambda = const$.

In table 2.3 coordinates of points of a surface (2.26) and (4.70) [9] on lines $\lambda = const$ for a dominating stretching – compression are presented $\lambda = const$, in work [9] given table is not presented, is visible that gives small enough quantity of points in a vicinity $Q_n \to 1$ with ordinates $x = y = 0{,}3876$, $z = 0{,}3872$. Tables 2.4 and 2.5 is other form of representation of results of calculation. Table 2.4 corresponds to a dominating bending of a shell, table 2.5 – to a case to a dominating stretching – to compression.

In relation (2.26) integrals (4.25) [9] are calculated under unified (unequivocal) formulas. The account of a dominating bending of a shell and a dominating stretching – compression is executed at level of communication of parametres μ and μ_{min}.

Let us show that a finite relation can be obtained using the parameters of A.A. Ilyushin and calculating the integrals (4.25) [9] along the coordinate z.

Quadratic forms according to (4.60) [9]:

$$hP_{\varepsilon\chi} = \frac{3}{8}\left(e_{i1}^2 - e_{i2}^2\right),$$

$$P_\varepsilon = \frac{3}{16}\left[\left(\sqrt{e_{i1}^2 - e_{i0}^2} \mp \sqrt{e_{i2}^2 - e_{i0}^2}\right)^2 + 4e_{i0}^2\right] =$$

$$= \frac{3}{16}\left[2\left(e_{i1}^2 + e_{i2}^2\right) - \left(\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right)^2\right],$$

$$\frac{h^2}{4}P_\chi = \frac{3}{16}\left(\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right)^2. \tag{2.43}$$

Substituting (2.43) into (2.10)–(2.13), we obtain the integrals J_1, J_2, J_3:

$$J_1 = \frac{\sigma_s h}{\left|\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right|} \times \ln \frac{e_{i2}\left|\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right| + \sqrt{e_{i2}^2 - e_{i0}^2}\cdot\left(\sqrt{e_{i2}^2 - e_{i0}^2} \pm \sqrt{e_{i1}^2 - e_{i0}^2}\right)}{e_{i1}\left|\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right| - \sqrt{e_{i1}^2 - e_{i0}^2}\cdot\left(\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right)}, \tag{2.44}$$

$$J_2 = \frac{\sigma_s h^2\left(e_{i2} - e_{i1}\right)}{\left(\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right)^2} + \frac{h\left(e_{i1}^2 - e_{i2}^2\right)}{2\left(\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right)^2} J_1, \tag{2.45}$$

$$J_3 = \frac{\sigma_s h^3\left(e_{i2} + e_{i1}\right)}{4\left(\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right)^2} + \frac{3h\left(e_{i1}^2 - e_{i2}^2\right)}{4\left(\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right)^2} J_2 - \frac{h^2\left[\left(\sqrt{e_{i1}^2 - e_{i0}^2} \mp \sqrt{e_{i2}^2 - e_{i0}^2}\right)^2 + 4e_{i0}^2\right]}{8\left(\sqrt{e_{i1}^2 - e_{i0}^2} \pm \sqrt{e_{i2}^2 - e_{i0}^2}\right)^2} J_1. \tag{2.46}$$

Taking into account the introduction of two basic parameters λ and μ according to (4.61) [9] $\lambda = \frac{e_{i2}}{e_{i1}}$, $\mu = \frac{e_{i0}}{e_{i1}}$, the relations (2.43) take the form

$$hP_{\varepsilon\chi} = \frac{3}{8e_{i1}^2}\left(1 - \lambda^2\right),$$

$$P_\varepsilon = \frac{3}{16e_{i1}^2}\left[\left(\sqrt{1-\mu^2} \mp \sqrt{\lambda^2 - \mu^2}\right)^2 + 4\mu^2\right],$$

$$\frac{h^2}{4}P_\chi = \frac{3}{16e_{i1}^2}\left(\sqrt{1-\mu^2} \pm \sqrt{\lambda^2 - \mu^2}\right)^2, \tag{2.47}$$

and the integrals J_1, J_2, J_3 are expressed in terms of the basic parameters λ and μ:

$$J_1 = \frac{\sigma_s h}{e_{i1}\left|\sqrt{1-\mu^2} \pm \sqrt{\lambda^2 - \mu^2}\right|} \times \ln \frac{\lambda\left|\sqrt{1-\mu^2} \pm \sqrt{\lambda^2 - \mu^2}\right| + \sqrt{\lambda^2 - \mu^2}\cdot\left(\sqrt{\lambda^2 - \mu^2} \pm \sqrt{1-\mu^2}\right)}{\left|\sqrt{1-\mu^2} \pm \sqrt{\lambda^2 - \mu^2}\right| - \sqrt{1-\mu^2}\cdot\left(\sqrt{1-\mu^2} \pm \sqrt{\lambda^2 - \mu^2}\right)}, \tag{2.48}$$

$$J_2 = \frac{\sigma_s h^2\left(\lambda - 1\right)}{e_{i1}\left(\sqrt{1-\mu^2} \pm \sqrt{\lambda^2 - \mu^2}\right)^2} + \frac{h^2\left(1 - \lambda^2\right)}{\left(\sqrt{1-\mu^2} \pm \sqrt{\lambda^2 - \mu^2}\right)^2} J_1, \tag{2.49}$$

Table 2.1. Coordinates curve Q_n, Q_m (the expanded version of table 4 [9]).

μ		0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.00	Q_n	0.0000	0.0905	0.2190	0.3473	0.4676	0.5781	0.6789	0.7706	0.8541	0.9303
	Q_m	1.0000	0.9502	0.8558	0.7447	0.6283	0.5122	0.3995	0.2914	0.1887	0.0916
0.01	Q_n	0.0028	0.1028	0.2320	0.3597	0.4791	0.5886	0.6885	0.7793	0.8621	0.9376
	Q_m	0.9990	0.9421	0.8452	0.7332	0.6166	0.5008	0.3884	0.2809	0.1787	0.0822
0.02	Q_n	0.0085	0.1153	0.2450	0.3721	0.4905	0.5990	0.6980	0.7880	0.8699	0.9448
	Q_m	0.9967	0.9336	0.8344	0.7216	0.6049	0.4894	0.3774	0.2704	0.1688	0.0728
0.03	Q_n	0.0159	0.1280	0.2580	0.3843	0.5018	0.6094	0.7073	0.7965	0.8777	0.9519
	Q_m	0.9933	0.9248	0.8236	0.7100	0.5933	0.4780	0.3665	0.2600	0.1590	0.0635
0.04	Q_n	0.0245	0.1409	0.2710	0.3965	0.5130	0.6196	0.7166	0.8050	0.8854	0.9589
	Q_m	0.9891	0.9156	0.8125	0.6984	0.5816	0.4666	0.3556	0.2497	0.1492	0.0543
0.05	Q_n	0.0341	0.1538	0.2839	0.4086	0.5241	0.6297	0.7258	0.8134	0.8931	0.9659
	Q_m	0.9841	0.9062	0.8014	0.6867	0.5700	0.4553	0.3448	0.2394	0.1395	0.0451
0.06	Q_n	0.0444	0.1667	0.2967	0.4206	0.5351	0.6397	0.7350	0.8217	0.9007	0.9729
	Q_m	0.9784	0.8966	0.7902	0.6751	0.5584	0.4441	0.3340	0.2291	0.1298	0.0360
0.07	Q_n	0.0553	0.1798	0.3094	0.4325	0.5460	0.6497	0.7440	0.8299	0.9082	0.9797
	Q_m	0.9721	0.8867	0.7790	0.6634	0.5468	0.4328	0.3233	0.2189	0.1201	0.0269
0.08	Q_m	0.0667	0.1928	0.3221	0.4443	0.5568	0.6595	0.7530	0.8380	0.9156	0.9865
	Q_m	0.9653	0.8766	0.7676	0.6517	0.5353	0.4217	0.3126	0.2088	0.1106	0.0179
0.09	Q_n	0.0784	0.2059	0.3347	0.4560	0.5675	0.6693	0.7618	0.8461	0.9230	0.9933
	Q_m	0.9580	0.8663	0.7562	0.6400	0.5237	0.4105	0.3020	0.1987	0.1011	0.0089
1.00	Q_n										1.0000
	Q_m										0.0000

$$J_3 = \frac{\sigma_s h^3(\lambda+1)}{4e_{i1}\left(\sqrt{1-\mu^2} \pm \sqrt{\lambda^2-\mu^2}\right)^2} + \frac{3h(1-\lambda^2)}{4\left(\sqrt{1-\mu^2} \pm \sqrt{\lambda^2-\mu^2}\right)^2} J_2 - \frac{h^2\left[\left(\sqrt{1-\mu^2} \mp \sqrt{\lambda^2-\mu^2}\right)^2 + 4\mu^2\right]}{8\left(\sqrt{1-\mu^2} \pm \sqrt{\lambda^2-\mu^2}\right)^2} J_1. \quad (2.50)$$

In case of dominating bending strains of the formula (2.44)-(2.46) become:

$$J_1 = \frac{\sigma_s h}{\left|\sqrt{e_{i1}^2-e_{i0}^2}+\sqrt{e_{i2}^2-e_{i0}^2}\right|} \times \ln\frac{\left(e_{i1}+\sqrt{e_{i1}^2-e_{i0}^2}\right)\left(e_{i2}+\sqrt{e_{i2}^2-e_{i0}^2}\right)}{e_{i0}^2}, \quad (2.51)$$

$$J_2 = \frac{\sigma_s h^2(e_{i2}-e_{i1})}{\left(\sqrt{e_{i1}^2-e_{i0}^2}+\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} + \frac{h(e_{i1}^2-e_{i2}^2)}{2\left(\sqrt{e_{i1}^2-e_{i0}^2}+\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} J_1, \quad (2.52)$$

$$J_3 = \frac{\sigma_s h^3(e_{i2}+e_{i1})}{4\left(\sqrt{e_{i1}^2-e_{i0}^2}+\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} + \frac{3h(e_{i1}^2-e_{i2}^2)}{4\left(\sqrt{e_{i1}^2-e_{i0}^2}+\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} J_2 - \frac{h^2\left[\left(\sqrt{e_{i1}^2-e_{i0}^2}-\sqrt{e_{i2}^2-e_{i0}^2}\right)^2+4e_{i0}^2\right]}{8\left(\sqrt{e_{i1}^2-e_{i0}^2}+\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} J_1. \quad (2.53)$$

In case of dominating lengthening of a middle surface from formulas (2.44)-(2.46) it is found:

$$J_1 = \frac{\sigma_s h}{\left|\sqrt{e_{i1}^2-e_{i0}^2}-\sqrt{e_{i2}^2-e_{i0}^2}\right|}\left|\ln\frac{\left(e_{i2}+\sqrt{e_{i2}^2-e_{i0}^2}\right)}{\left(e_{i1}+\sqrt{e_{i1}^2-e_{i0}^2}\right)}\right|, \quad (2.54)$$

$$J_2 = \frac{\sigma_s h^2(e_{i2}-e_{i1})}{\left(\sqrt{e_{i1}^2-e_{i0}^2}-\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} + \frac{h(e_{i1}^2-e_{i2}^2)}{2\left(\sqrt{e_{i1}^2-e_{i0}^2}-\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} J_1, \quad (2.55)$$

$$J_3 = \frac{\sigma_s h^3(e_{i2}+e_{i1})}{4\left(\sqrt{e_{i1}^2-e_{i0}^2}-\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} + \frac{3h(e_{i1}^2-e_{i2}^2)}{4\left(\sqrt{e_{i1}^2-e_{i0}^2}-\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} J_2 - \frac{h^2\left[\left(\sqrt{e_{i1}^2-e_{i0}^2}+\sqrt{e_{i2}^2-e_{i0}^2}\right)^2+4e_{i0}^2\right]}{8\left(\sqrt{e_{i1}^2-e_{i0}^2}-\sqrt{e_{i2}^2-e_{i0}^2}\right)^2} J_1. \quad (2.56)$$

Taking into account introduction of two key parametres λ and μ according to (4.61) [9] in case of dominating bending strains of the formula (2.51)-(2.53) become:

$$J_1 = \frac{\sigma_s h}{\left|\sqrt{1-\mu^2}+\sqrt{\lambda^2-\mu^2}\right|} \times \ln\frac{\left(1+\sqrt{1-\mu^2}\right)\left(\lambda+\sqrt{\lambda^2-\mu^2}\right)}{\mu^2}, \quad (2.57)$$

$$J_2 = \frac{\sigma_s h^2(\lambda-1)}{\left(\sqrt{1-\mu^2}+\sqrt{\lambda^2-\mu^2}\right)^2} + \frac{h(1-\lambda^2)}{2\left(\sqrt{1-\mu^2}+\sqrt{\lambda^2-\mu^2}\right)^2} J_1, \quad (2.58)$$

$$J_3 = \frac{\sigma_s h^3(\lambda+1)}{4\left(\sqrt{1-\mu^2}+\sqrt{\lambda^2-\mu^2}\right)^2} + \frac{3h(1-\lambda^2)}{4\left(\sqrt{1-\mu^2}+\sqrt{\lambda^2-\mu^2}\right)^2} J_2 - \frac{h^2\left[\left(\sqrt{1-\mu^2}-\sqrt{\lambda^2-\mu^2}\right)^2+4\mu^2\right]}{8\left(\sqrt{1-\mu^2}+\sqrt{\lambda^2-\mu^2}\right)^2} J_1. \quad (2.59)$$

In case of dominating lengthening of a middle surface from formulas (2.54)-(2.56) it is found:

$$J_1 = \frac{\sigma_s h}{\left|\sqrt{1-\mu^2}-\sqrt{\lambda^2-\mu^2}\right|}\left|\ln\frac{\left(\lambda+\sqrt{\lambda^2-\mu^2}\right)}{\left(1+\sqrt{1-\mu^2}\right)}\right| \quad (2.60)$$

$$J_2 = \frac{\sigma_s h^2(\lambda-1)}{\left(\sqrt{1-\mu^2}-\sqrt{\lambda^2-\mu^2}\right)^2} + \frac{h(1-\lambda^2)}{2\left(\sqrt{1-\mu^2}-\sqrt{\lambda^2-\mu^2}\right)^2} J_1, \quad (2.61)$$

$$J_3=\frac{\sigma_s h^3\left(\lambda+1\right)}{4\left(\sqrt{1-\mu^2}-\sqrt{\lambda^2-\mu^2}\right)^2}+$$
$$+\frac{3h\left(1-\lambda^2\right)}{4\left(\sqrt{1-\mu^2}-\sqrt{\lambda^2-\mu^2}\right)^2}J_2-$$
$$-\frac{h^2\left[\left(\sqrt{1-\mu^2}+\sqrt{\lambda^2-\mu^2}\right)^2+4\mu^2\right]}{8\left(\sqrt{1-\mu^2}-\sqrt{\lambda^2-\mu^2}\right)^2}J_1. \quad (2.62)$$

Intensity of deformations (2.1), taking into account (2.43) it is possible to present in a kind:

$$e_i=\sqrt{\begin{aligned}&\frac{1}{4}\left[\left(\sqrt{e_{i1}^2-e_{i0}^2}\mp\sqrt{e_{i2}^2-e_{i0}^2}\right)^2+4e_{i0}^2\right]-\\&-\frac{z}{h}\left(e_{i1}^2-e_{i2}^2\right)+\frac{z^2}{h^2}\left(\sqrt{e_{i1}^2-e_{i0}^2}\pm\sqrt{e_{i2}^2-e_{i0}^2}\right)^2\end{aligned}}. \quad (2.63)$$

According to (2.63) integrals in formulas (4.25) [9]:

$$J_1=\sigma_s\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{dz}{X^{\frac{1}{2}}},\ J_2=\sigma_s\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{zdz}{X^{\frac{1}{2}}},\ J_3=\sigma_s\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{z^2dz}{X^{\frac{1}{2}}},$$
$$X^{\frac{1}{2}}=\sqrt{c+bz+az^2},$$
$$c=\frac{1}{4}\left[\left(\sqrt{e_{i1}^2-e_{i0}^2}\mp\sqrt{e_{i2}^2-e_{i0}^2}\right)^2+4e_{i0}^2\right],$$
$$b=-\frac{1}{h}\left(e_{i1}^2-e_{i2}^2\right),\ a=\frac{1}{h^2}\left(\sqrt{e_{i1}^2-e_{i0}^2}\pm\sqrt{e_{i2}^2-e_{i0}^2}\right)^2. \quad (2.64)$$

Considering (2.8)-(2.9), transformations become less bulky and to receive (2.44)-(2.46) it is possible much more fast.

The relations (2.44)-(2.46) are equivalent to (4.38), (4.59), (4.60) [9]. This can be seen if (4.38) [9] leads to the form

$$J_1=\frac{\sqrt{3}}{2P_\chi^{\frac{1}{2}}}B,\ J_2=\frac{P_{\varepsilon\chi}}{P_\chi}J_1+\frac{3}{4P_\chi}A,$$
$$J_3=\frac{3\sqrt{3}}{8P_\chi^{\frac{3}{2}}}C+\frac{2P_{\varepsilon\chi}}{P_\chi}J_2-\frac{P_{\varepsilon\chi}^2}{P_\chi^2}J_1 \quad (2.65)$$

and to consider identities

$$hP_{\varepsilon\chi}=\frac{3}{8}\left(e_{i1}^2-e_{i2}^2\right),$$
$$P_\varepsilon=\frac{3}{16}\left[\left(\sqrt{e_{i1}^2-e_{i0}^2}\mp\sqrt{e_{i2}^2-e_{i0}^2}\right)^2+4e_{i0}^2\right]=$$
$$=\frac{3}{16}\left[2\left(e_{i1}^2+e_{i2}^2\right)-\left(\sqrt{e_{i1}^2-e_{i0}^2}\pm\sqrt{e_{i2}^2-e_{i0}^2}\right)^2\right],$$
$$\frac{h^2}{4}P_\chi=\frac{3}{16}\left(\sqrt{e_{i1}^2-e_{i0}^2}\pm\sqrt{e_{i2}^2-e_{i0}^2}\right)^2. \quad (2.66)$$

The relations (4.45) [9] – (2.26) can be given a different form if we introduce the new integral according to (4.28) [9]

$$J_0=J_1P_\varepsilon-2J_2P_{\varepsilon\chi}+J_3P_\chi=\frac{3}{4}\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_s e_i dz=$$
$$=\frac{\sqrt{3}}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_s\sqrt{P_\varepsilon-2zP_{\varepsilon\chi}+z^2P_\chi}\,dz,$$
$$J_0=\frac{\sqrt{3}}{2}\sigma_s\int_{-\frac{h}{2}}^{\frac{h}{2}}X^{\frac{1}{2}}dz,\ X^{\frac{1}{2}}=\sqrt{c+bz+az^2},$$
$$c=P_\varepsilon,\ b=-2P_{\varepsilon\chi},\ a=P_\chi. \quad (2.67)$$

This integral tabular. According to the formula 380.201 [42]

$$J_0=\frac{\sqrt{3}}{2}\sigma_s\left[\left(\frac{2az+b}{4a}\right)X^{\frac{1}{2}}+\left(\frac{4ac-b^2}{8a}\right)\int\frac{dz}{X^{\frac{1}{2}}}\right]_{-\frac{h}{2}}^{\frac{h}{2}},$$
$$X^{\frac{1}{2}}=\sqrt{c+bz+az^2},\ c=P_\varepsilon,\ b=-2P_{\varepsilon\chi},\ a=P_\chi. \quad (2.68)$$

From here follows

$$J_0=\frac{\sqrt{3}\sigma_s}{2}\cdot\frac{1}{4P_\chi}\times$$
$$\times\left(\begin{aligned}&\left(hP_\chi-2P_{\varepsilon\chi}\right)\sqrt{P_\varepsilon-hP_{\varepsilon\chi}+\frac{h^2}{4}P_\chi}+\\&+\left(hP_\chi+2P_{\varepsilon\chi}\right)\sqrt{P_\varepsilon+hP_{\varepsilon\chi}+\frac{h^2}{4}P_\chi}\end{aligned}\right)+$$
$$+\frac{P_\varepsilon P_\chi-P_{\varepsilon\chi}^2}{2P_\chi}J_1. \quad (2.69)$$

Considering (2.8), (2.68) it is possible to express an integral through integrals J_1 and J_2:

$$J_0=\frac{\sqrt{3}}{2}\sigma_s\left[\begin{aligned}&\frac{z}{2}\cdot X^{\frac{1}{2}}+\frac{b}{4}\left(\frac{X^{\frac{1}{2}}}{a}-\frac{b}{2a}\right)\int\frac{dz}{X^{\frac{1}{2}}}+\\&+\frac{c}{2}\int\frac{dz}{X^{\frac{1}{2}}}\end{aligned}\right]_{-\frac{h}{2}}^{\frac{h}{2}}=$$
$$=\frac{\sqrt{3}}{2}\sigma_s\left[\frac{z}{2}\cdot X^{\frac{1}{2}}\right]_{-\frac{h}{2}}^{\frac{h}{2}}+\frac{b}{4}J_2+\frac{c}{2}J_1,$$
$$X^{\frac{1}{2}}=\sqrt{c+bz+az^2},\ c=P_\varepsilon,\ b=-2P_{\varepsilon\chi},\ a=P_\chi. \quad (2.70)$$

Then (2.69) becomes (2.71)

$$J_0=\frac{\sqrt{3}\sigma_s}{2}\cdot\frac{1}{4}\times$$
$$\times\left(\sqrt{P_\varepsilon-hP_{\varepsilon\chi}+\frac{h^2}{4}P_\chi}+\sqrt{P_\varepsilon+hP_{\varepsilon\chi}+\frac{h^2}{4}P_\chi}\right)-$$
$$-\frac{P_{\varepsilon\chi}}{2}J_2+\frac{P_\varepsilon}{2}J_1 \quad (2.71)$$

Integral (2.69) taking into account (2.4)

$$J_0=\frac{\sigma_s h\left[\begin{array}{l}\left(e_{i1}^2+3e_{i2}^2-4e_{i0}^2\right)e_{i2}+\\+\left(3e_{i1}^2+e_{i2}^2-4e_{i0}^2\right)e_{i1}\end{array}\right]}{4\left(2e_{i1}^2+2e_{i2}^2-4e_{i0}^2\right)}+$$
$$+\frac{1}{2}\left[e_{i0}^2-\frac{\left(e_{i1}^2-e_{i2}^2\right)^2}{4\left(2e_{i1}^2+2e_{i2}^2-4e_{i0}^2\right)}\right]J_1. \quad (2.72)$$

According to (2.5) formula (2.72) for an integral J_0 becomes

$$J_0=\frac{\sigma_s h\left[\left(1+3\lambda^2-4\mu^2\right)\lambda\ +\left(3+\lambda^2-4\mu^2\right)\right]}{4e_{i1}\left(2+2\lambda^2-4\mu^2\right)}+$$
$$+\frac{1}{2}\left[\mu^2-\frac{\left(1-\lambda^2\right)^2}{4\left(2+2\lambda^2-4\mu^2\right)}\right]J_1. \quad (2.73)$$

The final relation (4.45) [9] – (2.26) taking into account (2.67) takes the form:

$$\begin{aligned}P_S&=J_1J_0-(J_1J_3-J_2^2)P_\chi\\P_H&=J_3J_0-(J_1J_3-J_2^2)P_\varepsilon\\P_{SH}&=J_2J_0-(J_1J_3-J_2^2)P_{\varepsilon\chi}\,.\end{aligned} \quad (2.74)$$

The relations (2.74), (2.26) and (4.70') [9] are equivalent.

2.2. Approximate dependencies of the final relation

The integrals J_1, J_2, J_3, J_0 can be found by the Simpson formula, performing integration within each half of the section, since the intensity of deformations e_i function can lose monotonicity at $z = 0$. According to (2.14–2.15), (2.67), the approximate values of the integrals:

$$J_1=\frac{\sigma_s h}{12}\left(\begin{array}{l}\frac{1}{e_{i1}}+\frac{1}{e_{i2}}+\frac{2}{e_{i0}}+\frac{16}{\sqrt{12e_{i0}^2+6e_{i1}^2-2e_{i2}^2}}+\\+\frac{16}{\sqrt{12e_{i0}^2+6e_{i2}^2-2e_{i1}^2}}\end{array}\right),$$

$$J_2=\frac{\sigma_s h^2}{24}\left(\begin{array}{l}-\frac{1}{e_{i1}}+\frac{1}{e_{i2}}-\frac{8}{\sqrt{12e_{i0}^2+6e_{i1}^2-2e_{i2}^2}}+\\+\frac{8}{\sqrt{12e_{i0}^2+6e_{i2}^2-2e_{i1}^2}}\end{array}\right),$$

$$J_3=\frac{\sigma_s h^3}{48}\left(\begin{array}{l}\frac{1}{e_{i1}}+\frac{1}{e_{i2}}+\frac{4}{\sqrt{12e_{i0}^2+6e_{i1}^2-2e_{i2}^2}}+\\+\frac{4}{\sqrt{12e_{i0}^2+6e_{i2}^2-2e_{i1}^2}}\end{array}\right), \quad (2.75)$$

$$J_0=\frac{3}{4}\cdot\frac{\sigma_s h}{12}\times$$
$$\times\left(\begin{array}{l}e_{i1}+e_{i2}+2e_{i0}+\\+\sqrt{12e_{i0}^2+6e_{i1}^2-2e_{i2}^2}+\sqrt{12e_{i0}^2+6e_{i2}^2-2e_{i1}^2}\end{array}\right). \quad (2.76)$$

Taking into account (2.5) formulas (2.75)-(2.76) become:

$$J_1=\frac{\sigma_s h}{12e_{i1}}\left(\begin{array}{l}1+\frac{1}{\lambda}+\frac{2}{\mu}+\frac{16}{\sqrt{12\mu^2+6-2\lambda^2}}+\\+\frac{16}{\sqrt{12\mu^2+6\lambda^2-2}}\end{array}\right),$$

$$J_2=\frac{\sigma_s h^2}{24e_{i1}}\left(\begin{array}{l}-1+\frac{1}{\lambda}-\frac{8}{\sqrt{12\mu^2+6-2\lambda^2}}+\\+\frac{8}{\sqrt{12\mu^2+6\lambda^2-2}}\end{array}\right),$$

$$J_3=\frac{\sigma_s h^3}{48e_{i1}}\left(\begin{array}{l}1+\frac{1}{\lambda}+\frac{4}{\sqrt{12\mu^2+6-2\lambda^2}}+\\+\frac{4}{\sqrt{12\mu^2+6\lambda^2-2}}\end{array}\right), \quad (2.77)$$

$$J_0=\frac{3}{4}\cdot\frac{\sigma_s h}{12e_{i1}}\left(\begin{array}{l}1+\lambda+2\mu+\sqrt{12\mu^2+6-2\lambda^2}+\\+\sqrt{12\mu^2+6\lambda^2-2}\end{array}\right). \quad (2.78)$$

Believing that within each half of section intensity of deformations e_i changes under the linear law

$$\begin{array}{l}e_i=e_{i0}+\frac{2z}{h}\left(e_{i2}-e_{i0}\right),\ \ 0\le z\le\frac{h}{2},\\e_i=e_{i0}-\frac{2z}{h}\left(e_{i1}-e_{i0}\right),\ \ -\frac{h}{2}\le z\le 0\end{array},$$ According to formulas (90.1, 91.1, 92.1) [42]

$$J_1=\frac{\sigma_s}{b}\left[\ln|a+bz|\right]_{-\frac{h}{2}}^{\frac{h}{2}},$$

$$J_2=\frac{\sigma_s}{b^2}\left[(a+bz)-a\ln|a+bz|\right]_{-\frac{h}{2}}^{\frac{h}{2}},$$

$$J_3=\frac{\sigma_s}{b^3}\left[\frac{(a+bz)^2}{2}-2a(a+bz)+a^2\ln|a+bz|\right]_{-\frac{h}{2}}^{\frac{h}{2}},$$

$$a=e_{i0},\ b=\frac{1}{h}(e_{i2}-e_{i0}),\ 0\le z\le\frac{h}{2},$$

$$b=-\frac{1}{h}(e_{i1}-e_{i0}),\ -\frac{h}{2}\le z\le 0. \tag{2.79}$$

From here follows:

$$J_1=\frac{\sigma_s h}{2(e_{i2}-e_{i0})}\ln\frac{e_{i2}}{e_{i0}}+\frac{\sigma_s h}{2(e_{i1}-e_{i0})}\ln\frac{e_{i1}}{e_{i0}},$$

$$J_2=-\frac{\sigma_s h^2}{4(e_{i1}-e_{i0})^2}\left[(e_{i1}-e_{i0})-e_{i0}\ln\frac{e_{i1}}{e_{i0}}\right]+$$

$$+\frac{\sigma_s h^2}{4(e_{i2}-e_{i0})^2}\left[(e_{i2}-e_{i0})-e_{i0}\ln\frac{e_{i2}}{e_{i0}}\right],$$

$$J_3=\frac{\sigma_s h^3}{16(e_{i1}-e_{i0})^3}\left[\begin{matrix}(3e_{i0}^2+e_{i1}^2-4e_{i0}e_{i1})+\\ +2e_{i0}^2\ln\dfrac{e_{i1}}{e_{i0}}\end{matrix}\right]+$$

$$+\frac{\sigma_s h^3}{16(e_{i2}-e_{i0})^3}\left[(3e_{i0}^2+e_{i2}^2-4e_{i0}e_{i2})+2e_{i0}^2\ln\frac{e_{i2}}{e_{i0}}\right], \tag{2.80}$$

$$J_0=\frac{3}{4}\cdot\frac{\sigma_s h}{4}(e_{i1}+e_{i2}+2e_{i0}). \tag{2.81}$$

Taking into account (2.5) formulas (2.80)-(2.81) become:

$$J_1=\frac{\sigma_s h}{2(\lambda-\mu)e_{i1}}\ln\frac{\lambda}{\mu}+\frac{\sigma_s h}{2(1-\mu)e_{i1}}\ln\frac{1}{\mu},$$

$$J_2=-\frac{\sigma_s h^2}{4(1-\mu)^2e_{i1}}\left[(1-\mu)-\mu\ln\frac{1}{\mu}\right]+$$

$$+\frac{\sigma_s h^2}{4(\lambda-\mu)^2e_{i1}}\left[(\lambda-\mu)-\mu\ln\frac{\lambda}{\mu}\right],$$

$$J_3=\frac{\sigma_s h^3}{16(1-\mu)^3e_{i1}}\left[(3\mu^2+1-4\mu)+2\mu^2\ln\frac{1}{\mu}\right]+$$

$$+\frac{\sigma_s h^3}{16(\lambda-\mu)^3e_{i1}}\left[(3\mu^2+\lambda^2-4\mu\lambda)+2\mu^2\ln\frac{\lambda}{\mu}\right], \tag{2.82}$$

$$J_0=\frac{3}{4}\cdot\frac{\sigma_s h}{4e_{i1}}(1+\lambda+2\mu). \tag{2.83}$$

Integrals J_1, J_2, J_3, J_0 (2.80) and (2.83) also can be found under Simpson's formula, executing integration within each half of section

$$J_1=\frac{\sigma_s h}{12}\left(\frac{1}{e_{i1}}+\frac{1}{e_{i2}}+\frac{2}{e_{i0}}+\frac{8}{(e_{i1}+e_{i0})}+\frac{8}{(e_{i2}+e_{i0})}\right),$$

$$J_2=\frac{\sigma_s h^2}{24}\left(-\frac{1}{e_{i1}}+\frac{1}{e_{i2}}-\frac{4}{(e_{i1}+e_{i0})}+\frac{4}{(e_{i2}+e_{i0})}\right),$$

$$J_3=\frac{\sigma_s h^3}{48}\left(\frac{1}{e_{i1}}+\frac{1}{e_{i2}}+\frac{2}{(e_{i1}+e_{i0})}+\frac{2}{(e_{i2}+e_{i0})}\right), \tag{2.84}$$

$$J_0=\frac{3}{4}\cdot\frac{\sigma_s h}{4}(e_{i1}+e_{i2}+2e_{i0}). \tag{2.85}$$

$$J_1=\frac{\sigma_s h}{12e_{i1}}\left(1+\frac{1}{\lambda}+\frac{2}{\mu}+\frac{8}{1+\mu}+\frac{8}{\lambda+\mu}\right),$$

$$J_2=\frac{\sigma_s h^2}{24e_{i1}}\left(-1+\frac{1}{\lambda}-\frac{4}{1+\mu}+\frac{4}{\lambda+\mu}\right),$$

$$J_3=\frac{\sigma_s h^3}{48e_{i1}}\left(1+\frac{1}{\lambda}+\frac{2}{1+\mu}+\frac{2}{\lambda+\mu}\right),$$

$$J_0=\frac{3}{4}\cdot\frac{\sigma_s h}{4e_{i1}}(1+\lambda+2\mu). \tag{2.86}$$

On the basis regression the analysis of a curve (2.41) (the minimum line Q_{nm}, table 2.1) are received versions of its approximation by polynoms of the second, third and fourth degree and its first derivative is found:

Polynom of the second degree.

$$Q_m=1.0099235-0.642635\cdot Q_n-0.3718551\cdot Q_n^2,$$

$$y=1.0099235-0.642635x-0.3718551x^2,$$

$$\frac{\partial y}{\partial x}=-0.642635-2\cdot0.3718551x,$$

$$\left.\frac{\partial y}{\partial x}\right|_{x=0}=-0.6426,\ \left.\frac{\partial y}{\partial x}\right|_{x=1}=-1.3864.$$

Polynom of the third degree.

$$Q_m = 1.0037431 - 0.5575663 \cdot Q_n - 0.586892 \cdot Q_n^2 + $$
$$+0.1423386 \cdot Q_n^3,$$
$$y = 1.0037431 - 0.5575663x \; - 0.586892x^2 + $$
$$+0.1423386x^3,$$
$$\frac{\partial y}{\partial x} = -0.5575663 - 2 \cdot 0.586892x + 3 \cdot 0.1423386x^2,$$
$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = -0.5576, \quad \left.\frac{\partial y}{\partial x}\right|_{x=1} = -1.3043.$$

Polynom of the fourth degree.

$$Q_m = 1.0019337 - 0.5128967 \cdot Q_n -$$
$$-0.7948953 \cdot Q_n^2 + 0.4669156 \cdot Q_n^3 -$$
$$-0.1613785 \cdot Q_n^4,$$
$$y = 1.0019337 - 0.5128967x - 0.7948953x^2 +$$
$$+0.4669156x^3 - 0.1613785x^4,$$
$$\frac{\partial y}{\partial x} = -0.5128967 - 2 \cdot 0.7948953x +$$
$$+3 \cdot 0.4669156x^2 - 4 \cdot 0.1613785x^3,$$
$$\left.\frac{\partial y}{\partial x}\right|_{x=0} = -0.5129, \quad \left.\frac{\partial y}{\partial x}\right|_{x=1} = -1.3475.$$

In fig. 2.1-2.3 the curve (2.41) (table 2.1) (the minimum line Q_{nm}) is presented, the variants of its approximation by polynomials of the second, third and fourth degree (the lines merge) and its first derivative on the basis of regression analysis. As you can see from the graphs, a polynomial of the second degree is sufficient.

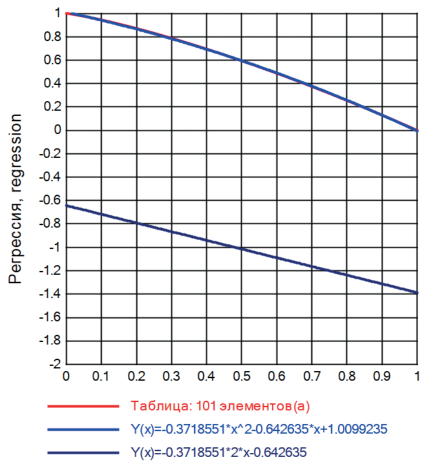

Figure 2.1. Curve (2.41) (table 2.1, a minimum line Q_{nm}), version of approximation by a polynom of the second degree and its first derivative on the basis regression the analysis.

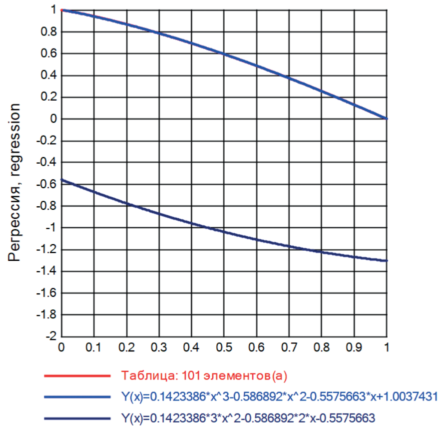

Figure 2.2. Curve (2.41) (table 2.1, a minimum line Q_{nm}), version of approximation by a polynom of the third degree and its first derivative on the basis regression the analysis.

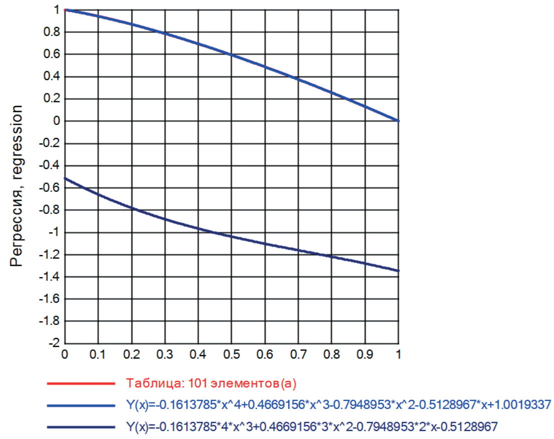

Figure 2.3. Curve (2.41) (table 2.1, a minimum line Q_{nm}), version of approximation by a polynom of the fourth degree and its first derivative on the basis regression the analysis.

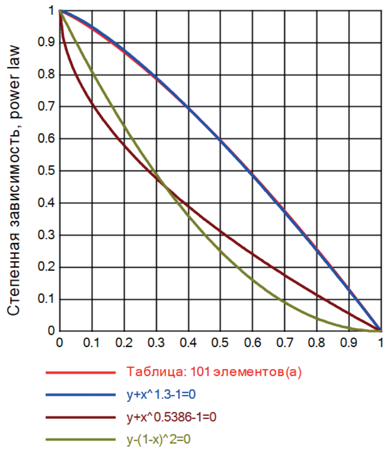

Figure 2.4. Curve (2.41) (table 2.1) (the minimum line Q_{nm}), variants of its approximation by a power law, and also the surface cross section (2.26) with a parabolic cylinder $Q_m - (1 - Q_n)^2 = 0$ (maximum line Q_{nm}).

<u>Table 2.2.</u> Coordinates of points of a surface Q_n, Q_m, Q_{nm} on lines λ = const for a dominating bending of a shell (the expanded version of table 5 [9]).

	μ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	λ											
Q_n	1.0	0.00000	0.09050	0.21897	0.34726	0.46759	0.57813	0.67891	0.77062	0.85414	0.93033	1.00000
Q_m		1.00000	0.95024	0.85582	0.74470	0.62830	0.51225	0.39946	0.29140	0.18871	0.09159	0.00000
Q_{nm}		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Q_n	0.9	0.00277	0.09967	0.23533	0.36942	0.49421	0.60812	0.71152	0.80540	0.89130	0.98296	
Q_m		0.99447	0.94107	0.84032	0.72261	0.60010	0.47868	0.36119	0.24887	0.14161	0.02263	
Q_{nm}		-0.05249	-0.04451	-0.03491	-0.02663	-0.01984	-0.01434	-0.00989	-0.00627	-0.00330	-0.00049	
Q_n	0.8	0.01235	0.11627	0.25958	0.39974	0.52919	0.64683	0.75362	0.85192	0.96525		
Q_m		0.97546	0.91947	0.81312	0.68917	0.56061	0.43347	0.31025	0.19067	0.04585		
Q_{nm}		-0.10974	-0.09179	-0.07067	-0.05278	-0.03831	-0.02672	-0.01740	-0.00979	-0.00212		
Q_n	0.7	0.03114	0.14275	0.29415	0.44066	0.57518	0.69753	0.81067	0.94710			
Q_m		0.93869	0.88187	0.77115	0.64145	0.50666	0.37264	0.23984	0.06914			
Q_{nm}		-0.17098	-0.14060	-0.10576	-0.07681	-0.05374	-0.03540	-0.02056	-0.00522			
Q_n	0.6	0.06250	0.18247	0.34238	0.49568	0.63638	0.76701	0.92891				
Q_m		0.87891	0.82409	0.71076	0.57572	0.43364	0.28818	0.09161				
Q_{nm}		-0.23438	-0.18858	-0.13757	-0.09607	-0.06339	-0.03727	-0.01015				
Q_n	0.5	0.11111	0.24010	0.40897	0.57016	0.72111	0.91146					
Q_m		0.79012	0.74153	0.62771	0.48690	0.33257	0.11168					
Q_{nm}		-0.29630	-0.23153	-0.16178	-0.10618	-0.06235	-0.01731					
Q_n	0.4	0.18367	0.32224	0.50097	0.67432	0.89616						
Q_m		0.66639	0.62989	0.51718	0.36587	0.12669						
Q_{nm}		-0.34985	-0.26211	-0.17111	-0.09927	-0.02699						
Q_n	0.3	0.28994	0.43862	0.63097	0.88572							
Q_m		0.50418	0.48669	0.37210	0.13201							
Q_{nm}		-0.38234	-0.26757	-0.15260	-0.03900							
Q_n	0.2	0.44444	0.60511	0.88564								
Q_m		0.30864	0.31363	0.11966								
Q_{nm}		-0.37037	-0.22529	-0.05138								
Q_n	0.1	0.66942	0.90868									
Q_m		0.10928	0.07641									
Q_{nm}		-0.27047	-0.05552									
Q_n	0.0	1.00000										
Q_m		0.00000										
Q_{nm}		0.00000										

Table 2.3. Coordinates of points of a surface Q_n, Q_m, Q_{nm} on lines λ = const for a dominating stretching – compression.

	μ	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	λ											
Q_n	1.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
Q_m		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Q_{nm}		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
Q_n	0.9	1.00000	0.99999	0.99996	0.99990	0.99980	0.99964	0.99938	0.99889	0.99766	0.98296	
Q_m		0.00000	0.00001	0.00006	0.00014	0.00027	0.00048	0.00082	0.00149	0.00312	0.02263	
Q_{nm}		0.00000	0.00000	0.00000	0.00000	-0.00001	-0.00001	-0.00002	-0.00003	-0.00007	-0.00049	
Q_n	0.8	1.00000	0.99995	0.99978	0.99947	0.99896	0.99810	0.99653	0.99302	0.96525		
Q_m		0.00000	0.00007	0.00029	0.00070	0.00138	0.00253	0.00461	0.00927	0.04585		
Q_{nm}		0.00000	0.00000	-0.00001	-0.00003	-0.00006	-0.00011	-0.00021	-0.00042	-0.00212		
Q_n	0.7	1.00000	0.99985	0.99936	0.99843	0.99683	0.99392	0.98772	0.94710			
Q_m		0.00000	0.00020	0.00085	0.00207	0.00420	0.00802	0.01619	0.06914			
Q_{nm}		0.00000	-0.00001	-0.00006	-0.00015	-0.00030	-0.00058	-0.00118	-0.00522			
Q_n	0.6	1.00000	0.99964	0.99846	0.99614	0.99182	0.98274	0.92891				
Q_m		0.00000	0.00048	0.00202	0.00506	0.01069	0.02253	0.09161				
Q_{nm}		0.00000	-0.00005	-0.00021	-0.00053	-0.00111	-0.00237	-0.01015				
Q_n	0.5	1.00000	0.99920	0.99654	0.99099	0.97904	0.91146					
Q_m		0.00000	0.00103	0.00446	0.01160	0.02693	0.11168					
Q_{nm}		0.00000	-0.00015	-0.00063	-0.00166	-0.00390	-0.01731					
Q_n	0.4	1.00000	0.99826	0.99223	0.97781	0.89616						
Q_m		0.00000	0.00219	0.00977	0.02780	0.12669						
Q_{nm}		0.00000	-0.00042	-0.00188	-0.00542	-0.02699						
Q_n	0.3	1.00000	0.99604	0.98065	0.88572							
Q_m		0.00000	0.00479	0.02327	0.13201							
Q_{nm}		0.00000	-0.00124	-0.00612	-0.03900							
Q_n	0.2	1.00000	0.98930	0.88564								
Q_m		0.00000	0.01197	0.11966								
Q_{nm}		0.00000	-0.00440	-0.05138								
Q_n	0.1	1.00000	0.90868									
Q_m		0.00000	0.07641									
Q_{nm}		0.00000	-0.05552									
Q_n	0.0	1.00000										
Q_m		0.00000										
Q_{nm}		0.00000										

Table 2.4. Coordinates of points of a surface Q_n, Q_m, Q_{nm} on lines $\lambda = const$ for a dominating bending of a shell.

λ	μ_{min}	μ	Q_n	Q_m	Q_{nm}
1.0	0.0	0.00000	0.00000	1.00000	0.00000
1.0	0.1	0.10000	0.09050	0.95024	0.00000
1.0	0.2	0.20000	0.21897	0.85582	0.00000
1.0	0.3	0.30000	0.34726	0.74470	0.00000
1.0	0.4	0.40000	0.46759	0.62830	0.00000
1.0	0.5	0.50000	0.57813	0.51225	0.00000
1.0	0.6	0.60000	0.67891	0.39946	0.00000
1.0	0.7	0.70000	0.77062	0.29140	0.00000
1.0	0.8	0.80000	0.85414	0.18871	0.00000
1.0	0.9	0.90000	0.93033	0.09159	0.00000
1.0	1.0	1.00000	1.00000	0.00000	0.00000
0.9	0.0	0.05000	0.00277	0.99447	-0.05249
0.9	0.1	0.11190	0.09967	0.94107	-0.04451
0.9	0.2	0.20640	0.23533	0.84032	-0.03491
0.9	0.3	0.30460	0.36942	0.72261	-0.02663
0.9	0.4	0.40380	0.49421	0.60010	-0.01984
0.9	0.5	0.50350	0.60812	0.47868	-0.01434
0.9	0.6	0.60350	0.71152	0.36119	-0.00989
0.9	0.7	0.70390	0.80540	0.24887	-0.00627
0.9	0.8	0.80550	0.89130	0.14161	-0.00330
0.9	0.9	0.92600	0.98296	0.02263	-0.00049
0.8	0.0	0.10000	0.01235	0.97546	-0.10974
0.8	0.1	0.14190	0.11627	0.91947	-0.09179
0.8	0.2	0.22480	0.25958	0.81312	-0.07067
0.8	0.3	0.31820	0.39974	0.68917	-0.05278
0.8	0.4	0.41530	0.52919	0.56061	-0.03831
0.8	0.5	0.51440	0.64683	0.43347	-0.02672
0.8	0.6	0.61510	0.75362	0.31025	-0.01740
0.8	0.7	0.71880	0.85192	0.19067	-0.00979
0.8	0.8	0.85440	0.96525	0.04585	-0.00212
0.7	0.0	0.15000	0.03114	0.93869	-0.17098
0.7	0.1	0.18120	0.14275	0.88187	-0.14060
0.7	0.2	0.25270	0.29415	0.77115	-0.10576
0.7	0.3	0.34030	0.44066	0.64145	-0.07681
0.7	0.4	0.43500	0.57518	0.50666	-0.05374

Table 2.5. Coordinates of points of a surface Q_n, Q_m, Q_{nm} on lines $\lambda = const$ for a dominating stretching – compression.

λ	μ_{min}	μ	Q_n	Q_m	Q_{nm}
1.0	0.0	1.00000	1.00000	0.00000	0.00000
1.0	0.1	1.00000	1.00000	0.00000	0.00000
1.0	0.2	1.00000	1.00000	0.00000	0.00000
1.0	0.3	1.00000	1.00000	0.00000	0.00000
1.0	0.4	1.00000	1.00000	0.00000	0.00000
1.0	0.5	1.00000	1.00000	0.00000	0.00000
1.0	0.6	1.00000	1.00000	0.00000	0.00000
1.0	0.7	1.00000	1.00000	0.00000	0.00000
1.0	0.8	1.00000	1.00000	0.00000	0.00000
1.0	0.9	1.00000	1.00000	0.00000	0.00000
1.0	1.0	1.00000	1.00000	0.00000	0.00000
0.9	0.0	0.95000	1.00000	0.00000	0.00000
0.9	0.1	0.95000	0.99999	0.00001	0.00000
0.9	0.2	0.94990	0.99996	0.00006	0.00000
0.9	0.3	0.94990	0.99990	0.00014	0.00000
0.9	0.4	0.94970	0.99980	0.00027	-0.00001
0.9	0.5	0.94950	0.99964	0.00048	-0.00001
0.9	0.6	0.94910	0.99938	0.00082	-0.00002
0.9	0.7	0.94840	0.99889	0.00149	-0.00003
0.9	0.8	0.94670	0.99766	0.00312	-0.00007
0.9	0.9	0.92600	0.98296	0.02263	-0.00049
0.8	0.0	0.90000	1.00000	0.00000	0.00000
0.8	0.1	0.89990	0.99995	0.00007	0.00000
0.8	0.2	0.89970	0.99978	0.00029	-0.00001
0.8	0.3	0.89930	0.99947	0.00070	-0.00003
0.8	0.4	0.89860	0.99896	0.00138	-0.00006
0.8	0.5	0.89740	0.99810	0.00253	-0.00011
0.8	0.6	0.89540	0.99653	0.00461	-0.00021
0.8	0.7	0.89070	0.99302	0.00927	-0.00042
0.8	0.8	0.85440	0.96525	0.04585	-0.00212
0.7	0.0	0.85000	1.00000	0.00000	0.00000
0.7	0.1	0.84980	0.99985	0.00020	-0.00001
0.7	0.2	0.84920	0.99936	0.00085	-0.00006
0.7	0.3	0.84800	0.99843	0.00207	-0.00015
0.7	0.4	0.84600	0.99683	0.00420	-0.00030

0.7	0.5	0.53420	0.69753	0.37264	-0.03540		0.7	0.5	0.84240	0.99392	0.00802	-0.00058
0.7	0.6	0.63900	0.81067	0.23984	-0.02056		0.7	0.6	0.83470	0.98772	0.01619	-0.00118
0.7	0.7	0.78580	0.94710	0.06914	-0.00522		0.7	0.7	0.78580	0.94710	0.06914	-0.00522
0.6	0.0	0.20000	0.06250	0.87891	-0.23438		0.6	0.0	0.80000	1.00000	0.00000	0.00000
0.6	0.1	0.22510	0.18247	0.82409	-0.18858		0.6	0.1	0.79960	0.99964	0.00048	-0.00005
0.6	0.2	0.28790	0.34238	0.71076	-0.13757		0.6	0.2	0.79820	0.99846	0.00202	-0.00021
0.6	0.3	0.37040	0.49568	0.57572	-0.09607		0.6	0.3	0.79550	0.99614	0.00506	-0.00053
0.6	0.4	0.46370	0.63638	0.43364	-0.06339		0.6	0.4	0.79050	0.99182	0.01069	-0.00111
0.6	0.5	0.56690	0.76701	0.28818	-0.03727		0.6	0.5	0.78010	0.98274	0.02253	-0.00237
0.6	0.6	0.72110	0.92891	0.09161	-0.01015		0.6	0.6	0.72110	0.92891	0.09161	-0.01015
0.5	0.0	0.25000	0.11111	0.79012	-0.29630		0.5	0.0	0.75000	1.00000	0.00000	0.00000
0.5	0.1	0.27160	0.24010	0.74153	-0.23153		0.5	0.1	0.74910	0.99920	0.00103	-0.00015
0.5	0.2	0.32860	0.40897	0.62771	-0.16178		0.5	0.2	0.74630	0.99654	0.00446	-0.00063
0.5	0.3	0.40830	0.57016	0.48690	-0.10618		0.5	0.3	0.74050	0.99099	0.01160	-0.00166
0.5	0.4	0.50500	0.72111	0.33257	-0.06235		0.5	0.4	0.72800	0.97904	0.02693	-0.00390
0.5	0.5	0.66140	0.91146	0.11168	-0.01731		0.5	0.5	0.66140	0.91146	0.11168	-0.01731
0.4	0.0	0.30000	0.18367	0.66639	-0.34985		0.4	0.0	0.70000	1.00000	0.00000	0.00000
0.4	0.1	0.31990	0.32224	0.62989	-0.26211		0.4	0.1	0.69830	0.99826	0.00219	-0.00042
0.4	0.2	0.37460	0.50097	0.51718	-0.17111		0.4	0.2	0.69260	0.99223	0.00977	-0.00188
0.4	0.3	0.45700	0.67432	0.36587	-0.09927		0.4	0.3	0.67910	0.97781	0.02780	-0.00542
0.4	0.4	0.60830	0.89616	0.12669	-0.02699		0.4	0.4	0.60830	0.89616	0.12669	-0.02699
0.5	0.5	0.66140	0.91146	0.11168	-0.01731		0.5	0.5	0.66140	0.91146	0.11168	-0.01731
0.4	0.0	0.30000	0.18367	0.66639	-0.34985		0.4	0.0	0.70000	1.00000	0.00000	0.00000
0.4	0.1	0.31990	0.32224	0.62989	-0.26211		0.4	0.1	0.69830	0.99826	0.00219	-0.00042
0.4	0.2	0.37460	0.50097	0.51718	-0.17111		0.4	0.2	0.69260	0.99223	0.00977	-0.00188
0.4	0.3	0.45700	0.67432	0.36587	-0.09927		0.4	0.3	0.67910	0.97781	0.02780	-0.00542
0.4	0.4	0.60830	0.89616	0.12669	-0.02699		0.4	0.4	0.60830	0.89616	0.12669	-0.02699
0.3	0.0	0.35000	0.28994	0.50418	-0.38234		0.3	0.0	0.65000	1.00000	0.00000	0.00000
0.3	0.1	0.36980	0.43862	0.48669	-0.26757		0.3	0.1	0.64670	0.99604	0.00479	-0.00124
0.3	0.2	0.42770	0.63097	0.37210	-0.15260		0.3	0.2	0.63410	0.98065	0.02327	-0.00612
0.3	0.3	0.56350	0.88572	0.13201	-0.03900		0.3	0.3	0.56350	0.88572	0.13201	-0.03900
0.2	0.0	0.40000	0.44444	0.30864	-0.37037		0.2	0.0	0.60000	1.00000	0.00000	0.00000
0.2	0.1	0.42290	0.60511	0.31363	-0.22529		0.2	0.1	0.59260	0.98930	0.01197	-0.00440
0.2	0.2	0.52920	0.88564	0.11966	-0.05138		0.2	0.2	0.52920	0.88564	0.11966	-0.05138
0.1	0.0	0.45000	0.66942	0.10928	-0.27047		0.1	0.0	0.55000	1.00000	0.00000	0.00000
0.1	0.1	0.50740	0.90868	0.07641	-0.05552		0.1	0.1	0.50740	0.90868	0.07641	-0.05552
0.0	0.0	0.50000	1.00000	0.00000	0.00000		0.0	0.0	0.50000	1.00000	0.00000	0.00000

In fig. 2.4 shows the curve (2.41) (table 2.1) (the minimum line Q_{nm}), variants of its approximation by a power law, and also the surface cross section (2.26) with a parabolic cylinder $Q_m - (1 - Q_n)^2 = 0$ (maximum line Q_{nm}). Other variants of approximation are given in part 3 of the article.

CONCLUSIONS

Alternative dependences of the finite relationship are developed, their equivalence to the relations A.A. Ilyushin is proved, approximate dependences of the final relationship are obtained. Based on the regression analysis of the minimum line , variants of its approximation by algebraic polynomials are obtained.

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