TRANSVERSE OSCILLATIONS OF THE BEAM ON AN ELASTIC BASE IF THE BOUNDARY CONDITIONS CHANGE

Yevgeny V. Leontiev

Federal autonomous institution "Main Department of State Exertise", Moscow, RUSSIA

Annotation: The article deals with the proper transverse oscillations of a beam with free edges while the conditions of support on an elastic base change, taking into account its own weight and the influence of the attached mass m1. The problem of determining the forces in the beam is being solved taking into account the dynamic load F(t) applied at an arbitrary point d while the conditions for the support of a part of the beam on an elastic base change.
The conditions that must be taken into account while analyzing the dynamic action of the structure under the influence of variable loads in the case of changes in the conditions of support on an elastic base are formulated.

Keywords: ground base, beam on an elastic foundation, the initial parameters method,
natural oscillation frequencies, forced oscillations, dynamic analysis.

ПОПЕРЕЧНЫЕ КОЛЕБАНИЯ БАЛКИ НА УПРУГОМ ОСНОВАНИИ
при изменении условий опирания

Е.В.Леонтьев

ФАУ «Главгосэкспертиза России», г. Москва, РОССИЯ

Аннотация: В работе изучаются собственные поперечные колебания балки со свободными краями при изменении условий опирания на упругое основание с учетом собственного веса и влияния присоединенной массы m1. Решается задача по определению усилий в балке с учетом динамической нагрузки F(t) приложенной в произвольной точке d при изменении условий опирания части балки на упругое основание.
Сформулированы условия, которые необходимо учитывать при анализе динамического поведения конструкции под действием переменных нагрузок в случае изменения условий опирания на упругое основание.

Ключевые слова: грунтовое основание, балка на упругом основании, метод начальных параметров,
свободные колебания, вынужденные колебания, динамический анализ.

1. INTRODUCTION

In order to fulfill the requirements of mechanical safety of buildings and structures, which are regulated by law [1] and have been developed in modern normative and technical documents [2, 3], it is urgent to study structural systems that change the design scheme for various reasons during local destruction [4, 5, 6]. Taking into account the affecting of sudden local destruction on the stress-strain state and dynamics of structures is an urgent need for predicting their work and assessing the bearing capacity and / or stability. Such structural systems include structures lying on the ground, which can be considered in their design as beams on an elastic foundation. To date, there are a number of works [7, 8, 9] devoted to the study of dynamic processes caused by the sudden formation of defects in beams with partial support on an elastic foundation.

2. MODELS AND METHODS

We consider a "beam-base" system, in which the beam was initially completely on an elastic foundation, but when a defect suddenly formed under a part of the beam, the base was excluded from power work of this structure (Figure 1). Figure 1 shows that the left side of the beam with length αL is located on an

elastic foundation with a constant coefficient r_0, the right side of the beam with length βL is cantilever. It is of interest to solve the problem of determining the natural frequencies and forms of transverse vibrations of a beam with free edges, in the case of an added mass m_1 and a dynamic load $F(t)$ applied at an arbitrary point d when a part of the base under the right part of the beam suddenly has been excluded. The differential equation of forced transverse vibrations of a beam on an elastic foundation of constant cross-section, taking into account the resistance forces for any law of change of the disturbing force $q(x, t)$, has the form [9–11]:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + \mu\frac{\partial^2 y(x,t)}{\partial x^2} +$$
$$+2\alpha\frac{\partial^5 y(x,t)}{\partial x^4 \partial t} + r_0 b\, y(x,t) = q(x,t), \quad (1)$$

where E is elasticity modulus of a beam material; I is inertia moment of a beam cross section, $y(x,t)$ is transverse deflection of the beam axis in the section x; $q(x,t)$ – disturbing load that changes its value in time t; $\mu = q/g$: q – evenly distributed load (dead load) attached along the beam; g – acceleration of gravity; α – coefficient characterizing internal friction of material; $r_0 b y(x,t)$ – the intensity of the reaction of the elastic Winkler foundation that varies its values along the length of the beam [10, 11, 12]; r_0 – modulus of subgrade reaction; b – width of the beam.

We solved the problem in three stages using the method of initial parameters.

At the first stage, we determined the natural transverse vibrations of the beam taking into account its own weight, and at the second stage – taking into account its own weight and the added mass m_1. At the third stage, we solved the problem taking into account the disturbing force, which varies in time according to the harmonic law $F(t) = F \sin \gamma t$ and is applied at an arbitrary point d. Here: F is the amplitude value of the disturbing force; γ is the angular frequency of change in the disturbing force.

<u>The first stage.</u>

Let us determine the circular frequencies and forms of natural transverse vibrations of a beam with free edges of length L and flexural rigidity EI (Figure 1).
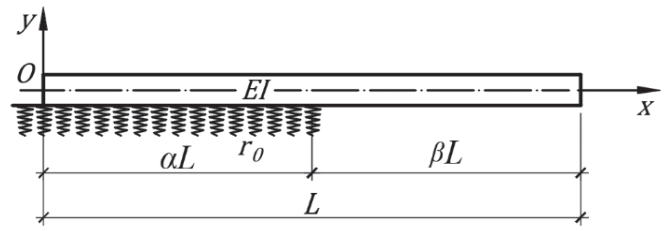

Figure 1. Beam with free edges, the left part of which αL is located on an elastic foundation.

It is known [13] that the dissipation of vibration energy on the frequencies and modes of natural vibrations of building structures affects only slightly, attenuation in their calculations is usually neglected.

A simple periodic solution to the equation of natural vibrations of the beam (1) is the main vibration, which changes according to the harmonic law:

$$y(x,t) = \varphi(x)\sin(\omega t + \alpha), \quad (2)$$

where $\varphi(x)$ – function that establishes the distribution law of the maximum deviations of the points of the beam axis from the equilibrium position; α – initial phase of oscillation; $\omega = \omega_\varkappa$ – the circular frequency of natural transverse vibrations of the beam at the base, and $\omega = \omega_k$ – circular frequency of natural transverse vibrations of a beam without a base, (rad / s).

Using the method of separation of variables, problem (2) can be reduced to the equation of natural vibrations for the left side of the beam αL on the basis of:

$$\varphi^{IV}(x) + \varkappa^4 \varphi(x) = 0, \quad (3)$$

where we accepted designation:

$$\varkappa^4 = \frac{\mu \omega_\varkappa^2 - r}{EI}. \quad (4)$$

For the right side of the beam βL without a base, the equation of natural vibrations is:

$$\varphi^{IV}(x) + k^4 \varphi(x) = 0, \quad (5)$$

where we accepted designation:

$$k^4 = \frac{\mu \omega_k^2}{EI}. \quad (6)$$

The solution of equations (3) and (5) is conveniently represented in the form of Krylov functions:

$$\left.\begin{array}{l} S(x) = \frac{1}{2}(ch\lambda x + cos\lambda x), \\ T(x) = \frac{1}{2}(sh\lambda x + sin\lambda x), \\ U(x) = \frac{1}{2}(ch\lambda x - cos\lambda x), \\ V(x) = \frac{1}{2}(sh\lambda x - sin\lambda x). \end{array}\right\}, \quad (7)$$

where $\lambda = \varkappa$ corresponds to the beam laying on an elastic foundation and $\lambda = k$ corresponds to the beam without foundation.

Let us write down the values of the boundary conditions for a beam with free edges on an elastic foundation:

$$x=0: M(0)=Q(0)=0 \atop x=L: M(L)=Q(L)=0 \Bigg\} \qquad (8)$$

For an arbitrary section of the beam in the first section $0 \leq x_1 \leq \alpha L$, which is located on an elastic foundation, displacements and forces are determined by the equations:

$$\left. \begin{aligned}
y_{1i}(x_1) &= y_{10i}\, S(\varkappa_i x_1) + \theta_{10i}\frac{1}{\varkappa_i}T(\varkappa_i x_1) \\
\theta_{1i}(x_1) &= y_{10i}\, \varkappa_i V(\varkappa_i x_1) + \theta_{10i}S(\varkappa_i x_1) \\
M_{1i}(x_1) &= -EJy_{10i}\, \varkappa_i^2 U(\varkappa_i x_1) - EJ\,\theta_{10i}\varkappa_i V(\varkappa_i x_1) \\
Q_{1i}(x_1) &= -EJy_{10i}\, \varkappa_i^3 T(\varkappa_i x_1) - EJ\theta_{10i}\varkappa_i^2 U(\varkappa_i x_1)
\end{aligned} \right\} \qquad (9)$$

Here $i=1, 2, 3$, etc.

In the second section of the beam without a base $0 \leq x_2 \leq \beta L$ displacements and forces for an arbitrary section are determined:

$$\left. \begin{aligned}
y_{2i}(x_2) &= y_{20i}\, S(k_i x_2) + \frac{\theta_{20i}}{k_i}T(k_i x_2) - \frac{M_{20i}}{k_i^2 EJ}U(k_i x_2) - \frac{Q_{20i}}{k_i^3 EJ}V(k_i x_2) \\
\theta_{2i}(x_2) &= y_{20i}\, k_i V(k_i x_2) + \theta_{20i}S(k_i x_2) - \frac{M_{20i}}{k_i EJ}T(k_i x_2) - \frac{Q_{20i}}{k_i^2 EJ}U(k_i x_2) \\
M_{2i}(x_2) &= -EJy_{20i}\, k_i^2 U(k_i x_2) - EJ\,\theta_{20i}k_i V(k_i x_2) + M_{20i}\, S(k_i x_2) + \frac{Q_{20i}}{k_i}T(k_i x_2) \\
Q_{2i}(x_2) &= -EJy_{20i}\, k_i^3 T(k_i x_2) - EJ\theta_{20i}k_i^2 U(k_i x_2) + M_{20i}\, k_i V(k_i x_2) + Q_{20i}S(k_i x_2)
\end{aligned} \right\} \qquad (10)$$

Using the conditions of conjugation of the sections αL and βL, we express the displacements and forces of the second section through the initial parameters of the first section:

$$\left. \begin{aligned}
y_{2i}(x_2) &= y_{10i}\left[S(\varkappa_i\,\alpha L)S(k_i x_2) + V(\varkappa_i\alpha L)\frac{\varkappa_i}{k_i}T(k_i x_2) + U(\varkappa_i\alpha L)U(k_i x_2)\frac{\varkappa_i^2}{k_i^2} + T(\varkappa_i\alpha L)V(k_i x_2)\frac{\varkappa_i^3}{k_i^3}\right] + \\
&\quad +\theta_{10i}\left[\frac{1}{\varkappa_i}T(\varkappa_i\,\alpha L)S(k_i x_2) + S(\varkappa_i\alpha L)\frac{1}{k_i}T(k_i x_2) + V(\varkappa_i\alpha L)U(k_i x_2)\frac{\varkappa_i}{k_i^2} + U(\varkappa_i\alpha L)V(k_i x_2)\frac{\varkappa_i^2}{k_i^3}\right] \\[4pt]
\theta_{2i}(x_2) &= y_{10i}\left[k_i S(\varkappa_i\,\alpha L)V(k_i x_2) + \varkappa_i V(\varkappa_i\alpha L)S(k_i x_2) + \frac{\varkappa_i^2}{k_i}U(\varkappa_i\alpha L)T(k_i x_2) + \frac{\varkappa_i^3}{k_i^2}T(\varkappa_i\alpha L)U(k_i x_2)\right] + \\
&\quad +\theta_{10i}\left[\frac{k_i}{\varkappa_i}T(\varkappa_i\,\alpha L)V(k_i x_2) + S(\varkappa_i\alpha L)S(k_i x_2) + \frac{\varkappa_i}{k_i}V(\varkappa_i\alpha L)T(k_i x_2) + U(\varkappa_i\alpha L)\frac{\varkappa_i^2}{k_i^2}U(k_i x_2)\right] \\[4pt]
M_{2i}(x_2) &= -EJy_{10i}\left[k_i^2 S(\varkappa_i\,\alpha L)U(k_i x_2) + \varkappa_i k_i\, V(\varkappa_i\alpha L)V(k_i x_2) + \varkappa_i^2 U(\varkappa_i\alpha L)S(k_i x_2) + \frac{\varkappa_i^3}{k_i}T(\varkappa_i\alpha L)T(k_i x_2)\right] - \\
&\quad -EJ\theta_{10i}\left[\frac{k_i^2}{\varkappa_i}T(\varkappa_i\,\alpha L)U(k_i x_2) + k_i S(\varkappa_i\alpha L)V(k_i x_2) + \varkappa_i\, V(\varkappa_i\alpha L)S(k_i x_2) + \frac{\varkappa_i^2}{k_i}U(\varkappa_i\alpha L)T(k_i x_2)\right] \\[4pt]
Q_{2i}(x_2) &= -EJy_{10i}\left[k_i^3 S(\varkappa_i\alpha L)T(k_i x_2) + \varkappa_i k_i^2 V(\varkappa_i\alpha L)U(k_i x_2) + \varkappa_i^2 k_i U(\varkappa_i\alpha L)V(k_i x_2) + \varkappa_i^3 T(\varkappa_i\alpha L)S(k_i x_2)\right] - \\
&\quad -EJ\theta_{10i}\left[\frac{k_i^3}{\varkappa_i}T(\varkappa_i\alpha L)T(k_i x_2) + k_i^2\, S(\varkappa_i\alpha L)U(k_i x_2) + \varkappa_i k_i V(\varkappa_i\alpha L)V(k_i x_2) + \varkappa_i^2 U(\varkappa_i\alpha L)S(k_i x_2)\right]
\end{aligned} \right\} \qquad (11)$$

Using the boundary conditions on the right edge (8) at $x_2 = \beta L$, we obtain the system of equations:

$$
\begin{cases}
M_{2i}(\beta L) = -EJy_{10i}\left[k_i^2 S(\varkappa_i\,\alpha L)U(k_i\beta L) + \varkappa_i k_i\,V(\varkappa_i\alpha L)V(k_i\beta L) + \varkappa_i^2 U(\varkappa_i\alpha L)S(k_i\beta L) + \dfrac{\varkappa_i^3}{k_i}T(\varkappa_i\alpha L)\,T(k_i\beta L)\right] - \\[2mm]
-EJ\theta_{10i}\left[\dfrac{k_i^2}{\varkappa_i}T(\varkappa_i\,\alpha L)U(k_i\beta L) + k_i S(\varkappa_i\alpha L)V(k_i\beta L) + \varkappa_i\,V(\varkappa_i\alpha L)\,S(k_i\beta L) + \dfrac{\varkappa_i^2}{k_i}U(\varkappa_i\alpha L)\,T(k_i\beta L)\right] = 0 \\[2mm]
Q_{2i}(\beta L) = -EJy_{10i}\left[k_i^3 S(\varkappa_i\alpha L)T(k_i\beta L) + \varkappa_i k_i^2 V(\varkappa_i\alpha L)\,U(k_i\beta L) + \varkappa_i^2 k_i U(\varkappa_i\alpha L)\,V(k_i\beta L) + \varkappa_i^3 T(\varkappa_i\alpha L)S(k_i\beta L)\right] - \\[2mm]
-EJ\theta_{10i}\left[\dfrac{k_i^3}{\varkappa_i}T(\varkappa_i\alpha L)T(k_i\beta L) + k_i^2\,S(\varkappa_i\alpha L)\,U(k_i\beta L) + \varkappa_i k_i V(\varkappa_i\alpha L)V(k_i\beta L) + \varkappa_i^2 U(\varkappa_i\alpha L)\,S(k_i\beta L)\right] = 0
\end{cases}
\tag{12}
$$

For a nontrivial solution of equations (12), it is necessary that the determinant, composed of the coefficients at arbitrary constants $EJ\,y_{10}$ and $EJ\,\theta_{10i}$, be equal to zero:

$$
\begin{aligned}
D &= \left[k_i^2 S(\varkappa_i\,\alpha L)U(k_i\beta L) + \varkappa_i k_i\,V(\varkappa_i\alpha L)V(k_i\beta L) + \varkappa_i^2 U(\varkappa_i\alpha L)S(k_i\beta L) + \dfrac{\varkappa_i^3}{k_i}T(\varkappa_i\alpha L)\,T(k_i\beta L)\right] * \\[2mm]
&* \left[\dfrac{k_i^3}{\varkappa_i}T(\varkappa_i\alpha L)T(k_i\beta L) + k_i^2\,S(\varkappa_i\alpha L)\,U(k_i\beta L) + \varkappa_i k_i V(\varkappa_i\alpha L)V(k_i\beta L) + \varkappa_i^2 U(\varkappa_i\alpha L)\,S(k_i\beta L)\right] - \\[2mm]
&- \left[\dfrac{k_i^2}{\varkappa_i}T(\varkappa_i\,\alpha L)U(k_i\beta L) + k_i S(\varkappa_i\alpha L)V(k_i\beta L) + \varkappa_i\,V(\varkappa_i\alpha L)\,S(k_i\beta L) + \dfrac{\varkappa_i^2}{k_i}U(\varkappa_i\alpha L)\,T(k_i\beta L)\right] * \\[2mm]
&* \left[k_i^3 S(\varkappa_i\alpha L)T(k_i\beta L) + \varkappa_i k_i^2 V(\varkappa_i\alpha L)\,U(k_i\beta L) + \varkappa_i^2 k_i U(\varkappa_i\alpha L)\,V(k_i\beta L) + \varkappa_i^3 T(\varkappa_i\alpha L)S(k_i\beta L)\right] = 0
\end{aligned}
\tag{13}
$$

The roots of equation (13) are the countless row of values k_i and $\varkappa$. In order to solve the equation, we introduce the relation $k_i = \varepsilon_i\,\varkappa_i$. Here ε is constant value. For each root value k_i and $\varkappa$ a certain angular frequency of natural transverse vibrations corresponds.

Using expression (4), we obtain a formula for determining $\omega_{i\varkappa}$ circular frequencies of natural transverse vibrations of a part of a beam αL on an elastic foundation:

$$
\omega_{i\varkappa} = \sqrt{\dfrac{EI\lambda_{i\varkappa}^4}{\mu(\alpha L)^4} + \dfrac{r}{\mu}}, \tag{14}
$$

where $\lambda_{i\varkappa} = \varkappa_i\alpha L$, and $i = 1, 2, 3$ etc. – frequency sequence number.

For a part of the beam βL without a base, using (6), we get:

$$
\omega_{ik} = \sqrt{\dfrac{EI\,\lambda_{ik}^4}{\mu(\beta L)^4}}, \tag{15}
$$

where $\lambda_{ik} = k_i\beta L$.

Let us determine the natural angular frequencies of transverse vibrations of the beam parts αL on the base and βL without the base, which form the spectra $\omega_{1\varkappa} < \omega_{2\varkappa} < ... < \omega_{n\varkappa}$ and $\omega_{1k} < \omega_{2k} < ... < \omega_{nk}$.

To determine the modes of natural vibrations, we substitute the values of the roots value k_i and $\varkappa_i$ into

the solution of the first equation (11), which will determine the values of the relative ordinates i-th of that form of natural vibrations.

The second stage.

Let us determine the natural angular frequencies and forms of transverse vibrations, taking into account the own weight and the added mass m_1 at point d (Figure 2).
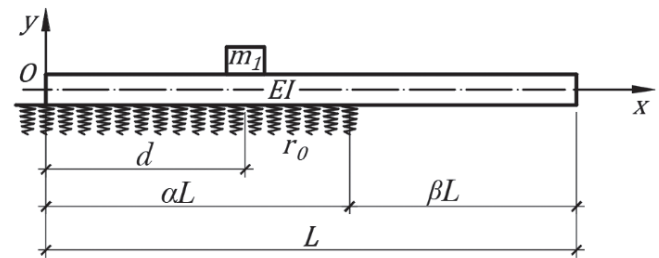

Figure 2. Beam with added mass m_1.

For an arbitrary section of the beam in the first section $0 \leq x_1 \leq \alpha L$ displacements and forces are determined by equations (9) only up to the point of application of the mass. For $x_1 > d$ free vibrations of the beam occur with the inertial force I. At point d we add the inertial force I and compose the system of equations:

$$
\left.\begin{array}{l}
y_{1i}(x_1) = y_{10i}\, S(\varkappa_i x_1) + \dfrac{\theta_{10i}}{\varkappa_i} T(\varkappa_i x_1) + \dfrac{I}{\varkappa_i^3 EI} V(\varkappa_i(x_1 - d)) \\[2mm]
\theta_{1i}(x_1) = y_{10i}\, \varkappa_i V(\varkappa_i x_1) + \theta_{10i} S(\varkappa_i x_1) + \dfrac{I}{\varkappa_i^2 EI} U(\varkappa_i(x_1 - d)) \\[2mm]
M_{1i}(x_1) = -EJ y_{10i}\, \varkappa_i^2 U(\varkappa_i x_1) - EJ\, \theta_{10i} \varkappa_i V(\varkappa_i x_1) - \dfrac{I}{\varkappa_i} T(\varkappa_i(x_1 - d)) \\[2mm]
Q_{1i}(x_1) = -EJ y_{10i}\, \varkappa_i^3 T(\varkappa_i x_1) - EJ \theta_{10i} \varkappa_i^2 U(\varkappa_i x_1) - IS(\varkappa_i(x_1 - d))
\end{array}\right\}, \tag{16}
$$

where $I = m_1 \omega_i^2 \left[y_0\, S(\varkappa_i d) + \dfrac{\theta_0}{\varkappa_i} T(\varkappa_i d) \right].$ (17)

Further, we have composed formulas for determining the deflections, angles of rotation, moments and shear forces of the second section of the beam without a base $0 \le x_2 \le \beta L$ using (9) and the conjugation conditions:

$$
\left.\begin{array}{l}
y_{2i}(x_2) = y_{10i}\left[S(\varkappa_i\,\alpha L)S(k_i x_2) + V(\varkappa_i\alpha L)\dfrac{\varkappa_i}{k_i}T(k_i x_2) + U(\varkappa_i\alpha L)U(k_i x_2)\dfrac{\varkappa_i^2}{k_i^2} + T(\varkappa_i\alpha L)V(k_i x_2)\dfrac{\varkappa_i^3}{k_i^3} \right] + \\[2mm]
\qquad + \theta_{10i}\left[\dfrac{1}{\varkappa_i}T(\varkappa_i\,\alpha L)S(k_i x_2) + S(\varkappa_i\alpha L)\dfrac{1}{k_i}T(k_i x_2) + V(\varkappa_i\alpha L)U(k_i x_2)\dfrac{\varkappa_i}{k_i^2} + U(\varkappa_i\alpha L)V(k_i x_2)\dfrac{\varkappa_i^2}{k_i^3} \right] + \\[2mm]
\qquad + \dfrac{I}{EI}\left[V(\varkappa_i(\alpha L - d))\dfrac{S(k_i x_2)}{\varkappa_i^3} + U(\varkappa_i(\alpha L - d))\dfrac{T(k_i x_2)}{\varkappa_i^2 k_i} + T(\varkappa_i(\alpha L - d))\dfrac{U(k_i x_2)}{\varkappa_i k_i^2} + S(\varkappa_i(\alpha L - d))\dfrac{V(k_i x_2)}{k_i^3} \right] \\[3mm]
\theta_{2i}(x_2) = y_{10i}\left[k_i S(\varkappa_i\,\alpha L) V(k_i x_2) + \varkappa_i V(\varkappa_i\alpha L) S(k_i x_2) + \dfrac{\varkappa_i^2}{k_i}U(\varkappa_i\alpha L) T(k_i x_2) + \dfrac{\varkappa_i^3}{k_i^2}T(\varkappa_i\alpha L)U(k_i x_2) \right] + \\[2mm]
\qquad + \theta_{10i}\left[\dfrac{k_i}{\varkappa_i}T(\varkappa_i\,\alpha L) V(k_i x_2) + S(\varkappa_i\alpha L) S(k_i x_2) + \dfrac{\varkappa_i}{k_i}V(\varkappa_i\alpha L)T(k_i x_2) + U(\varkappa_i\alpha L)\dfrac{\varkappa_i^2}{k_i^2}U(k_i x_2) \right] + \\[2mm]
\qquad + \dfrac{I}{EI}\left[V(\varkappa_i(\alpha L - d))\dfrac{k_i V(k_i x_2)}{\varkappa_i^3} + U(\varkappa_i(\alpha L - d))\dfrac{S(k_i x_2)}{\varkappa_i^2} + T(\varkappa_i(\alpha L - d))\dfrac{T(k_i x_2)}{\varkappa_i k_i} + S(\varkappa_i(\alpha L - d))\dfrac{U(k_i x_2)}{k_i^2} \right] \\[3mm]
M_{2i}(x_2) = -EJ y_{10i}\left[k_i^2 S(\varkappa_i\,\alpha L)U(k_i x_2) + \varkappa_i k_i\, V(\varkappa_i\alpha L)V(k_i x_2) + \varkappa_i^2 U(\varkappa_i\alpha L)S(k_i x_2) + \dfrac{\varkappa_i^3}{k_i}T(\varkappa_i\alpha L)\, T(k_i x_2) \right] - \\[2mm]
\qquad - EJ\theta_{10i}\left[\dfrac{k_i^2}{\varkappa_i}T(\varkappa_i\,\alpha L)U(k_i x_2) + k_i S(\varkappa_i\alpha L)V(k_i x_2) + \varkappa_i\, V(\varkappa_i\alpha L)\, S(k_i x_2) + \dfrac{\varkappa_i^2}{k_i}U(\varkappa_i\alpha L)\, T(k_i x_2) \right] - \\[2mm]
\qquad - \dfrac{I}{\varkappa_i^3}\left[k_i^2 V(\varkappa_i(\alpha L - d))U(k_i x_2) + \varkappa_i k_i U(\varkappa_i(\alpha L - d))V(k_i x_2) + \varkappa_i^2 T(\varkappa_i(\alpha L - d))S(k_i x_2) + \dfrac{\varkappa_i^3}{k_i}S(\varkappa_i(\alpha L - d))T(k_i x_2) \right] \\[3mm]
Q_{2i}(x_2) = -EJ y_{10i}[k_i^3 S(\varkappa_i\alpha L)T(k_i x_2) + \varkappa_i k_i^2 V(\varkappa_i\alpha L)\, U(k_i x_2) + \varkappa_i^2 k_i U(\varkappa_i\alpha L)\, V(k_i x_2) + \varkappa_i^3 T(\varkappa_i\alpha L)S(k_i x_2)] - \\[2mm]
\qquad - EJ\theta_{10i}\left[\dfrac{k_i^3}{\varkappa_i}T(\varkappa_i\alpha L)T(k_i x_2) + k_i^2\, S(\varkappa_i\alpha L)\, U(k_i x_2) + \varkappa_i k_i V(\varkappa_i\alpha L)V(k_i x_2) + \varkappa_i^2 U(\varkappa_i\alpha L)\, S(k_i x_2) \right] - \\[2mm]
\qquad - \dfrac{I}{\varkappa_i^3}[k_i^3 T(k_i x_2)V(\varkappa_i(\alpha L - d)) + \varkappa_i\, k_i^2 U(k_i x_2)U(\varkappa_i(\alpha L - d)) + \varkappa_i^2 k_i V(k_i x_2)T(\varkappa_i(\alpha L - d)) + \varkappa_i^3 S(k_i x_2)S(\varkappa_i(\alpha L - d))]
\end{array}\right\} \tag{18}
$$

We denote:

$$
a_1 = \left[k_i^2 S(\varkappa_i\,\alpha L)U(k_i \beta L) + \varkappa_i k_i\, V(\varkappa_i\alpha L)V(k_i \beta L) + \varkappa_i^2 U(\varkappa_i\alpha L)S(k_i \beta L) + \dfrac{\varkappa_i^3}{k_i}T(\varkappa_i\alpha L)\, T(k_i \beta L) \right];
$$

$$
a_2 = \left[\dfrac{k_i^2}{\varkappa_i}T(\varkappa_i\,\alpha L)U(k_i \beta L) + k_i S(\varkappa_i\alpha L)V(k_i \beta L) + \varkappa_i\, V(\varkappa_i\alpha L)\, S(k_i \beta L) + \dfrac{\varkappa_i^2}{k_i}U(\varkappa_i\alpha L)\, T(k_i \beta L) \right];
$$

$$
a_3 = \left[k_i^2 V(\varkappa_i(\alpha L - d))U(k_i \beta L) + \varkappa_i k_i U(\varkappa_i(\alpha L - d))V(k_i \beta L) + \varkappa_i^2 T(\varkappa_i(\alpha L - d))S(k_i \beta L) + \right.
$$
$$
\left. + \dfrac{\varkappa_i^3}{k_i}S(\varkappa_i(\alpha L - d))T(k_i \beta L) \right];
$$

$$
a_4 = \left[k_i^3 S(\varkappa_i\alpha L)T(k_i \beta L) + \varkappa_i k_i^2 V(\varkappa_i\alpha L)\, U(k_i \beta L) + \varkappa_i^2 k_i U(\varkappa_i\alpha L)\, V(k_i \beta L) + \varkappa_i^3 T(\varkappa_i\alpha L)S(k_i \beta L) \right];
$$

$$
a_5 = \left[\dfrac{k_i^3}{\varkappa_i}T(\varkappa_i\alpha L)T(k_i \beta L) + k_i^2\, S(\varkappa_i\alpha L)\, U(k_i \beta L) + \varkappa_i k_i V(\varkappa_i\alpha L)V(k_i \beta L) + \varkappa_i^2 U(\varkappa_i\alpha L)\, S(k_i \beta L) \right];
$$

$$
a_6 = \left[k_i^3 T(k_i \beta L)V(\varkappa_i(\alpha L - d)) + \varkappa_i\, k_i^2 U(k_i \beta L)U(\varkappa_i(\alpha L - d)) + \varkappa_i^2 k_i V(k_i \beta L)T(\varkappa_i(\alpha L - d)) + \right.
$$
$$
\left. + \varkappa_i^3 S(k_i \beta L)S(\varkappa_i(\alpha L - d)) \right].
$$

Using the boundary conditions on the right edge (8) at $x_2 = \beta L$, taking into account the inertial force I (17), we obtain the system of equations:

$$\begin{cases} y_{10i}\left[a_1 + \frac{m_1\omega_i^2}{\varkappa_i^3 EJ}S(\varkappa_i d)a_3\right] + \theta_{10i}\left[a_2 + \frac{m_1\omega_i^2}{\varkappa_i^4 EJ}T(\varkappa_i d)a_3\right] = 0 \\ y_{10i}\left[a_4 + \frac{m_1\omega_i^2}{\varkappa_i^3 EJ}S(\varkappa_i d)a_6\right] + \theta_{10i}\left[a_5 + \frac{m_1\omega_i^2}{\varkappa_i^4 EJ}T(\varkappa_i d)a_6\right] = 0 \end{cases} \quad (19)$$

The determinant of this system:

$$D = \left[a_1 + \frac{m_1\varkappa_i}{\mu}\left(1 + \frac{r}{\varkappa_i^4 EJ}\right)S(\varkappa_i d)a_3\right]\left[a_5 + \frac{m_1\varkappa_i}{\mu}\left(1 + \frac{r}{\varkappa_i^4 EJ}\right)T(\varkappa_i d)a_6\right] - $$
$$- \left[a_2 + \frac{m_1\varkappa_i}{\mu}(1 + \frac{r}{\varkappa_i^4 EJ})T(\varkappa_i d)a_3\right]\left[a_4 + \frac{m_1\varkappa_i}{\mu}(1 + \frac{r}{\varkappa_i^4 EJ})S(\varkappa_i d)a_6\right] = 0 \quad (20)$$

Defining a set of values k_i and $\varkappa$ we perform introducing constant ε_i. Using expressions (14) and (15), we determine the values $\omega_{i\varkappa}$ circular frequencies of natural transverse vibrations of a part of the beam αL on an elastic foundation and the values ω_{ik} for part of the βL beam without base.

In order to determine the modes of natural vibrations, the values of the roots k_i and $\varkappa$ substitute in the solution of the first equation (18), which determines the values of the relative ordinates of i-th form of natural vibrations.

The third stage.

Let us determine the efforts under the action of a dynamic load $F(t) = F\sin yt$, applied at an arbitrary point d (Figure 3) for the same beam.
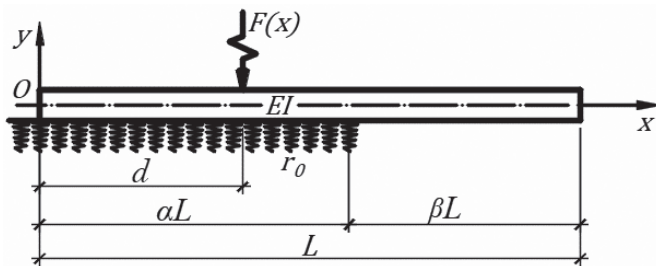

Figure 3. Beam with dynamic force F(t)

Let's return to the differential equation of forced vibrations of the beam (1). We assume that the disturbing force acts according to the law $q(x,t) = q(x)\sin yt$. Assuming that forced vibrations also change according to a harmonic law:

$$y(x,t) = \varphi(x)\sin(yt), \quad (21)$$

we obtain an inhomogeneous differential equation of forced vibrations of a beam on an elastic foundation:

$$\varphi^{IV}(x) + \varkappa^4\varphi(x) = q(x), \quad (22)$$

where: $\varkappa^4 = \frac{\mu y^2 - r}{EI}$. $\quad (23)$

For a beam without a base, the inhomogeneous differential equation of forced vibrations takes the form:

$$\varphi^{IV}(x) + k^4\varphi(x) = q(x), \quad (24)$$

where: $k^4 = \frac{\mu y^2}{EI}$. $\quad (25)$

We have obtain the general solutions of the inhomogeneous equations (22) and (24) as the sum of the general solutions of the homogeneous equation and the particular solution, which depends on the type of load. Further, using the method of initial parameters, we have obtain universal formulas for determining deflections, angles of rotation, moments and shear forces for an arbitrary section of the beam in the general case of the action of a disturbing load $q(x,t)$.

We use the values of the boundary conditions on the left and right edges of the beam (8).

For an arbitrary section of the beam in the first section $0 \le x_1 \le \alpha L$, that located on an elastic foundation and under the action of a dynamic load $F(t)$, that applied at an arbitrary point d, displacements and forces are determined by the equations:

$$
\left.\begin{array}{l}
y_{1i}(x_1) = y_{10i} S(\varkappa_i x_1) + \theta_{10i} \dfrac{1}{\varkappa_i} T(\varkappa_i x_1) + \dfrac{F \sin \gamma t}{\varkappa_i{}^3 EI} V[\varkappa_i (x_1 - d)] \\[2mm]
\theta_{1i}(x_1) = y_{10i} \varkappa_i V(\varkappa_i x_1) + \theta_{10i} S(\varkappa_i x_1) + \dfrac{F \sin \gamma t}{\varkappa_i{}^2 EI} U[\varkappa_i (x_1 - d)] \\[2mm]
M_{1i}(x_1) = -EJ y_{10i} \varkappa_i^2 U(\varkappa_i x_1) - EJ \theta_{10i} \varkappa_i V(\varkappa_i x_1) - \dfrac{F \sin \gamma t}{\varkappa_i} T[\varkappa_i (x_1 - d)] \\[2mm]
Q_{1i}(x_1) = -EJ y_{10i} \varkappa_i^3 T(\varkappa_i x_1) - EJ \theta_{10i} \varkappa_i^2 U(\varkappa_i x_1) - F \sin \gamma t \, S[\varkappa_i (x_1 - d)]
\end{array}\right\} \quad (26)
$$

In the second section of the beam without a base $0 \le x_2 \le \beta L$ the displacements and forces for an arbitrary section are determined by (10). Using the conditions of conjugation of the sections and, expressing the displacements and forces of the second section through the initial parameters of the first section, we get:

$$
\left.\begin{array}{l}
y_{2i}(x_2) = y_{10i}\left[S(\varkappa_i \alpha L) S(k_i x_2) + \dfrac{\varkappa_i}{k_i} V(\varkappa_i \alpha L) T(k_i x_2) + \dfrac{\varkappa_i^2}{k_i^2} U(\varkappa_i \alpha L) U(k_i x_2) + \dfrac{\varkappa_i^3}{k_i^3} T(\varkappa_i \alpha L) V(k_i x_2)\right] + \\[2mm]
\quad + \theta_{10i}\left[\dfrac{1}{\varkappa_i} T(\varkappa_i \alpha L) S(k_i x_2) + \dfrac{1}{k_i} S(\varkappa_i \alpha L) T(k_i x_2) + \dfrac{\varkappa_i}{k_i^2} V(\varkappa_i \alpha L) U(k_i x_2) + \dfrac{\varkappa_i^2}{k_i^3} U(\varkappa_i \alpha L) V(k_i x_2)\right] + \\[2mm]
\quad + \dfrac{F \sin \gamma t}{EI}\left[\dfrac{V[\varkappa_i(\alpha L - d)]}{\varkappa_i^3} S(k_i x_2) + \dfrac{U[\varkappa_i(\alpha L - d)]}{\varkappa_i^2 k_i} T(k_i x_2) + \dfrac{T[\varkappa_i(\alpha L - d)]}{\varkappa_i k_i^2} U(k_i x_2) + \dfrac{S[\varkappa_i(\alpha L - d)]}{k_i^3} V(k_i x_2)\right] \\[3mm]
\theta_{2i}(x_2) = y_{10i}\left[k_i S(\varkappa_i \alpha L) V(k_i x_2) + \varkappa_i V(\varkappa_i \alpha L) S(k_i x_2) + \dfrac{\varkappa_i^2}{k_i} U(\varkappa_i \alpha L) T(k_i x_2) + \dfrac{\varkappa_i^3}{k_i^2} T(\varkappa_i \alpha L) U(k_i x_2)\right] + \\[2mm]
\quad + \theta_{10i}\left[\dfrac{k_i}{\varkappa_i} T(\varkappa_i \alpha L) V(k_i x_2) + S(\varkappa_i \alpha L) S(k_i x_2) + \dfrac{\varkappa_i}{k_i} V(\varkappa_i \alpha L) T(k_i x_2) + \dfrac{\varkappa_i^2}{k_i^2} U(\varkappa_i \alpha L) U(k_i x_2)\right] + \\[2mm]
\quad + \dfrac{F \sin \gamma t}{EI}\left[\dfrac{V[\varkappa_i(\alpha L - d)]}{\varkappa_i^3} k_i V(k_i x_2) + \dfrac{U[\varkappa_i(\alpha L - d)]}{\varkappa_i^2} S(k_i x_2) + \dfrac{T[\varkappa_i(\alpha L - d)]}{\varkappa_i k_i} T(k_i x_2) + \dfrac{S[\varkappa_i(\alpha L - d)]}{k_i^2} U(k_i x_2)\right] \\[3mm]
M_{2i}(x_2) = -EJ y_{10i}\left[k_i^2 S(\varkappa_i \alpha L) U(k_i x_2) + \varkappa_i k_i V(\varkappa_i \alpha L) V(k_i x_2) + \varkappa_i^2 U(\varkappa_i \alpha L) S(k_i x_2) + \dfrac{\varkappa_i^3}{k_i} T(\varkappa_i \alpha L) T(k_i x_2)\right] - \\[2mm]
\quad - EJ \theta_{10i}\left[\dfrac{k_i^2}{\varkappa_i} T(\varkappa_i \alpha L) U(k_i x_2) + k_i S(\varkappa_i \alpha L) V(k_i x_2) + \varkappa_i V(\varkappa_i \alpha L) S(k_i x_2) + \dfrac{\varkappa_i^2}{k_i} U(\varkappa_i \alpha L) T(k_i x_2)\right] - \\[2mm]
\quad - \dfrac{F \sin \gamma t}{\varkappa_i^3}\left[k_i^2 U(k_i x_2) V[\varkappa_i(\alpha L - d)] + \varkappa_i k_i V(k_i x_2) U[\varkappa_i(\alpha L - d)] + \varkappa_i^2 T[\varkappa_i(\alpha L - d)] S(k_i x_2) + + \dfrac{\varkappa_i^3}{k_i} S[\varkappa_i(\alpha L - d)] T(k_i x_2)\right] \\[3mm]
Q_{2i}(x_2) = -EJ y_{10i}[k_i^3 T(k_i x_2) S(\varkappa_i \alpha L) + \varkappa_i k_i^2 U(k_i x_2) V(\varkappa_i \alpha L) + \varkappa_i^2 k_i V(k_i x_2) U(\varkappa_i \alpha L) + \varkappa_i^3 S(k_i x_2) T(\varkappa_i \alpha L)] - \\[2mm]
\quad - EJ \theta_{10i}\left[\dfrac{k_i^3}{\varkappa_i} T(\varkappa_i \alpha L) T(k_i x_2) + k_i^2 U(k_i x_2) S(\varkappa_i \alpha L) + \varkappa_i k_i V(\varkappa_i \alpha L) V(k_i x_2) + \varkappa_i^2 U(\varkappa_i \alpha L) S(k_i x_2)\right] - \\[2mm]
\quad - \dfrac{F \sin \gamma t}{\varkappa_i^3}\left[k_i^3 T(k_i x_2) V[\varkappa_i(\alpha L - d)] + \varkappa_i k_i^2 U[\varkappa_i(\alpha L - d)] U(k_i x_2) + \varkappa_i^2 k_i V(k_i x_2) T[\varkappa_i(\alpha L - d)] + + \varkappa_i^3 S(k_i x_2) S[\varkappa_i(\alpha L - d)]\right]
\end{array}\right\} \quad (27)
$$

We denote:

$$
a_1 = \left[k_i^2 S(\varkappa_i \alpha L) U(k_i \beta L) + \varkappa_i k_i V(\varkappa_i \alpha L) V(k_i \beta L) + \varkappa_i^2 U(\varkappa_i \alpha L) S(k_i \beta L) + \dfrac{\varkappa_i^3}{k_i} T(\varkappa_i \alpha L) T(k_i \beta L)\right];
$$

$$
a_2 = \left[\dfrac{k_i^2}{\varkappa_i} T(\varkappa_i \alpha L) U(k_i \beta L) + k_i S(\varkappa_i \alpha L) V(k_i \beta L) + \varkappa_i V(\varkappa_i \alpha L) S(k_i \beta L) + \dfrac{\varkappa_i^2}{k_i} U(\varkappa_i \alpha L) T(k_i \beta L)\right];
$$

$$
a_3 = \left[k_i^2 U(k_i \beta L) V[\varkappa_i(\alpha L - d)] + \varkappa_i k_i V(k_i \beta L) U[\varkappa_i(\alpha L - d)] + \varkappa_i^2 T[\varkappa_i(\alpha L - d)] S(k_i \beta L) + \right.
$$
$$
\left. + \dfrac{\varkappa_i^3}{k_i} S[\varkappa_i(\alpha L - d)] T(k_i \beta L)\right];
$$

$$
a_4 = \left[k_i^3 T(k_i \beta L) S(\varkappa_i \alpha L) + \varkappa_i k_i^2 U(k_i \beta L) V(\varkappa_i \alpha L) + \varkappa_i^2 k_i V(k_i \beta L) U(\varkappa_i \alpha L) + \varkappa_i^3 S(k_i \beta L) T(\varkappa_i \alpha L)\right];
$$

$$
a_5 = \left[\dfrac{k_i^3}{\varkappa_i} T(\varkappa_i \alpha L) T(k_i \beta L) + k_i^2 U(k_i \beta L) S(\varkappa_i \alpha L) + \varkappa_i k_i V(\varkappa_i \alpha L) V(k_i \beta L) + \varkappa_i^2 U(\varkappa_i \alpha L) S(k_i \beta L)\right];
$$

$$
a_6 = \left[k_i^3 T(k_i \beta L) V[\varkappa_i(\alpha L - d)] + \varkappa_i k_i^2 U[\varkappa_i(\alpha L - d)] U(k_i \beta L) + \varkappa_i^2 k_i V(k_i \beta L) T[\varkappa_i(\alpha L - d)] + \right.
$$
$$
\left. + \varkappa_i^3 S(k_i \beta L) S[\varkappa_i(\alpha L - d)]\right].
$$

Using the boundary conditions on the right edge (8) for $x_2 = \beta L$, we obtain a system of equations for determining y_{10i} and θ_{10i}:

$$\begin{cases} a_1 y_{10i} + a_2 \theta_{10i} = \dfrac{-F \sin \gamma t}{EJ \varkappa_i^3} a_3 \\ a_4 y_{10i} + a_5 \theta_{10i} = \dfrac{-F \sin \gamma t}{EJ \varkappa_i^3} a_6 \end{cases} \quad (28)$$

Using (26) and (28), at a given frequency of forced oscillations γ, we determine $\varkappa$ and k:

$$\varkappa = \sqrt[4]{\dfrac{\mu \gamma^2 - r}{EJ}}, \quad (29)$$

$$k = \sqrt[4]{\dfrac{\mu \gamma^2}{EJ}}. \quad (30)$$

Applying equations (26) and (27) taking into account certain values of the roots $\varkappa$ and k, that corresponds to given frequency γ of forced vibrations, and values of $F(t)$, we have determine forces in the beam under forced vibrations.

3. RESULTS AND ANALYSIS

Initial for calculations: beam width b=1.25 m, height h=1.5 m, length L=12.0 m, elasticity modulus of material E=2,1x10^6 t/m^2, modulus of subgrade reaction r_0=5000 t/m^3, force F=10.0 t, mass m_1 = F/g= 1.0194 t.

At the first and second stages, according to the results of calculations of the beam with $\alpha L = \beta L$, the values of the roots $\varkappa_i$ and k_i of the equations (13) and (20) are adopted such that ε_i from $k_i = \varepsilon_i \varkappa_i$ equal 0.5; 1.0 and 2.0. Also, the value ε_i is taken from the condition of equality of natural frequencies of transverse vibrations $\omega_{i\varkappa} = \omega_{ik}$ of two parts of beam. Root values $\varkappa_i$ and k_i, indicated in column 7 of tables 1 and 2 for a beam on a full base are defined in [15], in column 8 for a beam without a base – in [14]. The calculation results are presented in table 1.

At the first stage, the first three modes of beam vibrations were constructed with $\alpha L = \beta L$ without added mass m_1 (Figures 4, 5 and 6) corresponding to natural frequencies for ε.

Further, at the second stage, according to the results of calculations of a beam with an added mass m_1 located in a quarter of the beam $d = L / 4$ at $\alpha L =$
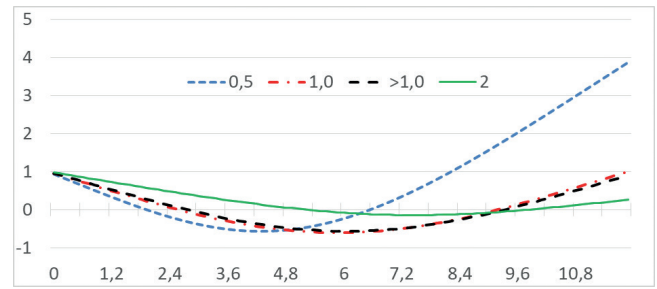

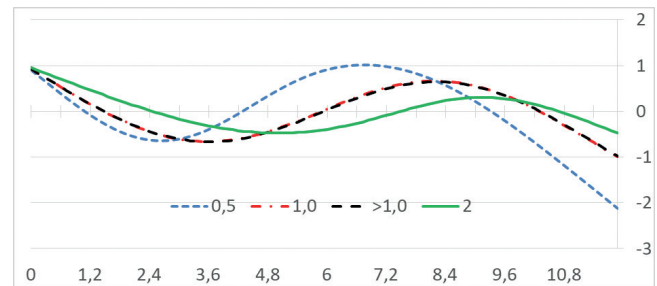Figure 4. *1st mode of vibration with αL = βL without mass m₁*
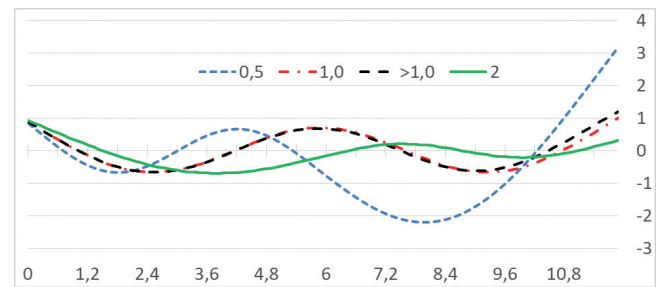

Figure 5. *The 2nd mode of vibration at αL = βL without mass m₁*

Figure 6. *The 3rd mode of vibration at αL = βL without mass m₁*

Table 1. Roots and natural angular frequencies (rad / sec) of transverse vibrations for αL= β.

		ε_i		$\omega_{i\varkappa} = \omega_{ik}$		Beam on the base	Beam without base
		0.5	1.0		2.0		
1	2	3	4	5	6	7	8
1 mode	$\varkappa_1$	0.5264	0.3942		0.3762	0.2632	0.3942	-
	k_1	0.2632			0.4108	0.5264	-	0.3942
	$\omega_1^\varkappa$	254.7	161.7	228.9	156.2	161.7	-
	ω_1^k	59.4	133.3		375.8	-	133.3
2 mode	$\varkappa_1$	0.8739	0.6544		0.6507	0.4369	0.6544	-
	k_2	0.4369			0.6582	0.8738	-	0.6544
	$\omega_2^\varkappa$	661.4	378.5	587.6	287.4	378.5	-
	ω_2^k	163.7	367.3		1035.5	-	367.3
3 mode	$\varkappa_1$	1.2566	0.9163		0.9149	0.6283	0.9163	-
	k_3	0.6283			0.9177	1.2566	-	0.9163
	$\omega_3^\varkappa$	1357.5	726.0	1142.2	549.7	726.0	-
	ω_3^k	338.6	720.2		2141.6	-	720.2

βL, we obtain the values of the roots ϰ and k_i. The calculation results are presented in Table 2.

The first three modes of vibrations of the beam are constructed for αL = βL with the added mass m1 located at the point d = L / 4. Vibration modes corresponding to natural frequencies for ε are presented in Figures 7, 8 and 9.
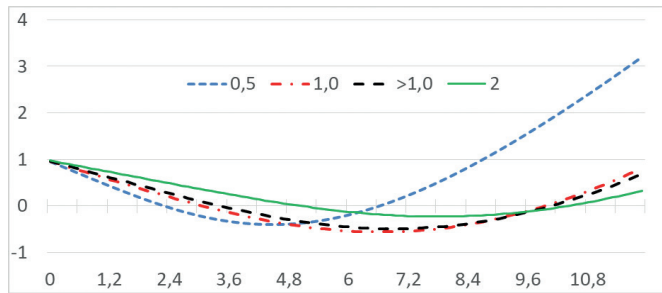

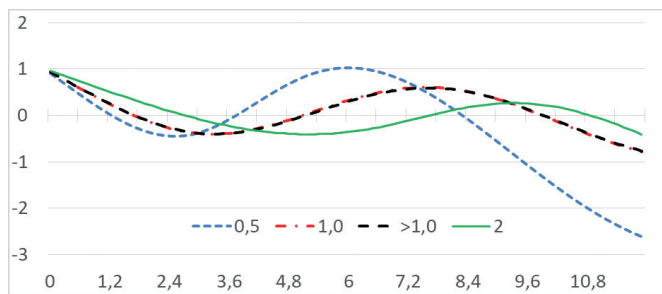*Figure 7. 1st mode of vibration at αL = βL with mass m_1 at point d = L / 4*

Figure 8. 2nd mode of vibration at αL = βL with mass m_1 at point d = L / 4

Table 2. Roots and natural angular frequencies (rad / s) of transverse vibrations at αL = β with mass m_1 at point d = L / 4

		ε_i		$\omega_{i\varkappa} = \omega_{ik}$		Beam on the base	Beam without base
		0.5	1.0		2.0		
1	2	3	4	5	6	7	8
1 mode	$\varkappa_1$	0.4927	0.3364	0.3034	0.2042	0.3925	-
	k_1	0.2463		0.3607	0.4085	-	0.3942
	$\omega_1{}^\varkappa$	352.1	197.8	176.5	137.0	160.7	-
	$\omega_1{}^k$	82.3	153.8		226.3	-	133.3
2 mode	$\varkappa_1$	0.8466	0.6521	0.6477	0.4168	0.6358	-
	k_2	0,4233		0.6553	0.8336	-	0.6544
	$\omega_2{}^\varkappa$	980.0	590.1	582.5	266.6	358.6	-
	$\omega_2{}^k$	243.0	576.7		942.4	-	367.3
3 mode	$\varkappa_1$	1.3749	0.8828	0.88	0.62	0.8936	-
	k_3	0.6874		0.8831	1.24	-	0.9163
	$\omega_3{}^\varkappa$	2565.7	1064.3	1057.7	536.1	691.0	-
	$\omega_3{}^k$	640.8	1057.0		2085.3	-	720.2

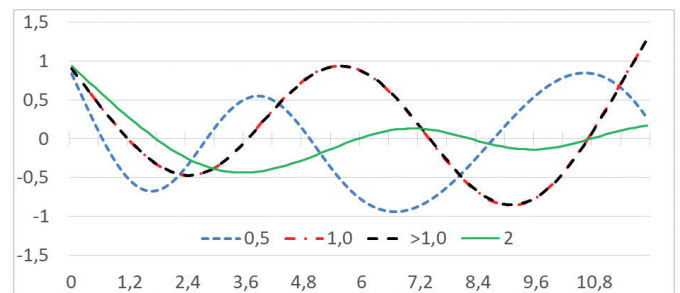*Figure 9. The 3rd mode of vibration at αL = βL with mass m_1 at point d = L / 4*

At the third stage, an example with the same beam under the action of a disturbing force F=10.0 t, applied in the points d=L/2 and d=L/4 is considered. Forced vibration frequencies are γ_1=220 rad/s and γ_2=400 rad/s. The displacements and forces in the beams are determined at various values αL. The figures 10–13 show the bending moment plots.
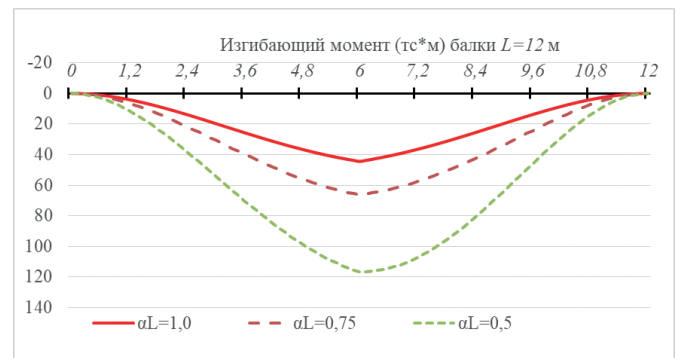

Figure 10. Diagrams of bending moments under the action of the force F (t) at the point d = L / 2 at γ_1 = 220 rad / s
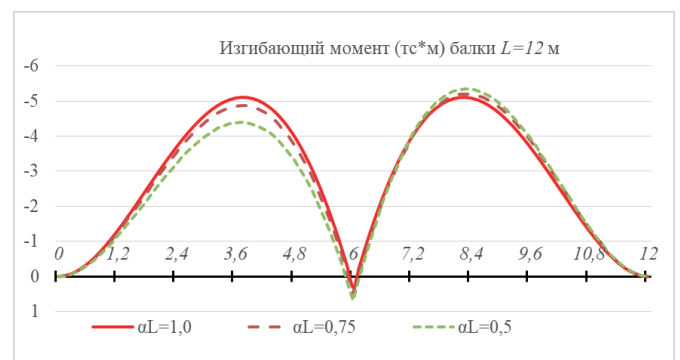

Figure 11. Diagrams of bending moments under the action of the force F (t) at the point d = L / 2 at γ_2 = 400 rad / s
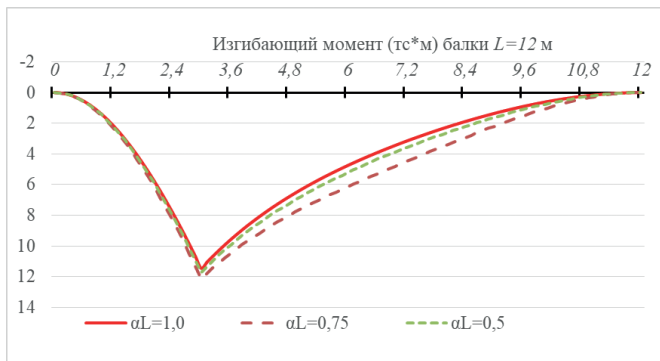
Figure 12. Diagrams of bending moments under the action of the force F (t) at the point d = L / 4 at γ₁ = 220 rad / s

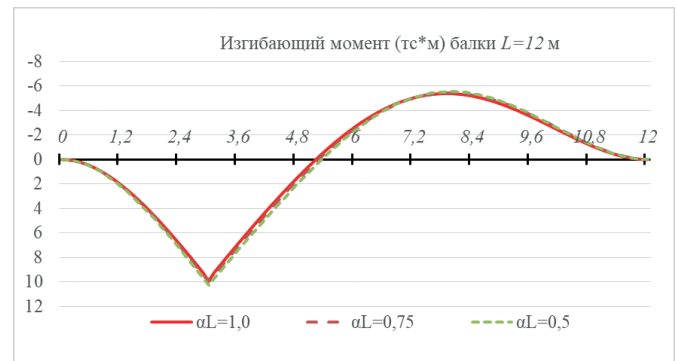*Figure 13. Diagrams of bending moments under the action of the force F (t) at the point d = L / 4 at γ₂ = 400 rad / s*

Analysis of the calculation results of the third stage allows us to draw the following conclusions.

The action of a disturbing force in the middle of the beam (d = L / 2) with a frequency of forced vibrations γ = 220 rad / s, close to the frequency of natural vibrations (for $\omega_{i\varkappa} = \omega_{ik}$) for the first mode of vibration, leads to an increase in displacements and efforts in sections of the beam more than three times when excluding part of the base from the work. Under similar conditions, the action of a disturbing force with a forced vibration frequency γ = 400 rad / s, close to the natural vibration frequency for the second form, does not lead to a significant change in the forces in the beam sections when part of the base is excluded from operation.

The action of a disturbing force in a quarter of the beam (d = L/4) with a frequency of forced vibrations γ = 220 rad/s and γ = 400 rad/s, when part of the base is excluded from the work, does not lead to a significant change in the forces in the beam sections.

4. CONCLUSIONS

1) under different conditions of support of the "beam-base" system, different frequencies of natural vibrations based on the results of calculating the roots of the secular equation can correspond to different parts of one beam. The values of the roots that determine the main modes of vibration of the beam as a whole are the values of the roots for a part of the beam on the base, while the natural frequency of the transverse vibrations of the part of the beam on the base is greater than or equal to the natural frequency of vibration of the part of the beam without the base;

2) under different conditions of support of the "beam-base" system, the values of the natural frequencies of transverse vibrations can be equal for different modes of vibration of two different parts of the beam. In this case, the action of a disturbing force with a frequency of forced vibrations equal to the frequency of natural vibrations leads to the formation of resonance for each of the two different modes of vibration of each part of the beam;

3) with the application of an additional mass m1 at the beam point, the vibration frequencies change its values. If the mass of the system increases, then the vibration frequencies of the system decrease and vice versa that corresponds to a similar conclusion for a beam on a full base [15];

4) when performing a dynamic calculation, it is necessary to consider all possible options for the application of masses, taking into account the points of their location in combinations with options for changing the conditions for supporting the beam on an elastic foundation. The number of determined frequencies and modes of natural vibrations for beams on an elastic foundation should not be less than two;

These conditions must be taken into account when analyzing the dynamic behavior of a structure under the action of variable loads in the event of a change in the conditions of bearing on an elastic foundation.

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Yevgeny V. Leontiev, Deputy Chief of Management of Construction Design, Chief of the Structural Reliability and Safety of Objects Department; Federal Autonomous Institution "Main Department of State Expertise"; 6 Furkasovsky pereulok, Moscow, 101000, Russia; phone +7 (495) 625-95-95, 540-70-96; E-mail: e.leontyev@gge.ru.

Леонтьев Евгений Владимирович, заместитель начальника Управления строительных решений – начальник отдела конструктивной надежности и безопасности объектов; ФАУ «Главгосэкспертиза России»; 101000, Россия, Москва, Фуркасовский пер., д. 6; тел. +7 (495) 625-95-95, E-mail: e.leontyev@gge.ru.