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NUMERICAL SOLUTION OF THE PROBLEM OF BEAM ANALYSIS WITH THE USE OF B-SPLINE FINITE ELEMENT METHOD

Pavel A. Akimov [1, 2, 3, 4], Marina L. Mozgaleva [1], Taymuraz B. Kaytukov [1]

[1] National Research Moscow State University of Civil Engineering, Moscow, RUSSIA
[2] Tomsk State University of Architecture and Civil Engineering, Tomsk, RUSSIA
[3] Peoples' Friendship University of Russia, Moscow, RUSSIA
[4] Russian Academy of Architecture and Construction Sciences, Moscow, RUSSIA

Abstract: Numerical solution of the problem of beam analysis (bending analysis of the Bernoulli beam) with the use of B-spline finite element method is under consideration in the distinctive paper. The original continual and finite element formulations of the problem are given, some actual aspects of construction of normalized basis functions of a B-spline are considered, the corresponding local constructions for an arbitrary finite element are described, some information about the numerical implementation and an example of analysis are presented.

Keywords: wavelet-based finite element method, B-spline finite element method, finite element method, B-spline, numerical solution, beam analysis

ЧИСЛЕННОЕ РЕШЕНИЕ ЗАДАЧИ О ПОПЕРЕЧНОМ ИЗГИБЕ БАЛКИ НА ОСНОВЕ ВЕЙВЛЕТ-РЕАЛИЗАЦИИ МЕТОДА КОНЕЧНЫХ ЭЛЕМЕНТОВ С ИСПОЛЬЗОВАНИЕМ B-СПЛАЙНОВ

П.А. Акимов [1,2,3,4], М.Л. Мозгалева [1], Т.Б. Кайтуков [1]

[1] Национальный исследовательский Московский государственный строительный университет, г. Москва, РОССИЯ
[2] Томский государственный архитектурно-строительный университет, г. Томск, РОССИЯ
[3] Российский университет дружбы народов, г. Москва, РОССИЯ
[4] Российская академия архитектуры и строительных наук, г. Москва, РОССИЯ

Аннотация: В настоящей статье рассматривается численное решение задачи о поперечном изгибе балки Бернулли на основе вейвлет-реализации метода конечных элементов с использованием B-сплайнов. Приведены исходные континуальная и конечноэлементная постановки задачи, рассмотрены некоторые актуальные вопросы построения нормализованных базисных функций B-сплайна, описаны соответствующие локальные построения для произвольного конечного элемента, представлены некоторые сведения о численной реализации и пример расчета.

Ключевые слова: вейвлет-реализация метода конечных элементов, метод конечных элементов, B-сплайны, численное решение, изгиб балки

INTRODUCTION

As is known, the B-spline in a given simple knot sequence can be constructed by employing piecewise polynomials between the knots and joining them together at the knots [1].

Compared with commonly used Daubechies wavelets [2-6] B-spline wavelet on interval (BSWI) has explicit expressions, facilitating the calculation of coefficient integration and differentiation [1]. Besides, the multiresolution and localization properties of BSWI can also supply some superiority for engineering structural analysis [1]. The early applications of spline can be found, for instance, in papers of H. Antes [7], J.G. Han [8, 9, 25], Y. Huang [8, 9], W.X. Ren [8, 9]. The spline wavelet finite element method was further developed in papers of D.P. Chen [26], X.F. Chen [10, 11, 13-16, 21, 22, 24], H.B. Dong [21], J.G. Han [23], Y.M. He [15], Z.H. He [16], Z.J. He [10, 11, 13-15, 21, 22, 24], Y. Huang [23, 25], Z.S. Jiang [20], B. Li [11, 13, 15, 21], M. Liang [17, 19], J.Q. Long [18], G. Ma [18], T. Matsumoto [18, 20], S.T. Mau [28], H.H. Miao [13], Q,M. Mo [16], T.H.H. Pian [26-28], K.Y. Qi [21], W.X. Ren [23, 25], K. Sumihara [27], P. Tong [28], Y.W. Wang [20], J.W. Xiang [10-12, 15-20], Z.B. Yang [13, 14, 22], X.W. Zhang [14, 22, 24], Y.H. Zhang [10], Y.T. Zhong [12].

The distinctive paper is devoted to numerical solution of the problem of beam analysis (bending analysis of the Bernoulli beam) with the use of B-spline finite element method.

1. FORMULATIONS OF THE PROBLEM

The unknown function of the beam deflections $y(x)$, caused by the load $q(x)$, can be defined using the condition for the minimum energy functional of the beam $\Phi(y)$ (i.e. unknown function provides a minimum value for this functional):

$$\Phi(y)=\frac{1}{2}\int_0^l [EJ(y'')^2+\beta y^2]dx-\int_0^l q(x)ydx\,, \quad (1.1)$$

where $EJ(x)$ is the bending stiffness of the beam; β is the coefficient of elasticity of the base (coefficient of bedding); $q(x)$ is the given load; l is the length of the beam; x is coordinate along the length of the beam.

Let us divide the interval [o, l], occupied by the beam into N_e parts (elements); $h_e = l/N_e$ is the length of the element. Let us also divide each element into N_k parts, for example, $N_k = 5$ (Figure 1). Let us introduce the following notation system: i_e is the element number; $x_1(i_e)$ is the coordinate of the starting point; $x_6(i_e)$ is the coordinate of the end point of the element number i_e, respectively. We take y_i and y'_i as unknowns at boundary points $i = 1,6$. We take y_i, $i = 2, 3, 4, 5$ as unknowns at the inner points. Thus, the number of unknowns per element with such discretization is defined by formula

$$N = N_k - 1 + 2 \cdot 2 = N_k + 3 = 8.$$

The number of boundary points for all elements is equal to

$$N_b = N_e + 1.$$

The number of interior points for all elements is equal to

$$N_p = N_e\,(N_k - 1).$$

The total (global) number of unknowns with such a discretization turns out to be equal to

$$N_g = N_p + 2N_b.$$

Thus, we have

$$\Phi(y)=\sum_{i_e=1}^{N_e}\Phi_{i_e}(y)\,,$$

$$\Phi_{i_e}(y)=\frac{1}{2}\int_{x_1(i_e)}^{x_5(i_e)}[EJ(y'')^2+\beta y^2]\,dx-\int_{x_1(i_e)}^{x_5(i_e)} qy\,dx\,; \quad (1.2)$$

2. SOME ASPECTS OF THE CONSTRUCTION OF NORMALIZED BASIS FUNCTIONS OF THE B-SPLINE

The construction of B-spline basic functions is determined by the recursive Cox-de Boer formulas:

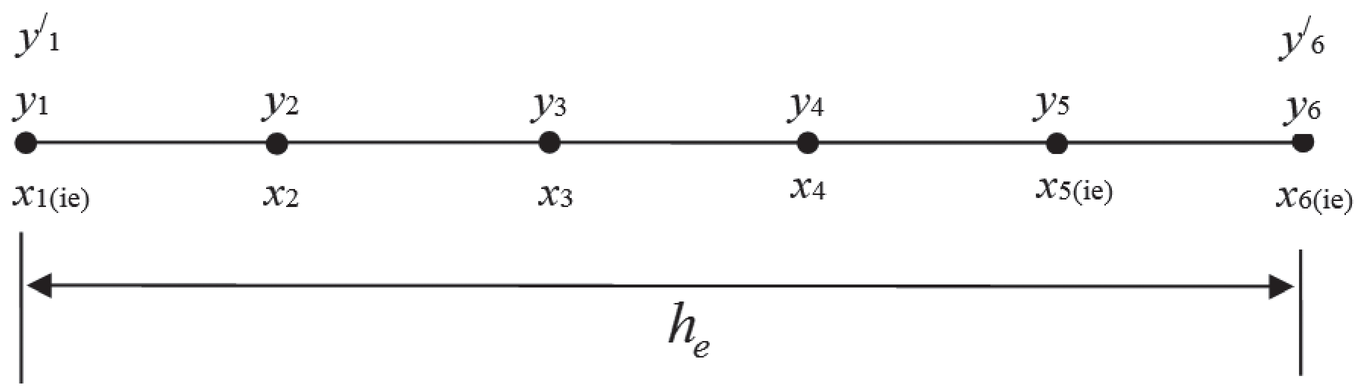

Figure 1. Finite element discretication (sample).

$$k=1: \quad \varphi_{i,1}(t)=\begin{cases}1\,, & x_i \le t < x_{i+1} \\ 0\,, & t<x_i \vee t \ge x_{i+1}\end{cases}, \tag{2.1}$$

$$k \ge 2: \quad \begin{aligned}\varphi_{i,k}(t) &= \frac{(t-x_i)\varphi_{i,k-1}(t)}{x_{i+k-1}-x_i}+ \\ &+\frac{(x_{i+k}-t)\varphi_{i+1,k-1}(t)}{x_{i+k}-x_{i+1}}.\end{aligned} \tag{2.2}$$

We will consider such a construction for the case $x_i = i$ are integers. Let us note that,

$$\varphi_{i,k}(t)=\varphi_{0,k}(t-i)$$

and therefore, recursive formulas (2.1)-(2.2) can be represented in the form

$$k=1: \quad \varphi_{0,1}(t)=\begin{cases}1\,, & 0 \le t < 1 \\ 0\,, & t<0 \vee t \ge 1\end{cases}, \tag{2.3}$$

$$k \ge 2: \quad \begin{aligned}\varphi_{0,k}(t) = \frac{1}{k-1}[t\cdot\varphi_{0,k-1}(t)+ \\ +(k-t)\varphi_{0,k-1}(t-1)].\end{aligned} \tag{2.4}$$

The function $\varphi_{0,1}(t)$ can be represented by formula

$$\varphi_{0,1}(t)=\frac{1}{2}[\text{sign}(t)-\text{sign}(t-1)]\,. \tag{2.5}$$

Let us denote by Δ_1 the operator of the first difference. Then we have

$$\varphi_{0,1}(t)=-\frac{1}{2}\Delta_1\ \text{sign}(t)\,. \tag{2.6}$$

We can substitute formula (2.5) into (2.4) in order to determine $\varphi_{0,2}(t)$:

$$\begin{aligned}\varphi_{0,2}(t) &= 1\cdot[t\cdot\varphi_{0,1}(t)+(2-t)\varphi_{0,1}(t-1)] = \\ &=\frac{1}{2}\{t\cdot[\text{sign}(t)-\text{sign}(t-1)]+(2-t)[\text{sign}(t-1)-\text{sign}(t-2)]\} = \\ &=\frac{1}{2}[t\,\text{sign}(t)-2(t-1)\,\text{sign}(t-1)+ \\ &(t-2)\,\text{sign}(t-2)]=\frac{1}{2}[|\,t\,|-2\,|\,t-1\,|+|\,t-2\,|.\end{aligned}$$

Let us denote by Δ_2 the operator of the second difference. Then we have

$$\varphi_{0,2}(t)=\frac{1}{2}[|\,t\,|-2\,|\,t-1\,|+|\,t-2\,|=\frac{1}{2}\Delta_2\,|\,t-1\,|\,. \tag{2.7}$$

We can define function $\varphi_{0,3}(t)$:

$$\varphi_{0,3}(t)=\frac{1}{2}[t\cdot\varphi_{0,2}(t)+(3-t)\varphi_{0,2}(t-1)]\,.$$

Omitting intermediate calculations, we get

$$\begin{aligned}\varphi_{0,3}(t) &= \frac{1}{4}[t\cdot|\,t\,|-3(t-1)\,|\,t-1\,|+ \\ &+3(t-2)\,|\,t-2\,|-(t-3)\,|\,t-3\,|] = \\ &=-\frac{1}{2!}\frac{1}{2}\Delta_1\Delta_2((t-1)\,|\,t-1\,|)\,.\end{aligned} \tag{2.8}$$

Based on formulas (2.8) and (2.4), we can define the function

$$\varphi_{0,4}(t)=\frac{1}{3}[t\cdot\varphi_{0,3}(t)+(4-t)\varphi_{0,3}(t-1)]\,.$$

Omitting intermediate calculations, we get

$$\begin{aligned}&\varphi_{0,4}(t) = \\ &=\frac{1}{2\cdot 3}\cdot\frac{1}{2}[t^2\cdot|\,t\,|-4(t-1)^2\,|\,t-1\,|+ \\ &+6(t-2)^2\,|\,t-2\,|-4(t-3)^2\,|\,t-3\,|+ \\ &+(t-4)^2\,|\,t-4\,|] = \\ &=\frac{1}{3!}\frac{1}{2}(\Delta_2)^2((t-2)^2\,|\,t-2\,|)\,.\end{aligned} \tag{2.9}$$

It can be proved that for even $k = 2m$ we have

$$\varphi_{0,k}(t)=\frac{1}{(2m-1)!}\frac{1}{2}(\Delta_2)^m((t-m)^{2m-2}\,|\,t-m\,|) \tag{2.10}$$

and for odd (uneven) $k = 2m + 1$ we have

$$\varphi_{0,k}(t)=-\frac{1}{(2m)!}\frac{1}{2}\Delta_1(\Delta_2)^m((t-m)^{2m-1}\,|\,t-1\,|)\,. \tag{2.11}$$

Note that $\varphi_{0,k}(t)$ is a polynomial of degree $k-1$ with bounded support and, as follows from the difference operator, this support is equal to the interval $[0, k]$. In addition, we should note the following property of B-spline basis functions:

$$\sum_i \varphi_{0,k}(t-i)\equiv 1 \ \text{ for arbitrary } t. \tag{2.12}$$

3. LOCAL CONSTRUCTIONS FOR ARBITRARY FINITE ELEMENT

Let us introduce local coordinates:

$$t=\frac{x-x_{1(i_e)}}{h_e}\,,\ x_{1(i_e)}\le x \le x_{6(i_e)}\,,\ 0\le t \le 1\,.$$

In this case, we have the following relations:

$$\begin{cases} x = x_{1(i_e)} & \Rightarrow t = 0 \\ x = x_2 & \Rightarrow t = 0.2 \\ x = x_3 & \Rightarrow t = 0.4 \\ x = x_4 & \Rightarrow t = 0.6 \\ x = x_5 & \Rightarrow t = 0.8 \\ x = x_{6(i_e)} & \Rightarrow t = 1 \end{cases},$$

$$\begin{matrix} \frac{d}{dx} = \frac{d}{dt} \cdot \frac{dt}{dx} = \frac{1}{h_e} \frac{d}{dt} \\ \frac{d^p}{dx^p} = \frac{1}{h_e^p} \frac{d^p}{dt^p} \end{matrix}, \quad dx = h_e \cdot dt. \tag{3.1}$$

Since the number of unknowns on the element is equal to $N = 8$, we use a B-spline of the seventh degree in order to represent the unknown deflection function. Let us use the following notation:

$$\varphi(t) = \varphi_{0,8}(t+4);$$

$$\begin{aligned} \varphi(t) &= \frac{1}{7!}\frac{1}{2}(\Delta_2)^4(t^6\,|\,t\,|) = \\ &= \frac{1}{2 \cdot 7!}[(t+4)^6\,|\,t+4\,| - \\ &-8(t+3)^6\,|\,t+3\,| + \\ &+28(t+2)^6\,|\,t+2\,| - \\ &-56(t+1)^6\,|\,t+1\,| + 70t^6\,|\,t\,| - \\ &-56(t-1)^6\,|\,t-1\,| + 28(t-2)^6\,|\,t-2\,| - \\ &-8(t-3)^6\,|\,t-3\,| + (t-4)^6\,|\,t-4\,|]. \end{aligned} \tag{3.2}$$

This function is a B-spline, symmetric with respect to $t = 0$ and its support is defined by an interval [–4, 4] (Figure 2).

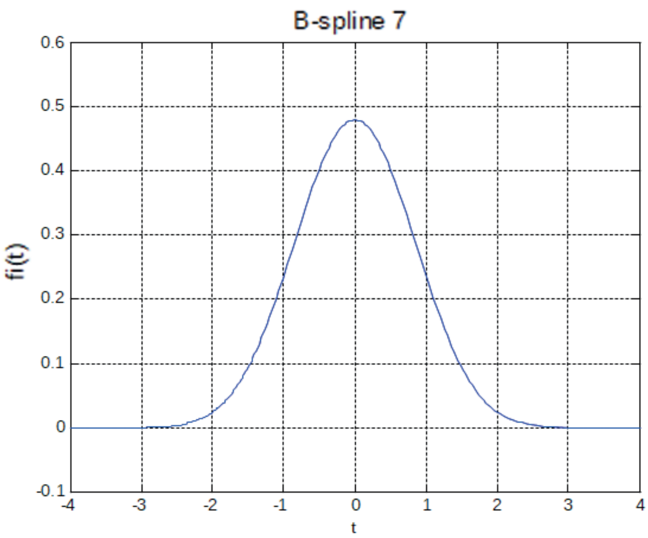

Figure 2. B-spline of the seventh order $\varphi(t) = \varphi_{0.8}(t+4)$.

Let us use the following notation system:

$$\begin{gathered} \varphi_1(t) = \varphi(t+3),\ \varphi_2(t) = \varphi(t+2), \\ \varphi_3(t) = \varphi(t+1),\ \varphi_4(t) = \varphi(t), \\ \varphi_5(t) = \varphi(t-1), \\ \varphi_6(t) = \varphi(t-2),\ \varphi_7(t) = \varphi(t-3), \\ \varphi_8(t) = \varphi(t-4),\ 0 \le t \le 1. \end{gathered} \tag{3.3}$$

We represent the unknown deflection function in the form

$$\begin{gathered} y(x) = w(t) = \sum_{k=1}^{N} \alpha_k \varphi_k(t), \\ x_{1(i_e)} \le x \le x_{6(i_e)},\ 0 \le t \le 1. \end{gathered} \tag{3.4}$$

We can substitute (3.4) into (1.3), taking into account relations (3.1).

$$\begin{aligned} \Phi_{i_e}(y) &= \frac{1}{2}\int_{x_1(i_e)}^{x_6(i_e)} \left(EJ\left(\frac{d^2y}{dx^2}\right)^2 + \beta y^2 \right) dx - \int_{x_1(i_e)}^{x_6(i_e)} qydx = \\ &= \frac{1}{2}\int_0^1 \left(\frac{EJ}{h_e^3}(w'')^2 + \beta h_e w^2 \right) dt - \int_0^1 h_e qwdt = \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i \alpha_j \times \\ &\times \int_0^1 \left(\frac{EJ}{h_e^3}\left(\varphi''_i(t)\varphi''_j(t)\right) + \beta h_e\left(\varphi_i(t)\varphi_j(t)\right) \right) dt - \\ &- \sum_{i=1}^{N} \alpha_i \int_0^1 h_e q \varphi_i(t) dt = \\ &= \frac{1}{2}(K_\alpha^{i_e}\overline{\alpha}, \overline{\alpha}) - (\overline{R}_\alpha^{i_e}, \overline{\alpha}) = \Phi_\alpha(\overline{\alpha}), \end{aligned} \tag{3.5}$$

where we have

$$K_\alpha^{i_e}(i,j) = \int_0^1 \left(\frac{EJ}{h_e^3}\left(\varphi''_i(t)\varphi''_j(t)\right) + \beta h_e\left(\varphi_i(t)\varphi_j(t)\right) \right) dt;$$

$$R_\alpha^{i_e}(i) = \int_0^1 \left(h_e q(t)\varphi_i(t)\right) dt.$$

Let's define the parameters through the nodal unknowns on the element:

$$\begin{cases} y_1 = w(0) = \sum_{k=1}^{N} \alpha_k \varphi_k(0) \\ \dfrac{dy_1}{dx} = \dfrac{1}{h_e} w'(0) = \dfrac{1}{h_e} \sum_{k=1}^{N} \alpha_k \varphi'_k(0) \\ y_2 = w(0.2) = \sum_{k=1}^{N} \alpha_k \varphi_k(0.2) \\ y_3 = w(0.4) = \sum_{k=1}^{N} \alpha_k \varphi_k(0.4) \\ y_4 = w(0.6) = \sum_{k=1}^{N} \alpha_k \varphi_k(0.6) \\ y_5 = w(0.8) = \sum_{k=1}^{N} \alpha_k \varphi_k(0.8) \\ y_6 = w(1) = \sum_{k=1}^{N} \alpha_k \varphi_k(1) \\ \dfrac{dy_6}{dx} = \dfrac{1}{h_e} w'(1) = \dfrac{1}{h_e} \sum_{k=1}^{N} \alpha_k \varphi'_k(1) \end{cases}$$

Therefor we have

$$\overline{y}^{i_e} = T\overline{\alpha}\,, \qquad (3.6)$$

where

$$\overline{y}^{i_e} = [y_1 \;\; \frac{dy_1}{dx} \;\; y_2 \;\; y_3 \;\; y_4 \;\; y_5 \;\; y_6 \;\; \frac{dy_6}{dx}]^{\mathrm{T}}\,;$$

$$\overline{\alpha} = [\alpha_1 \;\; \alpha_2 \;\; \alpha_3 \;\; \alpha_4 \;\; \alpha_5 \;\; \alpha_6 \;\; \alpha_7 \;\; \alpha_8]^{\mathrm{T}}\,;$$

$$D = diag(1 \;\; 1/h_e \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1/h_e)\,;$$

$$T = D\begin{bmatrix} \varphi_1(0) & \varphi_2(0) & \varphi_3(0) & \varphi_4(0) & \varphi_5(0) & \varphi_6(0) & \varphi_7(0) & \varphi_8(0) \\ \varphi'_1(0) & \varphi'_2(0) & \varphi'_3(0) & \varphi'_4(0) & \varphi'_5(0) & \varphi'_6(0) & \varphi'_7(0) & \varphi'_8(0) \\ \varphi_1(0.2) & \varphi_2(0.2) & \varphi_3(0.2) & \varphi_4(0.2) & \varphi_5(0.2) & \varphi_6(0.2) & \varphi_7(0.2) & \varphi_8(0.2) \\ \varphi_1(0.4) & \varphi_2(0.4) & \varphi_3(0.4) & \varphi_4(0.4) & \varphi_5(0.4) & \varphi_6(0.4) & \varphi_7(0.4) & \varphi_8(0.4) \\ \varphi_1(0.6) & \varphi_2(0.6) & \varphi_3(0.6) & \varphi_4(0.6) & \varphi_5(0.6) & \varphi_6(0.6) & \varphi_7(0.6) & \varphi_8(0.6) \\ \varphi_1(0.8) & \varphi_2(0.8) & \varphi_3(0.8) & \varphi_4(0.8) & \varphi_5(0.8) & \varphi_6(0.8) & \varphi_7(0.8) & \varphi_8(0.8) \\ \varphi_1(1) & \varphi_2(1) & \varphi_3(1) & \varphi_4(1) & \varphi_5(1) & \varphi_6(1) & \varphi_7(1) & \varphi_8(1) \\ \varphi'_1(1) & \varphi'_2(1) & \varphi'_3(1) & \varphi'_4(1) & \varphi'_5(1) & \varphi'_6(1) & \varphi'_7(1) & \varphi'_8(1) \end{bmatrix}$$

Therefor we have

$$\overline{\alpha} = T^{-1}\overline{y}^{i_e}\,. \qquad (3.7)$$

Substituting (3.7) into $\Phi_\alpha(\overline{\alpha})$, we obtain

$$\begin{aligned} \Phi_\alpha(\overline{\alpha}) &= \\ &= \frac{1}{2}(K_\alpha^{i_e} T^{-1}\overline{y}^{i_e}, T^{-1}\overline{y}^{i_e}) - (\overline{R}_\alpha^{i_e}, T^{-1}\overline{y}^{i_e}) = \\ &= \frac{1}{2}((T^{-1})^{\mathrm{T}} K_\alpha^{i_e} T^{-1}\overline{y}^{i_e}, \overline{y}^{i_e}) - ((T^{-1})^{\mathrm{T}}\overline{R}_\alpha^{i_e}, \overline{y}^{i_e}) = \\ &= \frac{1}{2}(K^{i_e}\overline{y}^{i_e}, \overline{y}^{i_e}) - (\overline{R}^{i_e}, \overline{y}^{i_e}) = \Phi_{i_e}(\overline{y}^{i_e}), \end{aligned} \qquad (3.8)$$

where

$$K^{i_e} = (T^{-1})^{\mathrm{T}} K_\alpha^{i_e} T^{-1}$$

is the local stiffness matrix;

$$\overline{R}^{i_e} = (T^{-1})^{\mathrm{T}} \overline{R}_\alpha^{i_e}$$

is the local load vector.

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4. INFORMATION ABOUT NUMERICAL IMPLEMENTATION

The presented algorithm can be implemented using MATLAB tools. The MATLAB system has convenient functions for working with polynomials. Moreover, the main parameter of these functions is the vector of coefficients of the polynomial. To determine the coefficients of basic polynomials φ_k on an interval [0 1], we can firstly determine their values at eight points of the interval $t = [t_1, t_2, ..., t_8]$, $t_i \in [0 \;\; 1]$, $i = 1, 2, ..., 8$;

$$F_k(i) = \varphi_k(t_i),\; i = 1, 2, ..., 8,\quad k = 1, 2, ..., 8\;.$$

Then, using the `polyfit` function, we define their coefficient vector:

```
pk=polyfit(t,Fk,7)
```

This function is used to determine the coefficients of the optimal polynomial using the least squares method. In the considering case, we are looking for a polynomial of the 7th degree (i.e. we have to define 8 coefficients of polynomial, according to its 8 values), therefore, we get a polynomial passing through the given values.

In order to calculate the derivatives we can sequentially use the `polyder` function:

```
dpk=polyder(pk)
```
is the vector of coefficients φ'_k;
```
d2pk=polyder(dpk)
```
is the vector of coefficients φ''_k.

In order to calculate the product of polynomials we can use the `conv` function:

```
pij=conv(pi,pj)
```
is the vector of coefficients $\varphi_i\varphi_j$;
```
d2pij=conv(d2pi, d2pj)
```
is the vector of coefficients $\varphi''_i\varphi''_j$.

In order to calculate the antiderivative of a polynomial we can use the `polyint` function:

```
Pi=polyint(pi)
```
is the vector of coefficients $\int\varphi_i dt$;
```
Pij=polyint(pij)
```
is the vector of coefficients $\int\varphi_i\varphi_j dt$;
```
d2Pij=polyint(d2pij)
```
is the vector of coefficients $\int\varphi''_i\varphi''_j dt$.

Then the calculation (formula (3.5))

$$K_\alpha^{i_e}(i,j) = = \int_0^1 \left(\frac{EJ}{h_e^3}\left(\varphi_i''(t)\varphi_j''(t)\right) + \beta h_e\left(\varphi_i(t)\varphi_j(t)\right) \right) dt\,.$$

can be summarized as follows:

$K_\alpha^{i_e}(i,j) = \frac{EJ}{h_e^3}$`( polyval(d2Pij,1)-polyval(d2Pij,0))+`
$+\,\beta h_e$`( polyval(Pij,1)-polyval(Pij,0)),`

where the function `polyval (p, t)` allows researcher to calculate the values of a polynomial with a vector of coefficients p at a given point t.

As for the calculation (see (3.5)),

$$R_\alpha^{i_e}(i) = \int_0^1 \left(h_e q(t)\varphi_i(t)\right)dt$$

here, for example, the following options are possible:
– point load setting (using delta functions);
– setting the load averaged on the element,

$R_\alpha^{i_e}(i) = h_e q_{ie}$`(polyval (Pi, 1) - - polyval (Pi, 0))`

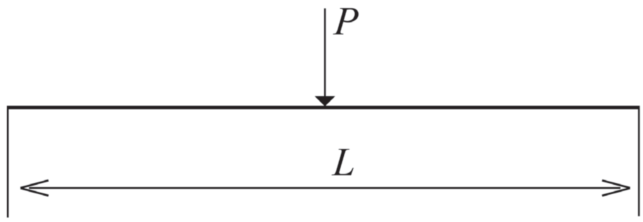


Figure 3. Example of analysis.

If q is represented by a polynomial, then, as in the case of calculating the elements of a local matrix $K_\alpha^{i_e}$, here researcher can use the function of multiplying polynomials `conv` followed by determining the antiderivative of the product using the `polyint` functions and calculating the definite integral using the `polyval` function.

5. EXAMPLE OF ANALYSIS

As a model example let us consider a beam on an elastic foundation with the following parameters:

$$q(x) = P\delta(x - \frac{L}{2}),\ \ P = 100\text{ kN}$$

is load given at the midpoint (Figure 3);

$$L = 8m;\ h_b = 1.3m;\ b_b = 1m;$$
$$E = 2560 \cdot 10^4\ kN/m^2;\ k = 75 \cdot 10^3 kN/m^3.$$

In this case we should consider the following boundary conditions:

$$\begin{cases} y(0) = y''(0) = 0 \\ y(L) = y''(L) = 0 \end{cases}$$

– the beam is hingedly supported on both sides (the first case);

$$\begin{cases} y(0) = y'(0) = 0 \\ y(L) = y'(L) = 0 \end{cases}$$

– the beam is rigidly fixed on both sides (the second case);

$$\begin{cases} y(0) = y''(0) = 0 \\ y'''(L) = y''(L) = 0 \end{cases}$$

– the beam is hingedly supported on the left end, the right end is free (the third case);

$$\begin{cases} y(0) = y'(0) = 0 \\ y'''(L) = y''(L) = 0 \end{cases}$$

the beam is rigidly fixed to the left end, the right end is free (the fourth case).
Let us set $N_e = 4$ (the number of elements).
Then we have

$$N_g = N_p + 2N_b = 4 \cdot (5 - 1) + 2 \cdot (4 + 1) = 26;$$

is the total number of unknowns;

$$h_e = L / N_e = 8 / 4 = 2$$

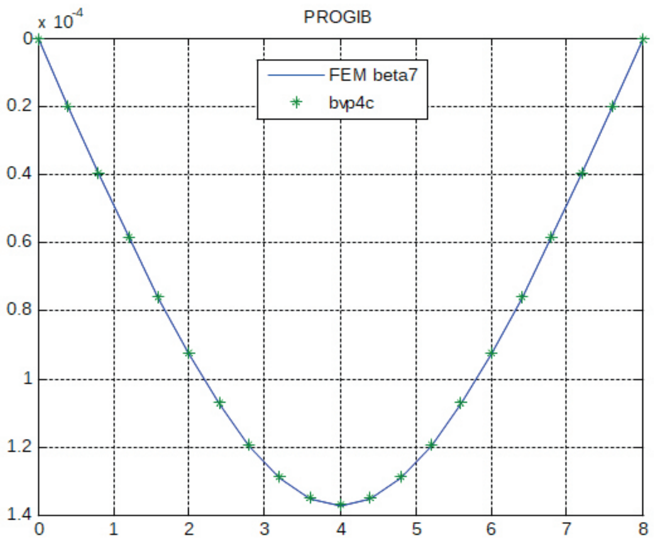
is the length of the element;

$$h_p = h_e / 5 = 2 / 5 = 0.4$$

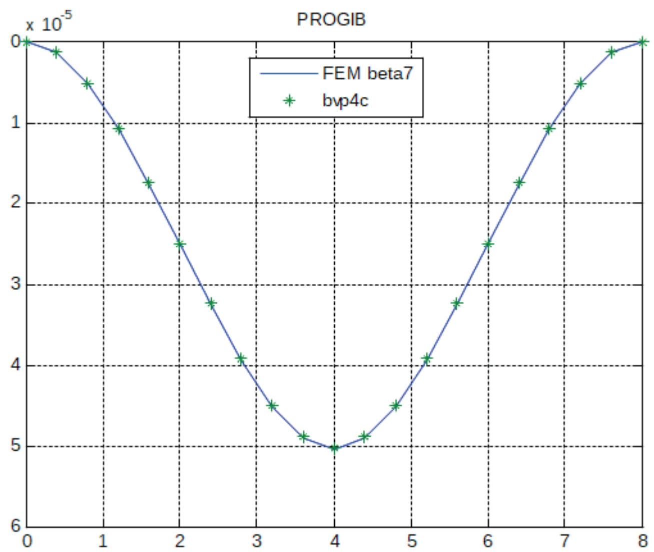
is the step between the coordinates of the nodes;

$$N_x = L / h_p + 1 = 8 / 0.4 + 1 = 21$$

is the total number of nodes.
Several results of analysis are presented at Figures 4, 5, 6 and 7.



Figures 4. Comparison of results for the first case.



Figures 5. Comparison of results for the second case.

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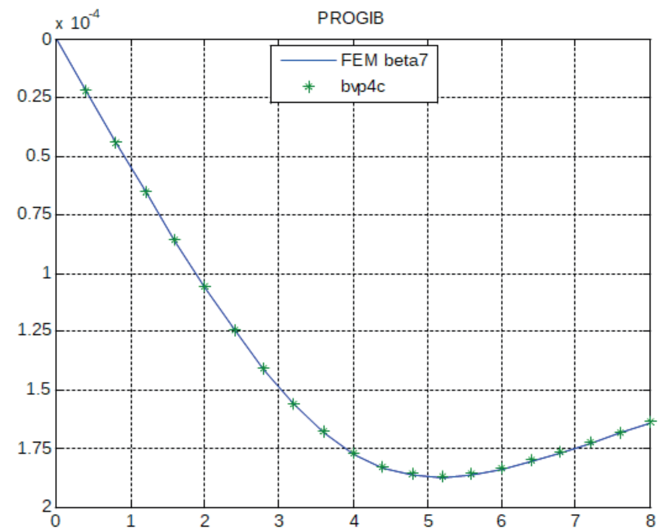
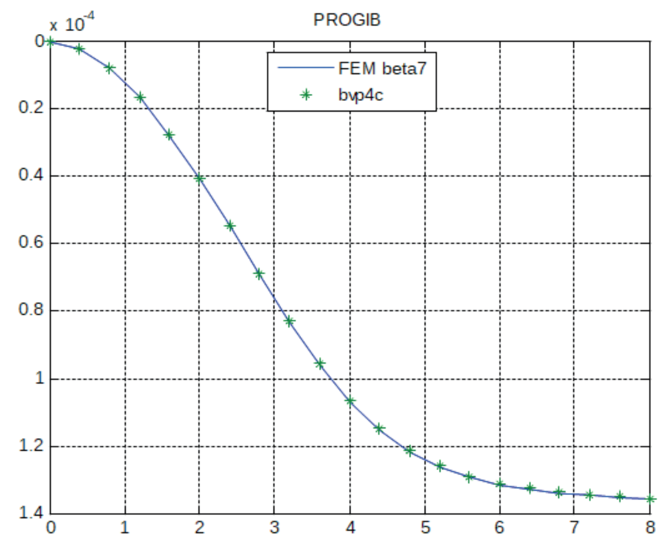


Figures 6. Comparison of results for the third case.



Figures 7. Comparison of results for the fourth case.

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Акимов Павел Алексеевич, академик РААСН, профессор, доктор технических наук; временно исполняющий обязанности ректора Национального исследовательского Московского государственного строительного университета; профессор Департамента архитектуры и строительства Российского университета дружбы народов; профессор кафедры строительной механики Томского государственного архитектурно-строительного университета; исполняющий обязанности вице-президента Российской академии архитектуры и строительных наук; 129337, Россия, г. Москва, Ярославское шоссе, дом 26; телефон: +7(495) 651-81-85; факс: +7(499) 183-44-38; Email: AkimovPA@mgsu.ru, rector@mgsu.ru, pavel.akimov@gmail.com.

Мозгалева Марина Леонидовна, старший научный сотрудник, доктор технических наук; профессор кафедры прикладной математики Национального исследовательского Московского государственного строительного университета; 129337, Россия, г. Москва, Ярославское шоссе, дом 26; телефон/факс: +7(499) 183-59-94; Email: marina.mozgaleva@gmail.com.

Кайтуков Таймураз Батразович, советник РААСН, доцент, кандидат технических наук; проректор, профессор кафедры прикладной математики Национального исследовательского Московского государственного строительного университета; 129337, Россия, г. Москва, Ярославское шоссе, дом 26; телефон: +7(499) 929-52-29; факс: +7(499) 183-44-38; Email: KaytukovTB@mgsu.ru.

Pavel A. Akimov, Full Member of the Russian Academy of Architecture and Construction Sciences, Professor, Dr.Sc.; Acting Rector of National Research Moscow State University of Civil Engineering; Professor of Department of Architecture and Construction, Peoples' Friendship University of Russia; Professor of Department of Structural Mechanics, Tomsk State University of Architecture and Building; Acting Vice-President of the Russian Academy of Architecture and Construction Sciences; 26, Yaroslavskoe Shosse, Moscow, 129337, Russia; phone: +7(495) 651-81-85; Fax: +7(499) 183-44-38; E-mail: AkimovPA@mgsu.ru, rector@mgsu.ru, pavel.akimov@gmail.com.

Marina L. Mozgaleva, Senior Scientist Researcher, Dr.Sc.; Professor of Department of Applied Mathematics, National Research Moscow State University of Civil Engineering; 26, Yaroslavskoe Shosse, Moscow, 129337, Russia; phone/fax +7(499) 183-59-94; Fax: +7(499) 183-44-38; Email: marina.mozgaleva@gmail.com.

Taymuraz B. Kaytukov, Advisor of the Russian Academy of Architecture and Construction Sciences, Associate Professor, Ph.D.; Vice-Rector, Associate Professor Department of Applied Mathematics, National Research Moscow State University of Civil Engineering; 26, Yaroslavskoe Shosse, Moscow, 129337, Russia; phone: +7(499) 929-52-29; fax: +7(499) 183-59-94; Email: KaytukovTB@mgsu.ru.