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INTEGRAL PARAMETERS OF CONCRETE DIAGRAMS FOR CALCULATIONS OF STRENGTH OF REINFORCED CONCRETE ELEMENTS USING THE DEFORMATION MODEL

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Abstracts: In accordance with the requirements of regulatory documents, restrictions are introduced on stress levels at the end of the falling branch of the diagrams at the maximum normalized strain values. We have developed mathematical models that establish a uniform sequence for calculating the unambiguous values of deformations at the base points of concrete diagrams, taking into account the accepted functional relationships and the rules for their use according to the tables of normative documents. It was shown that for equal values of deformations and stresses at base points, analytical expressions of diagram recommended by regulatory documents, even if it differs in structure, give identical outlines, diagram branches coincide. The correlation between the calculation models by Russian and foreign regulatory documents was established by comparing the values of the integral parameters of the diagrams and the ultimate forces obtained by calculating the reinforced concrete element according to the deformation model. As integral parameters of concrete deformation diagrams, it was recommended to use areas bounded by diagram branches and diagram completeness coefficients. Analytical modeling of integral parameters allowed us to exclude the procedure for numerically summing stresses along elementary strips in a section and solving nonlinear equations by the method of successive approximations when calculating the strength of an element.

Keywords: strength, deformations, concrete diagram, integral parameters, deformation model

ИНТЕГРАЛЬНЫЕ ПАРАМЕТРЫ ДИАГРАММ БЕТОНА В РАСЧЕТАХ ПРОЧНОСТИ ЖЕЛЕЗОБЕТОННЫХ ЭЛЕМЕНТОВ ПО ДЕФОРМАЦИОННОЙ МОДЕЛИ

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Аннотация: В соответствии с требованиями нормативных документов, введены ограничения на уровни напряжений в конце ниспадающей ветви диаграмм при максимальных нормированных значениях деформаций. Разработаны математические модели, устанавливающие единообразную форму вычисления однозначных значений деформаций в базовых точках диаграмм бетона, с учетом принятых функциональных связей и правил их назначения по таблицам нормативных документов. Показано, что при равных значениях деформаций и напряжений в базовых точках, рекомендованные нормативными документами аналитические выражения описания диаграмм, разные по своей структуре, дают одинаковые их очертания, ветви диаграмм совпадают. Соотношение между расчетными моделями в редакции российских и зарубежных нормативных документов устанавливается сравнением значений интегральных параметров диаграмм и предельных усилий, полученных расчетом железобетонного элемента по деформационной модели. В качестве интегральных параметров диаграмм деформирования бетона рекомендуется использовать площади областей, ограниченных ветвями диаграмм и коэффициенты полноты диаграмм. Аналитическое моделирование интегральных параметров позволяет исключить из расчета прочности элемента процедуры численного суммирования напряжений по элементарным полоскам в сечении и решения нелинейных уравнений путем последовательного приближения.

Ключевые слова: прочность, деформации, диаграмма бетона, интегральные параметры, деформационная модель.

INTRODUCTION

The regulatory documents [1, 2, 3, 4] recommend different types of concrete deformation diagrams and analytical dependencies that establish the relationship between deformations and stresses "$\varepsilon_b - \sigma_b$" under axial compression and tension. The curvilinear diagram with ascending and descending deformation branches corresponds to the physical properties of concrete and the experimental test data for standard concrete specimens most fully. When describing curved diagrams of concrete deformation under compression, the authors of Russian and foreign publications [5, 6, 7, 8, 9] use the base points: at the top of the diagram on the ascending branch; at the end of the falling branch, in which the deformations reach their maximum values. The differences between analytical dependencies of the diagrams, the differences between calculation methods for determining of deformations and design values of concrete strength in the base points that is contained in regulatory documents leads to a mutual discrepancy between the values of ultimate forces in the strength calculations of reinforced concrete elements. In addition, difficulties arise in the comparative evaluation of the efficiency of computational models. In calculations by the deformation model, the numerical integration of stresses in the selected elementary strips of concrete over the thickness of the element and the solution of nonlinear equations satisfying the condition of equilibrium of forces by the method of successive approximation (iterations) is a laborious procedure in the calculations of complex engineering systems. The transition from the real stress diagram to the conventional stress diagram of a rectangular shape for the compressed zone of an element is important to simplify the computer modeling technique in the calculations of generalized internal forces. The performed studies are important for the discrete-continuum approach in numerical modeling of the behavior of the load-bearing systems of high-rise buildings [10], the improvement of computational models of power resistance of reinforced concrete [11] and the development of the survivability theory of structural systems of buildings and structures [12, 13,14].

THE PURPOSE AND OBJECTIVES OF THE RESEARCH

The first purpose of this research is developing of a mathematical model for calculating deformations at the base points of concrete diagrams, taking into account the accepted functional relationships and the rules for their accepting in accordance with the tables of normative documents. The second purpose is to include the integral parameters of concrete diagrams in the calculation method based on the deformation model and establish the relationships between the ultimate forces for the respective classes of concrete using the compressive strength. The third purpose is to propose a simplified method for calculating the strength of an element, excluding the procedure of the numerical integration of stresses over the thickness and solving nonlinear equations by the iteration method. Finally, it is to establish a relationship between the parameters of the deformation model and the method of ultimate forces for the ultimate state of an element.

METHOD

The normative documents [2, 3] sign the concrete class for the axial compression strength by the letter C and numbers, for example, C12 / 15. The first number means the value of normative resistance f_{ck} i.e. the compressive strength of cylinders of 150 mm in diameter and 300 mm in height, tested in age 28 days. The second number is the value of the guaranteed strength of the concrete cube of 150 x 150 x 150 mm with a statistical security of 0.95 ($f^{G}_{c,cube}$). Russian standards are based on the strength of the cube. In accordance with these principles, we established the correspondence between classes C and B (table 1). For example, concrete class B15 corresponds to class C12/15, etc. Further, we found respectively the normative concrete resistance under axial compression R_{bn} (prismatic strength) and f_{ck} (cilindric strength) for compressive strength classes of concrete B and C using tables of regulatory documents. The design values of concrete resistance R_b and f_{cd} (Table 1) are calculated dividing a value of the normative concrete resistance under compression, respectively, R_{bn} by the reliability coefficient for concrete under compression γ_b = 1.3 and f_{ck} by the safety coefficient for concrete γ_c = 1.5. When calculating RC elements for the limit states of the first group for high-strength concrete of class C, the work [2] takes into account the partial coefficient γ_{HSC}. The values of the initial modulus of elasticity of concrete E_b and E_{cm} for the compressive strength class of concrete B and C are taken according to the tables of normative documents. When evaluating the deformation properties of concrete, the works [2, 3] introduce the average values of compressive strength f_{cm}. Concrete compression diagrams are plotted in the coordinates "$\varepsilon_b(\varepsilon_c)-\sigma_b(f_c)$". Here, parentheses contain the denotations of deformations and stresses accepted in [2,3]. The base points of curvilinear diagrams for strength calculations are the following ones: the top of the ascending branch of the diagram which takes coordinates $\hat{\varepsilon}_b(\varepsilon_{c1}), R_b(f_{cd})$; the end of the descending branch which takes the maximum strain value and coordinates $\varepsilon_{bu}(\varepsilon_{cu1}), \sigma_{bu}(f_{cu})$. The work [3] (table 6.1) normalizes the strain values at the base points ε_{c1} and ε_{cu1} which uses when calculating the stresses for concrete compression class C. The dependence stress – strain is constructed using the current values of strains

$$\eta_c = \varepsilon_c/\varepsilon_{c1}\ (|\varepsilon_c| \leq\ |\varepsilon_{cu1}|).$$

The stress value f_c takes its maximum value at $\eta_c = 1$ in the top of the diagram:

$$f_c = f_{cd}$$

– applied at the calculations for the first limiting state and

$$f_c = f_{ck}$$

– applied at the calculations for the second limiting state. Normative document [1] normalizes the magnitude of maximum strains ε_{bu}. Deformations $\hat{\varepsilon}_b$ at the top of the diagram, in contrast to [2, 3], are not assigned according to the tables of norms, but it is calculated by the formula, which takes into account the class and type of concrete. The relative stress level

$$\eta_b = \eta_{bu} = 0{,}85$$

($\eta_{bu} = 1$ for high-strength concrete) limits the descending branch of the diagram. Transforming the formula that describes the diagram, calculations can be performed both through stress and through deformation.

Table 1. Calculation parameters of concrete deformation diagrams.

	Compressive class of concrete	C12	C25	C35	C50	C60	C70	C80	C90
Building Code of Belarus SNB [3]	f_{cd}, MPa	8.0	16,7	23.3	33.3	39.2	42.6	47.6	50.2
	ε_{c1} [%o]	1.9	2,16	2.3	2.48	2.58	2.67	2.76	2.83
	ε_{cu1} [%o]	3.5	3.5	3.47	3.35	3.24	3.11	2.98	2.83
	S_{dc}	24.9	50,37	67.75	89.4	99.2	101.6	105.1	102.9
	ω_{dc}	0.89	0,86	0.845	0.8	0.78	0.768	0.744	0.725
	ε_{cc} [%o]	1.89	1.95	1.96	1.94	1.9	1.84	1.79	1.71
	$M_{c,ult}$, kN m	309	630	861	1191	1367	1450	1560	1590
Building Code of Russia SP [1]	Compressive class of concrete	B15	B30	B45	B60	B75	B85	B95	B105
	R_b, МПа	8.5	17.0	25.0	33.0	39.0	42.5	45.75	49.0
	$\hat{\varepsilon}_b$ [%o]	1.9	2.18	2.36	2.5	2.62	2.68	2.75	2.8
	ε_{bu} [%o]	3.5	3.5	3.44	3.31	3.2	3.04	2.92	2.8
	S_{db}	26.04	50.9	71.0	87.2	95.1	97.6	98.2	97.6
	ω_{db}	0.875	0.855	0.826	0.8	0.762	0.755	0.735	0.711
	ε_{bc} [%o]	1.88	1.948	1.95	1.92	1.87	1.82	1.77	1.71
	σ_{bc} MPa	8.41	15.3	21.3	25.4	27.8	28.8	29.5	29.9
	$M_{b,ult}$, kN m	321	636	916	1173	1341	1422	1482	1528

Currently, a curvilinear diagram is effectively used in structural calculations for the second limiting state, in which the accuracy of the calculation in comparison with the experimental data is determined by the analytical description of the ascending branch of the diagram. It should be noted that some discrepancy between the strain values $\hat{\varepsilon}_b$ and ε_{c1} at the top of the diagram for concrete classes B and C as amended by normative documents [1] and [2] does not lead to significant differences in the outline of the ascending branch of the diagrams and, respectively, the stress values for given strains. Strength calculations use the full concrete deformation diagram for compression. There are increasing requirements for the description of the descending branch of the diagram, for compliance with the recommendations of the norms on limiting the values of both stresses and strains.

Analytical expressions for the description of concrete deformation diagrams characterize short-term loading models. The standard is the test mode of specimens at constant strain growth rates, which allows you to identify two branches of concrete deformation diagrams. In experiments, the rate of change in the load on the test equipment can be accepted arbitrary, the descending branch may appear partially or completely absent. The parameters of the diagram in the edition of normative documents [2, 3] were investigated in experiments with monotonically increasing compression strains, at a speed $\varepsilon_c^* \approx 0{,}015$ %o / sec. It is assumed that the nonlinear properties of concrete for the corresponding concrete classes B and C for a given compression test mode of concrete specimens of prisms and cylinders are manifested equally, and deformations at the base points have the same values: $\hat{\varepsilon}_b = \varepsilon_{c1}; \varepsilon_{bu} = \varepsilon_{cu1}$.

Deformation values at base points are determined according to the rules of the rules depending on the average stresses f_{cm} in the formulas (1), (2) and concrete class B – in the formula (3). This means that the strain values at the base points can be used in the calculations for the limiting states of both the first and second groups.

According to the analytical dependencies presented in the regulatory documents [1,2,3,4], taking into account (5), concrete diagrams "$\varepsilon_b(\varepsilon_c) - \sigma_b(f_c)$" are constructed. The branches of these diagrams pass through the base points. whose values are calculated from expressions (1), (2), (3) and (4). The shape of the concrete diagrams corresponds to the shape of the stress diagrams in the compressed zone of the element (Figures 1, 2).

The dependences for the calculating of deformations at base points. When conducting calculations in software systems, it is more convenient to use analytical dependencies in which the functional relationship is preserved when assigning normalized parameters from the tables. Deformations ε_{c1} increase with increasing concrete strength at maximum compression stress. Meyer (1998) proposed a mathematical model for their calculation:

$$\varepsilon_{c1} = 1{,}6(f_{cm}/10 M\Pi a)^{0{,}25}/1000, \quad (1)$$

where $f_{cm} = f_{ck} + \Delta f$ (Δf = 8 MPa).

It is proposed calculating the ultimate compressive strain of concrete ε_{cu1}, normalized in tabular form [2, 3], by the formula:

$$\varepsilon_{cu1} = \varepsilon_{c1}\left(1 - \frac{f_{cm} - f_{cm}^*}{81 M\Pi a}\left(\frac{10 M\Pi a}{f_{cm}}\right)^{0.2}\right), \quad (2)$$

where f_{cm}^* is the fixed value of the average concrete strength for the concrete class, in which the descending branch is excluded from the calculation and the equalities $|\varepsilon_{c1}| = |\varepsilon_{cu1}|$ and $f_{cd} = f_{cu}$ are satisfied (assumed that f_{cm}^* = 98 MPa).

The analytical dependencies uniform by the structure with (1) and (2), are introduced for heavy concrete in order to determine deformations at base points $\hat{\varepsilon}_b$ and ε_{bu} (Table 1):

$$\hat{\varepsilon}_b = 1{,}75\left(\frac{B}{10 M\Pi a}\right)^{0{,}2}/1000; \quad (3)$$

$$\varepsilon_{bu} = \hat{\varepsilon}_b\left(1 - \frac{B - B^*}{98 M\Pi a}\left(\frac{10 M\Pi a}{B}\right)^{0.2}\right)$$

where B^* is a fixed class of concrete, in which the descending branch is excluded from the calculation and the equalities

$$|\hat{\varepsilon}_b| = |\varepsilon_{bu}| \quad \text{and} \quad \sigma_{bu} = R_b$$

are satisfied (assumed that B^*= 105 MPa).

When working with diagrams, there is a general rule. If deformations are assigned and stresses are calculated during the construction of diagrams, then the maximum values of deformations are limited by values ε_{cu1} (2) and ε_{bu} (4). If stresses are assigned and deformations are calculated [1, 4], then the minimum stress values on the descending branch are limited by the relative stress value η_{bu} calculated by the formula:

$$\eta_{bu} = 1 + \lambda_b \frac{B - B^*}{B + B^*}, \quad (4)$$

where, $\eta_{bu} = \sigma_{bu}/R_b$, here, B^* is a fixed class of concrete, in which the descending branch of a diagram is excluded from the calculation (assumed that B^* = 105 MPa).

If we take into account foreign experience, then from formula (1) it follows that the minimum value of the relative stresses on the descending branch

$$\eta_{cu} = f_{cu} / f_{cd}$$

for $\sigma_c = f_{cu}$.

When increasing the class of concrete accepted by compressive strength, it varies linearly from 0.9 to 1. In norms [1, 4], it is recommended to take the value of 0.85 for low-strength concrete, then

$$\lambda_b = 0.2$$

in the formula (4) and the linear relationship for η_{bu} is maintained for concrete classes ranging from 0.85 to 1.

A drop-down branch is carried out from the expression:

$$\eta_b = 1 + (\eta_{bu} - 1)\left(\frac{\eta_d - 1}{\eta_{du} - 1}\right)^2, \qquad (5)$$

where,

$$\eta_{du} = \varepsilon_{bu} / \hat{\varepsilon}_b, \; \eta_d = \varepsilon_b / \hat{\varepsilon}_b$$

are the current values of strains.

The values of deformations at the base points are determined according to the rules of norms depending on the average stresses f_{cm} in the formula (1, 2) and concrete class B in the formula (3). This means that the strain values at the base points can be used in the calculations for the limiting states of both the first and second groups.

According to the analytical dependencies presented in the regulatory documents [1, 2, 3, 4] taking into account (5), concrete diagrams "$\varepsilon_b(\varepsilon_c) - \sigma_b(f_c)$" are constructed. The branches of these diagrams pass through the base points, whose values are calculated from expressions (1), (2), (3) and (4). The outline of the stress diagrams in the compressed zone of the element corresponds to outline of the concrete deformation diagrams (Figures 1, 2).

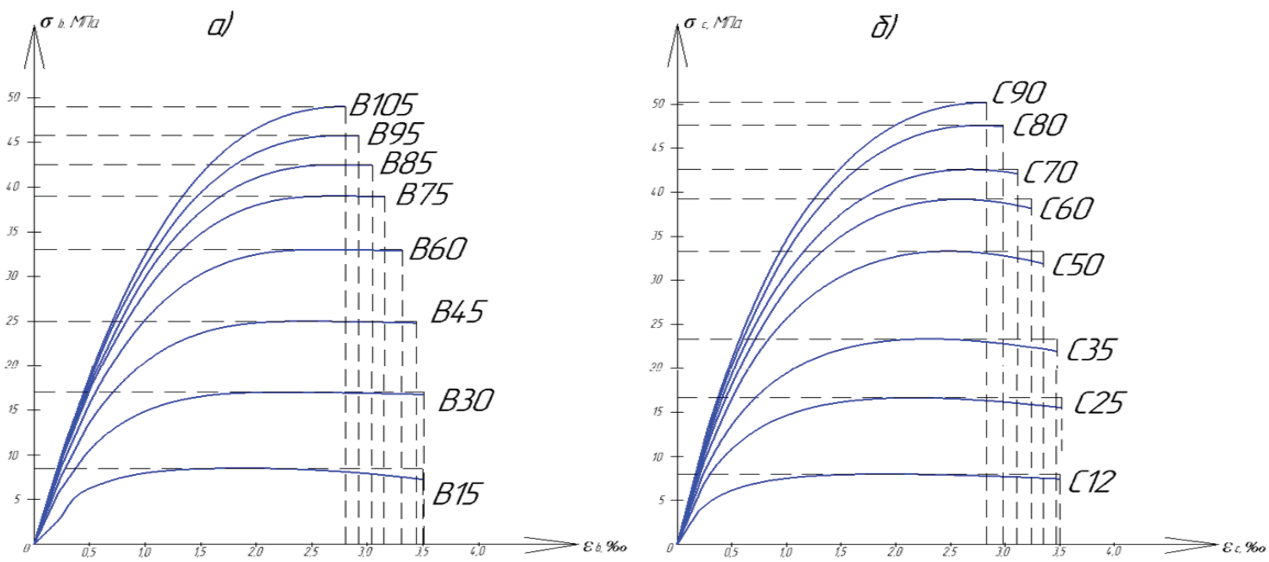

Figure 1. Diagrams of deforming of concrete by regulatory documents: (a) Building Code of Russia SP 63.13330.2012 and (b) Buiding Codeof Belarus SNB 5.03.01-02 taking in account formulas (1) – (5).

ENERGY MODEL FOR CALCULATING THE STRENGTH OF A REINFORCED CONCRETE ELEMENT USING MATERIAL DEFORMATION DIAGRAMS

Figure 2 (d) presents the stress diagram and diagram of internal forces for a rectangular cross-section with reinforcement in the lower zone A_s and in the upper zone $A_s^/$ (Fig. 2b), taking into account the distribution of the deformations of concrete and reinforcement according to the linear law (Fig. 2c). The relations for curvature based on the linear law of the deformations' distribution along the height of the element takes the following form,

$$\frac{1}{\rho}=\chi=\frac{\varepsilon_{sn}}{h_0-x}=\frac{\varepsilon_{bn}}{x}=\frac{\varepsilon_{bn}+\varepsilon_{sn}}{h_0}, \qquad (6)$$

where h_0 is the working height of the section; x is the height of the compressed zone; $\varepsilon_{bn}-$ is deformations of the outer fiber of the compressed zone of concrete; χ - curvature of the element; ρ - radius of curvature; ε_{sn} - deformations in tensile reinforcement.

The values of the internal forces in the reinforcement, respectively in the stretched and compressed zone, are

$$N_s=R_sA_s\,,\quad N_s^/=\sigma_s^/A_s^/=\varepsilon_s^/E_sA_s^/\,.$$

Here the deformation of the reinforcement is determined by the formula:

$$\varepsilon_s^/=\varepsilon_{bn}-\chi a^/\,. \qquad (7)$$

The value of the force N_b perceived by a concrete strip of unit width (b = 1) in the compressed zone at the limiting state is calculated by the formula

$$N_b=S_{db}/\chi\,. \qquad (8)$$

Taking into account the obtained dependences, the equilibrium equation for the limiting state for a symmetric section of width b is written in the form

$$\frac{S_{db}b}{\chi}+\sigma_s^/A_s^/-R_sA_s=0$$

or

$$\frac{S_{db}xb}{\varepsilon_{bu}}+\sigma_s^/A_s^/-R_sA_s=0\,. \qquad (9)$$

In the general case, when the ascending and descending branches of the diagram are described by nonlinear equations, small sections are plotted along the deformation axis using computer simulation (Figure 2a) $\Delta\varepsilon_{b,i}$ (i section numbers).

The height of the elementary area of the section

$$\Delta h_{b,i}=\Delta\varepsilon_{b,i}/\chi$$

with the value of the stress $\sigma_{b,i}$ corresponds to deformations on the diagrams $\Delta\varepsilon_{b,i}$ in the compressed zone of the element.

For each i-th section, it can be determined the following parameters using the diagrams: $\sigma_{b,i}$ - stress value; $\varepsilon_{b,i}$ - deformations in the coordinate system $\varepsilon_b 0\sigma_b$;

$$A_{b,i}=\Delta\varepsilon_{b,i}\sigma_{b,i}$$

- area of the i-th section;

$$S_{db}=\sum_{i=1}^{n}A_{b,i}=\sum_{i=1}^{n}\sigma_{b,i}\Delta\varepsilon_{b,i}$$

- the area of the field bounded by the branches of the diagram.

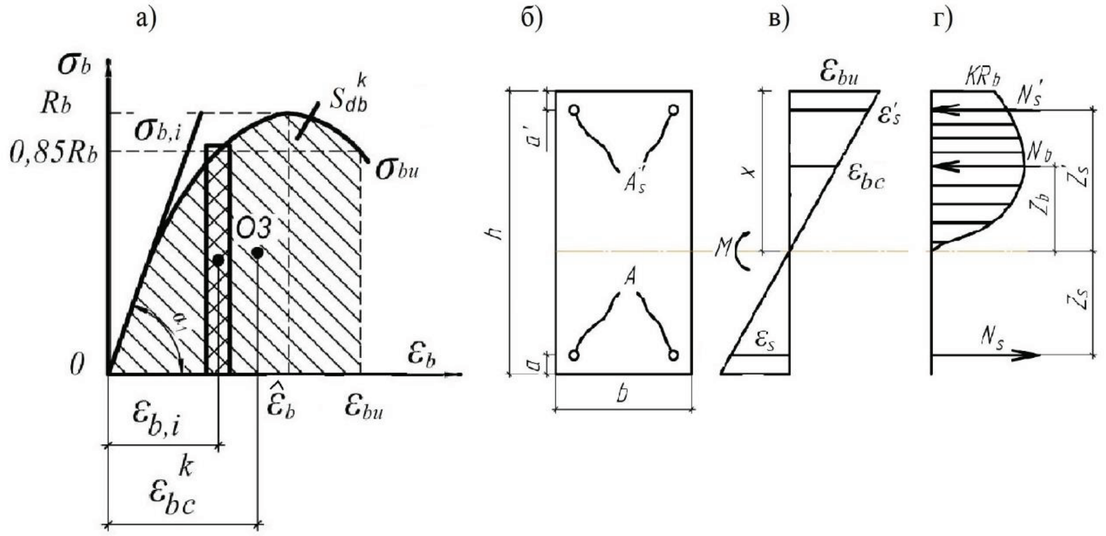

Figure 2. Schemes for explaining the methodic for calculating strength of an element using deformation model (in accordance with regulatory documents [2,3] index 'b' changed by index 'c'): (a) deformation diagram of concrete under compression and scheme for determining the integral parameters; (b) cross-section of an element; (c) linear distribution of deformations along the height of a cross-section; (d) stress diagram for compressed zone and scheme of internal forces into concrete and reinforcement.

The verification of the equilibrium equation (9) is performed by the method of successive approximations (iteration method), in which the variable is the element curvature χ determined from relations (6).

Strength calculation uses the complete concrete diagram (Fig. 2, a). The area of the field bounded by the branches of the diagram S_{db} (S_{dc}) remains constant. An integral characteristic of a concrete deformation diagram is the coefficient of completeness of the diagram ω_{db} (ω_{dc}). This coefficient characterizes the deviation of the actual area of the curved diagram S_{db} (S_{dc}) from the area of the rectangle S^*_{db} (S^*_{dc}) that describes the diagram by base points. The area of the complete diagram S_{db} (S_{dc}) for each class of concrete is calculated by numerical methods or using graphical computer programs (Table). The area of the rectangular diagram is calculated by the formula

$$S^*_{db} = R_b \varepsilon_{bu} \quad \text{or} \quad S^*_{dc} = f_{cd} \varepsilon_{cu1},$$

where R_b, f_{cd} are the design concrete resistances for the limiting states of the first group for concrete of compressive strength classes B and C, respectively; ε_{bu} (ε_{cu1}) - normalized values of ultimate strains are calculated by formulas (2) and (3). Coefficients of completeness of the diagram

$$\omega_{db} = S_{db} / S^*_{db}$$

and

$$\omega_{dc} = S_{dc} / S^*_{dc}$$

are calculated by the formulas

$$\omega_{db} = 0{,}71 - 0{,}2 \frac{B - B^*}{B^*};$$

$$\omega_{dc} = 0{,}724 - 0{,}2 \frac{f_{cm} - f^*_{cm}}{f^*_{cm}}, \tag{10}$$

where B^* is a fixed class of concrete, for which the descending branch is excluded from the calculation and the equalities

$$\left|\hat{\varepsilon}_b\right| = \left|\varepsilon_{bu}\right| \text{ and } \sigma_{bu} = R_b$$

are satisfied (assumed that B^*=105 MPa); f_{cm}^* - a fixed value of the average concrete strength for the concrete class, for which the descending branch is excluded from the calculation and the equalities

$$\left|\varepsilon_{c1}\right| = \left|\varepsilon_{cu1}\right| \quad \text{and} \quad f_{cd} = f_{cu}$$

are satisfied (assumed that f_{cm}^* = 98 MPa).

For an increase of the class of concrete, the curvature of the diagram decreases, approaching to the elastic one (Table), however $\omega_{db} > 0.5$. If condition (9) is satisfied the value of the ultimate bending moment M_{ult} perceived by the cross-section of an element is determined relatively to a fixed zero line:

$$M_{ult} = \frac{S_{db}}{\chi} b z_b + R_s A_s z_s + \sigma_s^/ A_s^/ z_s^/ . \quad (11)$$

The distances from the generalized forces $N_s^/, N_s$ and N_b in the reinforcement and concrete to the neutral axis, respectively, are:

$$z_s^/ = \frac{\varepsilon_b^{(k)} - a^/ \chi^{(k)}}{\chi^{(k)}};$$
$$z_s = \frac{\chi^{(k)} h_0 - \varepsilon_b^{(k)}}{\chi^{(k)}}; \quad (12)$$
$$z_b = \frac{W_{db}}{\chi^{(k)} S_{db}} = \frac{\varepsilon_{bc}}{\chi^{(k)}},$$

where

$$W_{db} = \sum_{i=1}^{n} A_{b,i}\varepsilon_{b,i} = \sum_{i=1}^{n} \sigma_{b,i}\Delta\varepsilon_{b,i}\varepsilon_{b,i}$$

is the moment that numerically equal to the sum of the products of the areas of the elementary section on the concrete diagrams and the distances of their centers of gravity to the stress axis σ_b;

$$\varepsilon_{bc} = W_{db} / S_{db}$$

- deformations at the level of the center of gravity of the diagram O_3 (Fig. 2a); $\chi^{(k)}$ - the curvature of an element after satisfying the equilibrium condition (9) at the *k*-th iteration.

From the formula (12) for z_b, it follows that the deformations at the level of the center of gravity of the stress diagram in the concrete of the compressed zone of an element are equal to the deformations ε_{bc} at the center of gravity of the full diagram. Studies indicate that the ratios between the values of strains at the center of gravity of the diagrams and strains at the top of the diagrams

$$\eta_{bc} = \varepsilon_{bc} / \hat{\varepsilon}_b \; (\eta_{cc} = \varepsilon_{cc} / \hat{\varepsilon}_{c1})$$

are a monotonically decreasing functions (for increasing concrete class B and average concrete strength f_{cm}) that can be described by analytical expressions:

$$\eta_{bc} = \left(\frac{0{,}75 МПа}{B}\right)^{0{,}1} - 0{,}29\frac{B - B^*}{B^*};$$
$$\eta_{cc} = \left(\frac{0{,}65 МПа}{B}\right)^{0{,}1} - 0{,}35\frac{f_{cm} - f_{cm}^*}{f_{cm}^*}, \quad (13)$$

where the parameters B^* and f_{cm}^* are taken from (10).

Modeling of the parameters of the diagrams by analytical dependencies allows us to exclude from the calculations of the strength of elements the procedure of the numerical integration of the areas of elementary sections and the solution of nonlinear equations by the iteration method. The calculation of the strength of the element, taking into account the proposed dependencies, is performed in the following sequence:

- it is assigned a class of concrete, section, reinforcement: $A_s, A_s^/$;
- for a given class of concrete, the coefficient of completeness of the concrete deformation diagram ω_{db} is calculated by formula (10), for the area of a rectangular diagram S_{db}^*, the area S_{db} of the region bounded by the branches of the diagram is calculated;
- taking into account (6) and (7), equation (9) is converted into a quadratic equation with respect to the actual height of the compressed zone *x*:

$$x^2 \frac{S_{db} b}{\varepsilon_{bn}} + x(\varepsilon_{bn} E_s A_s^/ - R_s A_s) - a^/ \varepsilon_{bn} E_s A_s^/ = 0; \tag{14}$$

- according to formula (11) and taking into account (12), the moment value in the limiting state is calculated, where the force distances to the neutral axis are not determined with the parameter $\chi^{(k)}$ obtained by the sequential approximation procedure, but by solving the quadratic equation for the height of the compressed zone (14) and calculating the element curvature from formula (6).

TRANSITION TO THE METHOD OF ULTIMATE FORCES

For calculation by the method of ultimate forces, a simple rectangular diagram of normal stresses in the compressed zone of concrete was adopted. The relationship between the curvilinear stress diagram and the rectangular stress diagram is established from the condition of equality of the forces in these diagrams

$$\frac{S_{db} x b}{\varepsilon_{bu}} = R_b x^* b, \tag{15}$$

from which a relationship between the heights of the compressed zone, respectively *x* and *x** is established for a given cross-section, reinforcement and class of concrete compressive strength. The value of the bending moment M_{ult}^* perceived by the cross-section of the element, according to ultimate forces, is calculated by the formula:

$$M_{ult}^* = R_b b x^* (h_0 - 0{,}5x) + R_{sc} A_s^/ (h_0 - a^/). \tag{16}$$

A comparative analysis of the methods for calculating strength is performed for a reinforced concrete section with dimensions *h* = 60 cm, *b* = 30 cm. Reinforcement in the stretched zone is periodical steel rebars of A400 class. The condition of the equilibrium of forces in the normal section is satisfied by the reinforcement saturation of the stretched zone at given strain values: in the reinforcement

$$\varepsilon_s = R_s / E_s \text{, where } R_s = 355 \text{ MPa;}$$

in the outer concrete fiber of the compressed zone ε_{bu}, calculated by the formula (3). For the simple case of bending, the calculations are carried out in the same sequence, just for given deformations using the formulas (6), the element curvature and the actual height of the compressed zone *x* are calculated. Using the equilibrium equation (9) and without taking into account the reinforcement in the compressed zone, the reinforcement area A_s is determined. The forces in concrete for given concrete class of compressive strength are equal to the forces in the reinforcement.

Table 2. Design values of parameters for limiting state by deformation model (A) and the method of ultimate forces (B)

Compressive class of concrete		B15	B30	B45	B60	B75	B85	B95	B105
A	x, cm	37.3	37.2	37.0	36.4	35.8	35.4	34.9	34.3
	M_{ult}, kN m	317.7	644.8	824.3	1178.7	1342.7	1419.5	1478.0	1523.1
B	x^*, cm	32.2	32.0	30.7	29.2	27.7	26.6	25.5	24.3
	M^*_{ult}, kN m	327	653	834.8	1196.4	1366.0	1449.8	1514.2	1566.5

The curvature of the element and the height of the compressed zone x decrease due to a reduction in the limit values of nonlinear deformations in high-strength concrete when increasing the class of concrete, and the value of the ultimate moment M_{ult} increases (Table 2, A). The height of the compressed zone x^* of rectangular shape is smaller than the actual height of the compressed zone x, however, the increase in the shoulder of the inner pair of forces compensates the difference between the values of the limiting moments calculated by formula (16) without taking into account the reinforcement in the compressed zone (Table 2, B).

CONCLUSION

The ratio of ultimate efforts when calculating the strength of elements according to the deformation model is determined by the integral parameters of the diagrams of concrete deformation under compression, the analytical modeling of which allows us to exclude from the calculation of strength the procedure for numerically summing of stresses along elementary strips in a section and solving nonlinear equations by successive approximations. Replacing a curvilinear stress diagram with a rectangular one does not introduce a significant error in the calculation of ultimate forces, since a decrease in the height of the compressed zone with a rectangular diagram is compensated by an increase in the shoulder of the internal pair of forces.

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