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SOLUTION OF THE PROBLEM OF THERMOELASTICITY FOR NONLINEAR ELASTIC INHOMOGENEOUS THICK-WALL CYLINDRICAL SHELL

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Abstract: The distinctive paper presents the calculation of a thick-walled cylindrical shell with hinged and free ends on the temperature effect. The shell consists of three layers: two layers of heat-resistant concrete and steel outer layer. The calculation takes into account the piecewise linear inhomogeneity of the shell due to its three-layer construction and the continuous inhomogeneity caused by the action of a stationary temperature field. To take into account the nonlinear nature of concrete deformation, the problem was solved using the method of successive approximations described in [1]. A comparative analysis of the results of the calculation of the shell with and without taking into account the continuous inhomogeneity and the nonlinear nature of the deformation of concrete is given. Comparison of the results showed a significant decrease in circumferential stresses in the most loaded concrete layers when calculating the shell with regard to physical nonlinearity and heterogeneity of materials.

Keywords: nonlinearity, inhomogeneity, concrete, elevated temperature, cylindrical shell

РЕШЕНИЕ ЗАДАЧИ ТЕРМОУПРУГОСТИ ДЛЯ НЕЛИНЕЙНО УПРУГОЙ НЕОДНОРОДНОЙ ТОЛСТОСТЕННОЙ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКИ

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Аннотация: В настоящей статье приводится расчет толстостенной цилиндрической оболочки с шарнирно закрепленными и свободными торцами на температурное воздействие. Оболочка состоит из трех слоев: два слоя из жаростойкого бетона и стальной наружный слой. При расчете учитывается кусочно-линейная неоднородность оболочки, обусловленная ее трехслойной конструкцией и непрерывная неоднородность, вызванная воздействием стационарного температурного поля. Для учета нелинейного характера деформирования бетона задача решалась методом последовательных приближений, описанном в [1]. Приведен сравнительный анализ результатов расчета оболочки с учетом и без учета непрерывной неоднородности и нелинейного характера деформирования бетона. Сравнение результатов показало значительное снижение окружных напряжений в наиболее нагруженных слоях бетона при расчете оболочки с учетом физической нелинейности и неоднородности материалов.

Ключевые слова: нелинейно упругий материал, неоднородность, бетон, повышенные температуры, цилиндрическая оболочка

INTRODUCTION

Structural elements in the form of hollow cylinders are widely used in technological equipment of the chemical and energy industries. Such structures often work in conditions of elevated and high temperatures, aggressive environments (reactors and regenerators for many catalytic processes). In order to protect the steel casing of the apparatus, it is lined inside with heat-resistant concrete, which is coated by a corrosion-resistant layer.

This design of the reactor allows to protect the metal against corrosion, reduce the metal consumption of the apparatus and reduce heat loss. Modern building standards [2] require taking into account changes in the mechanical and elastoplastic properties of concrete depending on the temperature of exposure, and suggest using a two-line or three-line deformation diagram for assessing the stress-strain state of compressed concrete. This article proposes a solution to the problem of thermoelasticity taking into account changes in the properties of concrete depending on temperature and using experimental deformation diagrams.

1. FORMULATION OF THE PROBLEM

The problem of calculating a three-layer cylindrical shell on the temperature effect is considered. Shell materials are: internal corrosion-resistant layer of heat-resistant concrete made of alumina cement (concrete No. 1) of 50 mm thick, the middle layer of heat-resistant concrete made of Portland cement (concrete No. 2) - of 100 mm, the outer layer made of steel - of 40 mm. A constant temperature of 500 ° C is maintained inside.

Two solutions are considered: with hinged end of the cylinder and with a free end (Figure 1 a, b).
The temperature distribution inside the multilayer wall is determined by solving the heat equation. The following initial data are used in the solution: $T_i = 500\,°\mathrm{C}$ is temperature inside the shell; $T_o = 20\,°\mathrm{C}$ is the temperature of the outside air; $\alpha_1 = 162{,}8\,\mathrm{W/m^2{°}C}$ is heat transfer coefficient from the internal area to the concrete wall; $\lambda_1 = 0{,}8\,\mathrm{W/m°C}$ is the thermal conductivity of the first concrete layer; $\lambda_2 = 0{,}85\,\mathrm{W/m°C}$ is the thermal conductivity of the second concrete layer; $\lambda_3 = 25\,\mathrm{W/m°C}$ - coefficient of thermal conductivity of steel; $\alpha_2 = 7{,}6\ \mathrm{W/m^2{°}C}$ - heat transfer coefficient from the outer surface of the shell to the air; $r_1 = 0{,}55\,\mathrm{m}$, $r_2 = 0{,}6\,\mathrm{m}$, $r_3 = 0{,}7\,\mathrm{m}$, $r_4 = 0{,}74\,\mathrm{m}$.
The temperature distribution in the three-layer wall is shown in Figure 1: $T_i = 500\,°\mathrm{C}$, $T_1 = 488{,}9\,°\mathrm{C}$, $T_2 = 380{,}4\,°\mathrm{C}$, $T_3 = 199{,}5\,°\mathrm{C}$, $T_4 = 197{,}3\,°\mathrm{C}$, $T_o = 20\,°\mathrm{C}$.

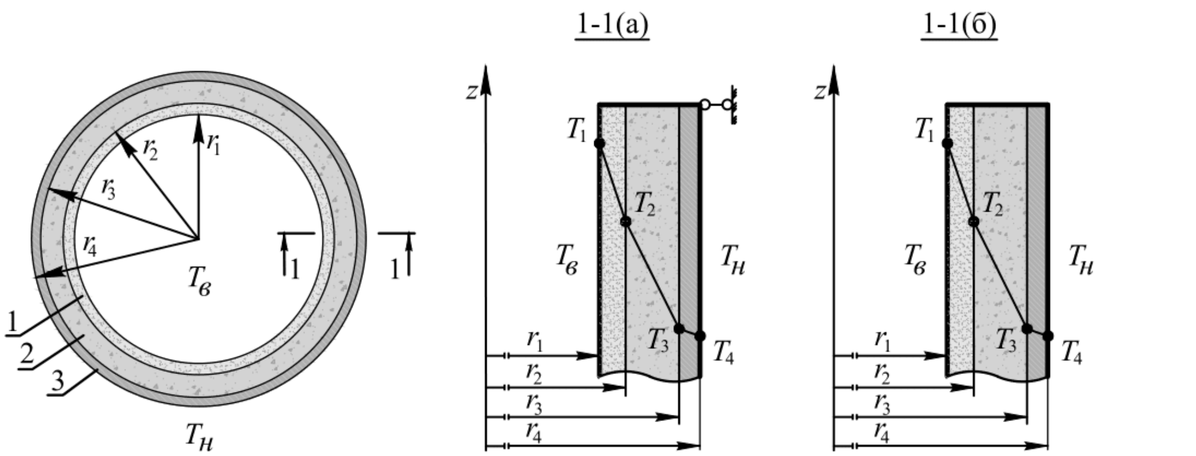


Figure 1. Distribution of temperature into a three-layer shell:
1 – concrete No. 1; 2 - concrete No. 2; 3 – steel.

Function of the temperature changes through the radius of each layer takes the form:

$$T_j(r) = \ln(r/a)\,\frac{T_{j+1}-T_j}{\ln(r_{j+1}/r_j)} + \frac{T_j \ln(r_{j+1}/a) - T_{j+1}\ln(r_j/a)}{\ln(r_{j+1}/r_j)}, \quad (1)$$

where j is layer number (1, 2, …, n), a = 1 m.
Thus, the following temperature distribution functions were obtained for the first and second concrete layers:

$$T_1(r) = -1247\ln(r) - 256{,}4,$$
$$T_2(r) = -1173\ln(r) - 218{,}9. \qquad (2)$$

The change in temperature over the thickness of the steel layer is not taken into account in the solution, the average temperature value $T_3 = 198{,}4\,^{\circ}\mathrm{C}$. is taken instead it.

In order to describe the nonlinear nature of concrete deformation, the experimental deformation diagrams of heat-resistant concrete given in [3] are used. The solution uses a diagram $\sigma_i - \varepsilon_i$ that is described by a dependence with three constants, proposed in [4]:

$$\sigma_i = E\varepsilon_i - A\varepsilon_i^{\alpha}, \qquad (3)$$

The heterogeneity of concrete resulting from exposure under elevated temperatures is taken into account by replacing the constants E, A, and α with the functions $E\,(T)$, $A\,(T)$, and $\alpha\,(T)$. In order to describe the deformation diagrams of heat-resistant concrete made of Portland cement, the following functions were used:

$$E_1(T) = E_{01} + k_{E1}\left(\frac{\Delta T}{T_0}\right)^{0,5} + m_{E1}\left(\frac{\Delta T}{T_0}\right)^{2,5},$$
$$A_1(T) = A_{01} + k_{A1}\left(\frac{\Delta T}{T_0}\right)^{0,8} + m_{A1}\left(\frac{\Delta T}{T_0}\right)^{1,5}, \qquad (4)$$
$$\alpha_1(T) = \alpha_{01} + k_{\alpha 1}\left(\frac{\Delta T}{T_0}\right)^{1,2} + m_{\alpha 1}\left(\frac{\Delta T}{T_0}\right)^{1,5}.$$

The following functions were used to describe the deformation diagrams of heat-resistant concrete on alumina cement:

$$E_2(T) = E_{02}\cdot\exp\left[k_{E2}\left(\frac{\Delta T}{T_0}\right)^{0,9} + m_{E2}\left(\frac{\Delta T}{T_0}\right)^{0,2}\right],$$
$$A_2(T) = A_{02}\cdot\exp\left[k_{A2}\left(\frac{\Delta T}{T_0}\right)^{0,7} + m_{A2}\left(\frac{\Delta T}{T_0}\right)^{0,3}\right], \qquad (5)$$
$$\alpha_2(T) = \alpha_{02}\cdot\exp\left[k_{\alpha 2}\left(\frac{\Delta T}{T_0}\right)^{0,9} + m_{\alpha 2}\left(\frac{\Delta T}{T_0}\right)^{0,15}\right].$$

We accept the following designations in formulas (4) and (5): $\Delta T = T - T_0$, where $T_0 = 20\,^{\circ}C$ is the normal temperature of concrete; E_0, A_0, α_0, k_E, k_A, k_α, m_E, m_A, m_α are the coefficients obtained by approximating the experimental deformation diagrams.

The work [2] presents the values of the coefficient of linear temperature deformation of concretes of various compositions depending on temperature. We accepted the values α_b that correspond to the continuous heating mode upon repeated exposure to temperature. To approximate the data from [2], the following functions were used:

$$\alpha_{b1}(T) = k_1\left(\frac{\Delta T}{T_0}\right) + k_2\left(\frac{\Delta T}{T_0}\right)^{1,5} + k_3\left(\frac{\Delta T}{T_0}\right)^{2},$$
$$\alpha_{b2}(T) = \alpha_t + k_4\left(\frac{\Delta T}{T_0}\right)^{1,1} + k_5\left(\frac{\Delta T}{T_0}\right)^{1,2} + k_6\left(\frac{\Delta T}{T_0}\right)^{1,5}, \qquad (6)$$

Where α_{b1} is the coefficient of linear temperature deformation of concrete No. 1; α_{b2} is coefficient of linear temperature deformation of concrete No. 2; $k_1 \ldots k_6$ are the coefficients obtained by approximating the data from [2].

The values of the coefficients from formulas (4), (5), (6), as well as the diagram $\sigma_i - \varepsilon_i$ of the deformation of concrete of two different compositions are given in [5], where a solution to a similar problem for an infinite cylindrical shell was considered.

2. METHOD OF ANALYSIS

Based on the method of successive approximations, we developed a numerical method for solving plane axisymmetric and centrally symmetric problems for thick-walled shells made of physically nonlinear radially inhomogeneous material with arbitrary dependences of mechanical characteristics through the radius. We obtained numerical solutions to test problems. A comparison of the results of numerical and analytical solutions showed sufficient accuracy of the developed method [6]. The solution for the condition of a planar deformed state assumes that the cylinder is very long and stresses arising at a sufficient distance from the ends are considered. This article proposes a solution to the problem taking into account local disturbances near the ends of the cylinder.

The solution obtained for the conditions of planar deformable state requires that normal stresses σ_z^* be distributed at the ends of the cylinder. The work [7] proposed determining the stresses in the final cylindrical shell by summing up the solution for the planar deformable state condition and the solution for the cylindrical shell, at the ends of which forces, that are equal in value and opposite in sign to the stresses σ_z^*, are applied.

In order to determine the stresses caused by these forces ($-\sigma_z^*$), we consider a longitudinal strip of unit width cut from a cylindrical shell. Such a strip can be considered as a beam on an elastic base, for which the equation of deflection has the form:

$$D\frac{d^4u}{dz^4}+Ku=p(z)\,, \qquad (7)$$

where $p(z)=0$ is the intensity of the radial axisymmetric load;

$$D=\frac{Eh^3}{12(1-\nu^2)}$$

is the flexural stiffness of the cylindrical shell, replacing the flexural stiffness of the rod *EI*;

$$K=\frac{Eh}{R^2}$$

is the stiffness of the shell in tension-compression in the circumferential direction, replacing the stiffness coefficient of the elastic base.

In the case of a radially inhomogeneous shell, the equation takes the form:

$$\frac{d^4u}{dz^4}\frac{h^2}{12}\int_{r_1}^{r_4}\frac{E(r)dr}{1-\nu(r)^2}+u\int_{r_1}^{r_4}\frac{E(r)dr}{r^2}=0\,, \qquad (8)$$

where *h* is the thickness of the shell.

For the convenience of integrating equation (8), we introduce the dimensionless variable: $\xi=\lambda z$, where

$$\lambda=\sqrt[4]{\frac{K_1}{4D_1}}\,.$$

The parameter λ depends on the flexural stiffness of the shell and on the stiffness of the shell in tension-compression in the circumferential direction. Derivatives with respect to variables ξ and *z* are related by the ratio:

$$\frac{d^n}{dz^n}=\lambda^n\cdot\frac{d^n}{d\xi^n}\,. \qquad (9)$$

After replacing the variable, the differential equation takes the form:

$$\frac{d^4u(\xi)}{dz^4}+4u(\xi)=0\,. \qquad (10)$$

Solution to the homogeneous equation:

$$u(\xi)=\sin\xi\,\left(c_1\exp\xi+c_3\exp(-\xi)\right)+\cos\xi\,\left(c_2\exp\xi+c_4\exp(-\xi)\right), \qquad (11)$$

where

$$\xi=z\sqrt[4]{\frac{K_1}{4D_1}}\,.$$

Stresses arising near the ends of the cylinder quickly descend with increasing distance from the end z, and already at a distance of 1.5 m from the edge tend to zero, therefore, with a cylinder length of more than L = 3 m, the beam can be considered as semi-infinite, on the end of which stresses $-\sigma_z^*$ are attached. Then, assuming that the integration constants C_1 and C_2 equal to zero, we obtain the equation:

$$u(\xi)=c_3\exp(-\xi)\sin(\xi)+c_4\exp(-\xi)\cos(\xi). \quad (12)$$

The constants C_3 and C_4 are determined from the boundary conditions. Let us consider the case of hinge supporting and free edge.
For hinge supporting it is

$$u(0)=0,$$

then $c_4=0$. Also it is known normal stresses σ_z distributed over the ends of the cylinder, which are equal to the stresses calculated in the planar deformable state condition, taken with the opposite sign:

$$\sigma_z(0)=-\sigma_z^*.$$

Stresses σ_z are determined through displacements:

$$\sigma_z=-\frac{E}{1-\nu^2}\left(r-\frac{r_1+r_4}{2}\right)\frac{d^2u}{dz^2}. \quad (13)$$

From the condition $\sigma_z(0)=-\sigma_z^*$, we get:

$$c_3=\frac{1}{2\lambda^2}\frac{\int_{r_1}^{r_4}\sigma_z^*(r)dr}{\int_{r_1}^{r_4}\frac{E(r)}{(1-\nu(r))^2}\left(r-\frac{r_1+r_4}{2}\right)dr}. \quad (14)$$

The final expression for the displacement:

$$u(z)=\frac{1}{2\lambda^2}\frac{\int_{r_1}^{r_4}\sigma_z^*(r)dr}{\int_{r_1}^{r_4}\frac{E(r)}{(1-\nu(r))^2}\left(r-\frac{r_1+r_4}{2}\right)dr}\times \quad (15)$$

$$\times\exp(-\lambda z)\sin(\lambda z).$$

For the condition of a free edge, the conditions

$$\varphi(0)=0,\ \varphi=\frac{du}{dz}$$

are satisfied, then $c_4=-c_3$, the expression for the displacements takes the form:

$$u(z)=\frac{1}{2\lambda^2}\frac{\int_{r_1}^{r_4}\sigma_z^*(r)dr}{\int_{r_1}^{r_4}\frac{E(r)}{(1-\nu(r))^2}\left(r-\frac{r_1+r_4}{2}\right)dr}\times \quad (16)$$

$$\times\left[\exp(-\lambda z)\cos(\lambda z)-\exp(-\lambda z)\sin(\lambda z)\right].$$

Since we have a formula for the deflection curve, it is possible to calculate the corresponding bending stresses σ_z^u and tangential stresses σ_θ^u for any value of z. The component of deformation in the tangential direction is equal for each layer:

$$\varepsilon_\theta^u=\frac{u}{r}.$$

The component of stresses in the tangential direction is determined from Hooke's law:

$$\sigma_\theta^u=E\varepsilon_\theta^u+\nu\sigma_z^u=$$
$$=E\frac{u}{r}+\nu\frac{E}{1-\nu^2}\left(\frac{r_1+r_4}{2}-r\right)\frac{d^2u}{dz^2}. \quad (17)$$

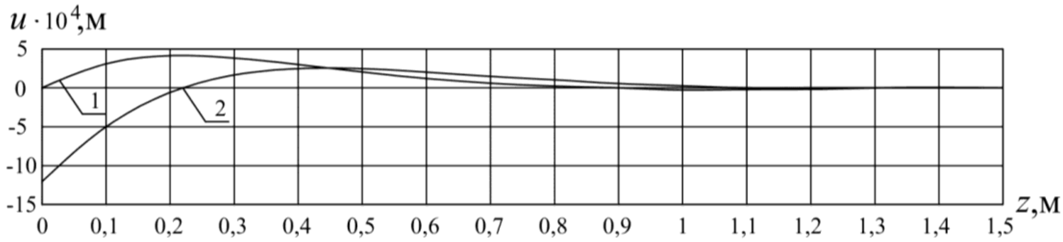


Figure 2. Curvature of deflections caused by acting of $-\sigma_z^*$.

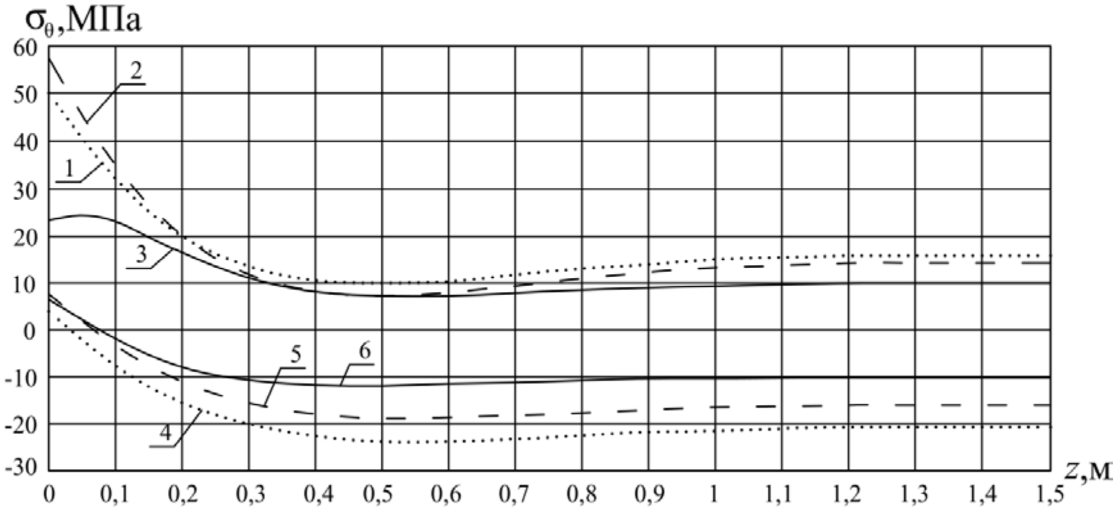


Figure 3. Stresses σ_θ *near the free end of cylinder for* $r = 0,55$ *m and* $r = 0,7$ *m.*

The final stresses in the cylinder are obtained by summing the stresses obtained assuming that stress-strain state is $\sigma_z^*, \sigma_\theta^*, \sigma_r^*$, and the stresses arising due to the action of the force $-\sigma_z^*$ attached at the ends of the cylinder are $\sigma_z^u, \sigma_\theta^u$.

3. RESULTS

Figure 2 shows the curve of the deflections that occur near the end of the cylinder: 1 - deflection for hinge supporting case, 2 - for the free end.

Figures 3-6 show stresses σ_θ calculated both for homogeneous materials and in comparison with the linear calculation. In the solution for homogeneous materials, the influence of elevated temperature on the properties of concrete was taken into account, but the values of the basic elastic characteristics of concrete were adopted for average temperature over the layer.

Figure 3 shows stresses σ_θ near the free end of the cylinder in the most stressed annular concrete layers $r = 0,55$ m and $r = 0,7$ m: 1 - linear homogeneous material, $r = 0,7$ m; 2 - linear heterogeneous material, $r = 0,7$ m; 3 - nonlinear heterogeneous material, $r = 0,7$ m; 4 - linear homogeneous material, $r = 0,55$ m; 5 - linear heterogeneous material, $r = 0,55$ m; 6 - nonlinear inhomogeneous material, $r = 0,55$ m.

Figure 4 shows the stresses σ_θ near the hinged end of the cylinder in the most stressed annular concrete layers $r = 0,55$ m and $r = 0,7$ m: 1 - linear homogeneous material, $r = 0,7$ m; 2 - linear heterogeneous material, $r = 0,7$ m; 3 - nonlinear heterogeneous material, $r = 0,7$ m; 4 - linear homogeneous material, $r = 0,55$ m; 5 - linear heterogeneous material, $r = 0,55$ m; 6 - nonlinear inhomogeneous material, $r = 0,55$ m.

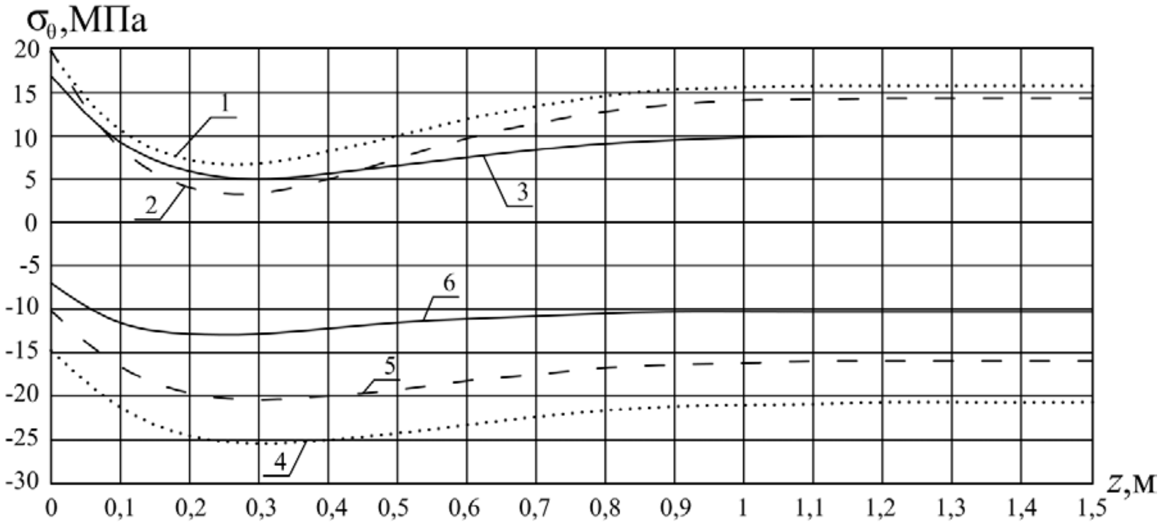


Figure 4. Stresses σ_θ near the hinged end of cylinder for $r = 0,55\,m$ and $r = 0,7\,m$.

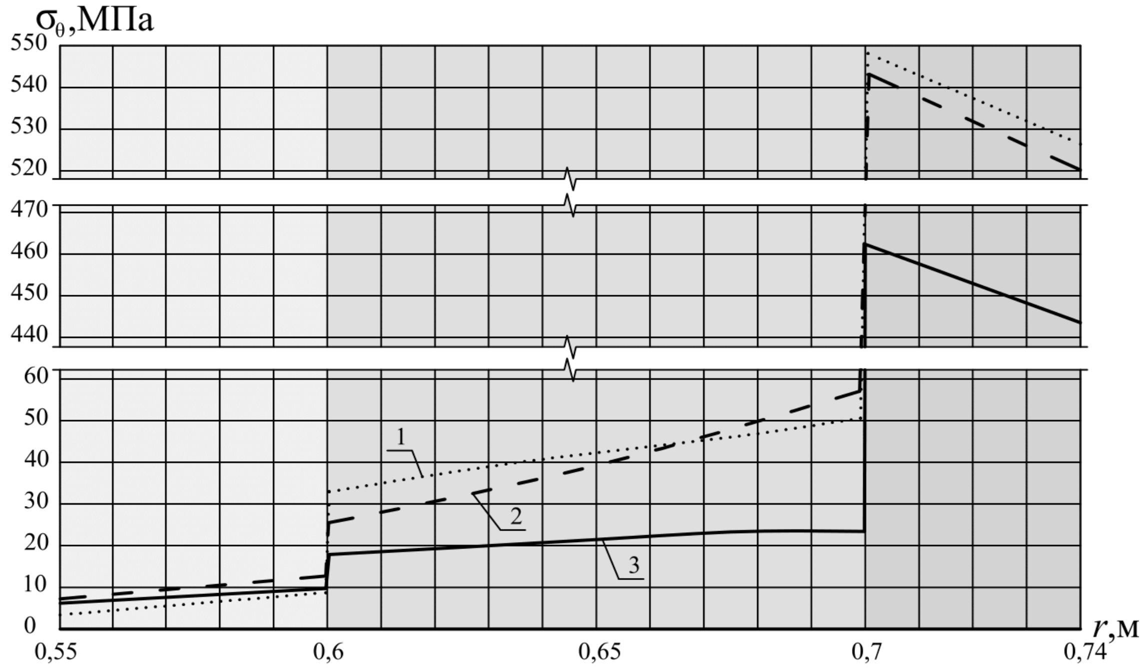


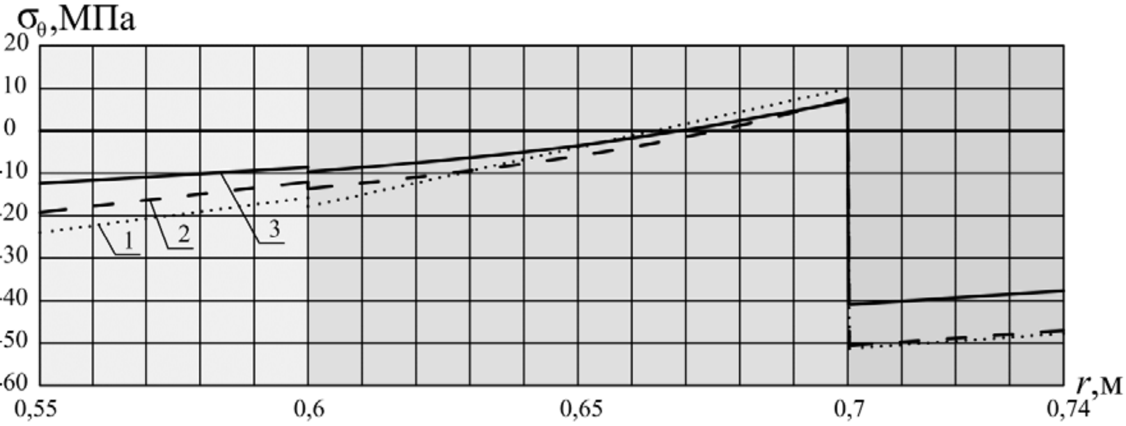
Figure 5. Stresses σ_θ for $z = 0\,m$.

Figures 5 and 6 show the stresses σ_θ over the entire wall thickness, at the free ends of the cylinder, at distances $z = 0$ m and $z = 0,55$ m: 1 - linear homogeneous material; 2 - linear heterogeneous material; 3 - nonlinear heterogeneous material.

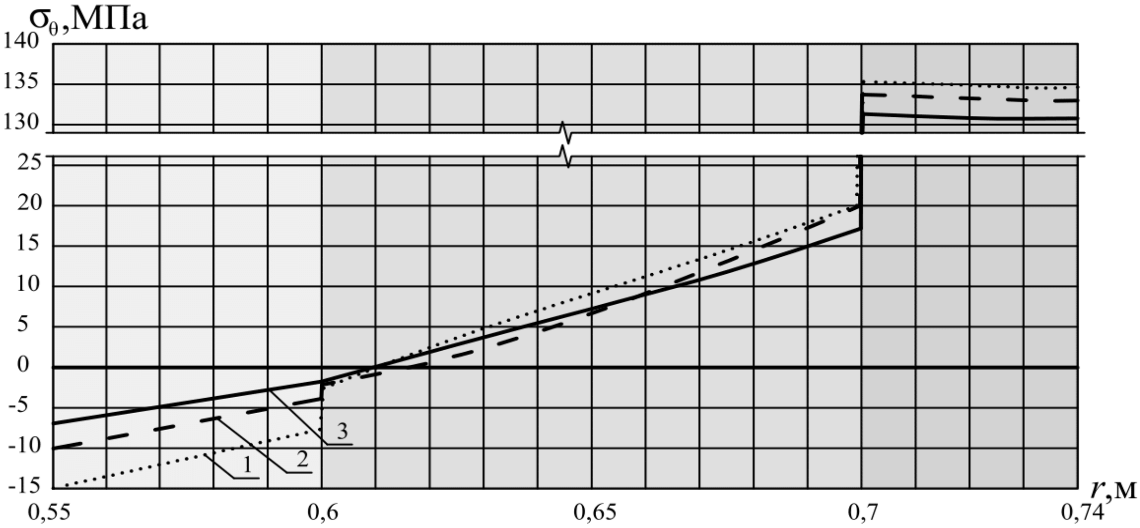
Figures 7 and 8 show stresses σ_θ over the entire wall thickness, with cylinder ends pivotally attached, at distances $z = 0\,m$ and $z = 0,32$ m: 1 – linear homogeneous material; 2 - linear heterogeneous material; 3 - nonlinear heterogeneous material.

CONCLUSIONS

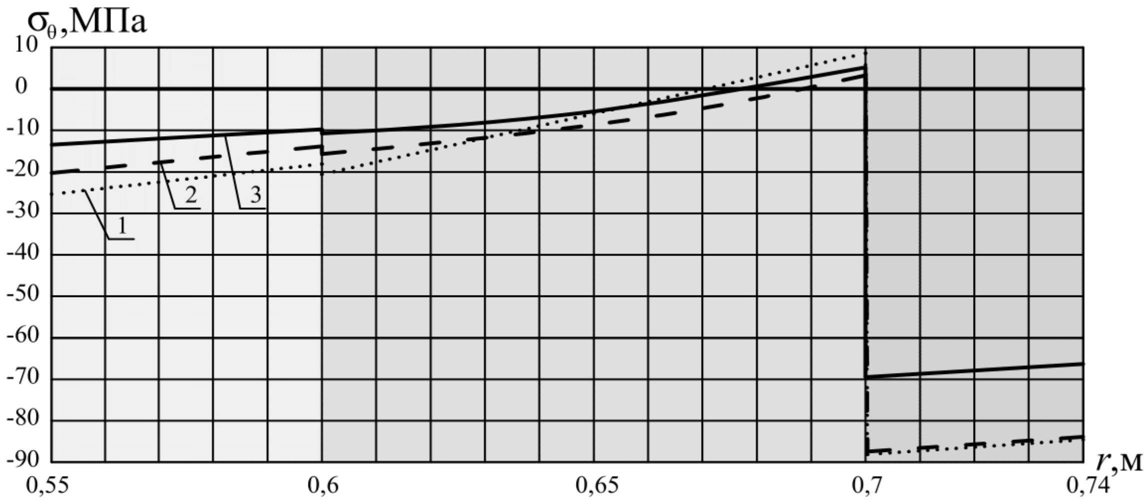
When analyzing the results stresses in the compressed zone of concrete are of greatest interest, since tensile stresses on the outside of the lining should be perceived by steel reinforcement.



<u>Figure 6.</u> Stresses σ_θ *for* $z = 0,55\,m$*.*



<u>Figure 7.</u> Stresses σ_θ *for* $z = 0\,m$*.*



<u>Figure 8.</u> Stresses σ_θ *for* $z = 0,32\,m$*.*

Taking into account only the heterogeneity of concrete gives a reduction in circumferential stresses of about 20% in the compressed zone. When taking into account the heterogeneity and physical nonlinearity of concrete, the maximum circumferential stresses are significantly reduced in both the stretched and compressed zones, by about 2 times.

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