OPTIMIZATION OF SIMPLY SUPPORTED CASTELLATED I-BEAMS LOADED BY A UNIFORMLY DISTRIBUTED LOAD

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Abstract: The paper discusses the problem of optimizing the geometric parameters of simply supported I-beams in order to maximize their load carrying capacity. Numerical simulation of various types of failure of castellated I-beams with ideal elastic-plastic steel is carried out. The stability of the wall, the strength of the welds and flanges, depending on the geometric parameters investigated. Using the coordinate descent method, the optimization problem is solved for nine design schemes with respect to the section height and the weld length. It was revealed that in short beams the section height should be less and the weld length longer, in contrast to long beams.

Keywords: castellated I-beam, numerical simulation, optimization, ideal elastic-plastic material

ОПТИМИЗАЦИЯ СВОБОДНО ОПЕРТЫХ РАЗВИТЫХ ДВУТАВРОВ, НАГРУЖЕННЫХ РАВНОМЕРНО РАСПРЕДЕЛЕННОЙ НАГРУЗКОЙ

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Аннотация: В статье рассматривается проблема оптимизации геометрических параметров развитых двутавров с перфорированной стенкой с целью максимизации их несущей способности. Проведено численное моделирование различных типов разрушения развитых двутавров в идеальной упругопластической постановке. Исследована устойчивость стенки, прочность швов и полок в зависимости от геометрических параметров. Используя метод покоординатного спуска, решена проблема оптимизации для девяти расчетных схем относительно двух параметров: высоты сечения и длины сварного шва. Выявлено, что в коротких балках высота сечения должна быть меньше, а длина шва больше, чем в длинных балках.

Ключевые слова: развитые двутавры, численное моделирование, оптимизация, идеальный упругопластический материал

INTRODUCTION

Existing analytical methods for calculating strength, rigidity and stability of castellated I-beams give a very approximate result [1-2]. These methods bad take in account a complexity of the geometric shape of the castellated I-beams and plastic deformation steel in angles of holes. These factors can be taken account with sufficient accuracy only by experiment and numerical methods. Of practical interest is the question of choosing such geometric parameters zigzag cutting I-beam that will provide maximum load carrying capacity of castellated I-beam. But experiment methods are too expensive and time-consuming for their application in full optimization problem.
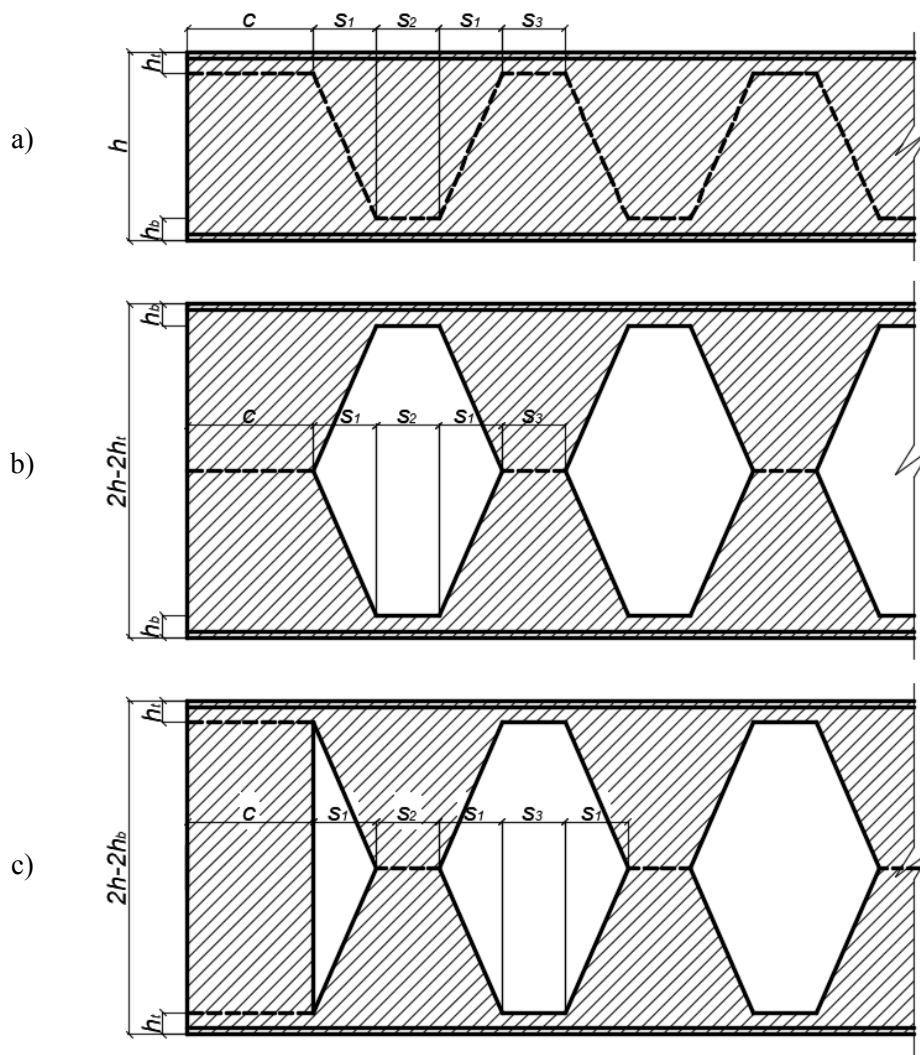
Figure 1. Scheme method for cutting (a) and subsequent welding of halves
(b, c) of the original I-beam.

In this paper, to solve the nonlinear optimization problem, numerical methods are used. Stress-strain state and buckling is calculated by the finite element method. Using FEM analysis results, the ultimate load on the castellated I-beam is calculated according to several criteria (more about the criteria will be further in the text). To search the optimal geometric parameters, the coordinate descent method is used.

ANSYS Mechanical is used to solve the problem. APDL macros allow automated load carrying capacity calculation and use powerful capabilities of ANSYS in the finite element analysis [3].

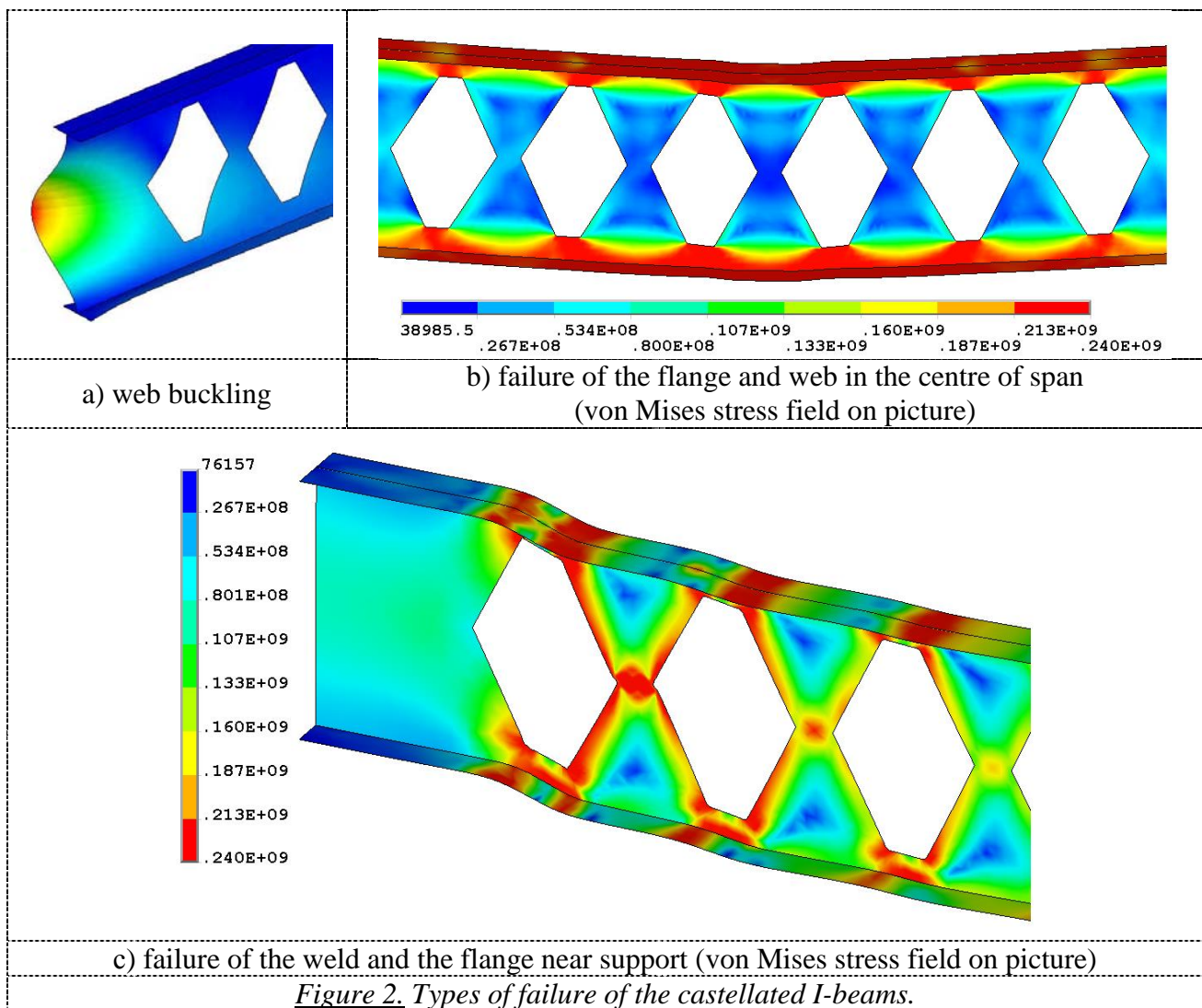There are several methods for cutting and subsequent welding of halves of the I-beam. Differ-ent methods give different relationship between geometric parameter. The paper adopts the method illustrated in the Figure 1.

The article discusses the optimization of the load carrying capacity of the castellated I-beam composed of the lower halves of the original I-beams (Figure 1a). In order for the holes to be the same in both versions of the castellated I-beam (Figure 1b, 1c)

$$s_2 = s_3.$$

Radius of fillet in the corners due to cutting – $r = 1cm$.

The design scheme: beam supported at its ends and loaded by a uniformly distributed load Q.

a) web buckling

b) failure of the flange and web in the centre of span
(von Mises stress field on picture)

c) failure of the weld and the flange near support (von Mises stress field on picture)

Figure 2. Types of failure of the castellated I-beams.

The material model (steel): ideal elastic-plastic (stress-strain curve with elastic modulus $E = 206$ GPa, yield strength $\sigma_y = 240$ MPa, Poisson's ratio $v = 0.3$).

Geometric parameters of the original I-beams are taken according to GOST 8239-89 [4].

1. LOAD CARRYING CAPACITY PARAMETERS

There are 4 types of failure of the castellated I-beam supported at its ends and loaded by a uniformly distributed load:

1. Web buckling (Figure 2a).
2. Failure of the flange and web in the field of angles in the centre of span (Figure 2b).
3. Failure of the flange and web in the field of angles near support (Figure 2c).
4. Failure of the weld (Figure 2c).

For all types of failure, three parameters can be introduced:

1. $F_b = F_{cr}/1.5$, where F_{cr} − the first critical load, *1.5* − safety factor [1].
2. F_f − load at which plastic strain completely fills the section of the flange.
3. F_w − load at which plastic strain completely fills the weld between two holes.

To automatic calculate the F_b, a buckling analysis (by the Block Lanczos method, linear material model) was first performed. Then the first critical load F_{cr} devided by safety factor *1.5*, due to the possible eccentricity of the load and the initial imperfections of the castellated I-beam.

The top flange was fixed against possible lateral movement and rotation around the longitudinal axis, thereby simulating the mounting of the castellated I-beam to overlying structures. The top flange boundary condition us used only in buckling analysis, since it strongly affects the forms of buckling.

To automatic calculate the F_f and F_w, a linear static calculation (linear material model) was initially performed to determine the load F_p at which plastic appears in the castellated I-beam flange. Then a load equal $1.2F_p$ was applied and the static calculation was performed with a nonlinear material with a load step of 0.01. The load $1.2F_p$ is likely to lead to the failure of the I-beam, because according to analytical decisions, $\approx 1.18F_p$ is enough. Then, in the Postprocessor, a special algorithm at each load step checks for plastic strain in the nodes of the flange and nodes of the weld. If at some load step in a some section of the flange or in the weld in all nodes there are plastic strain, then this load step is recognized as the moment of failure. So,

$$F_f = 1.2F_p[i/100+(i-1)/100]/2,$$
$$F_w = 1.2F_w[j/100+(j-1)/100]/2,$$

where i – load step is recognized as the moment of failure flange, j – load step is recognized as the moment of failure weld.

Ideal elastoplastic material behaves unphysically in the support zone. If fix the beam pointwise on the edge, then near the fastening large plastic strain will quickly develop and the solution will fall apart. Therefore, in a nonlinear analysis at the edges of the beam, all nodes in the section were fixed in the direction of deflection. This avoids problems with the boundary effects.

2. FINITE ELEMENT MODELS

Geometric and finite element model are created automatically using developed APDL macros [3]. In the area between the last holes, and also above them and central holes, the mesh is more detailed. Failure occurs in these areas. Only the top flange for searching for plastic strain is considered, as it does not have boundary effect of supports (Figure 3).

The weld does not differ geometrically and in properties from the steel of original I-beam.

Elements type – *Shell 181*. In the buckling analysis, to combat non-physical buckling forms, full integration in the wall elements was used. Also, a coarser grid was used in the linear and buckling analyses.
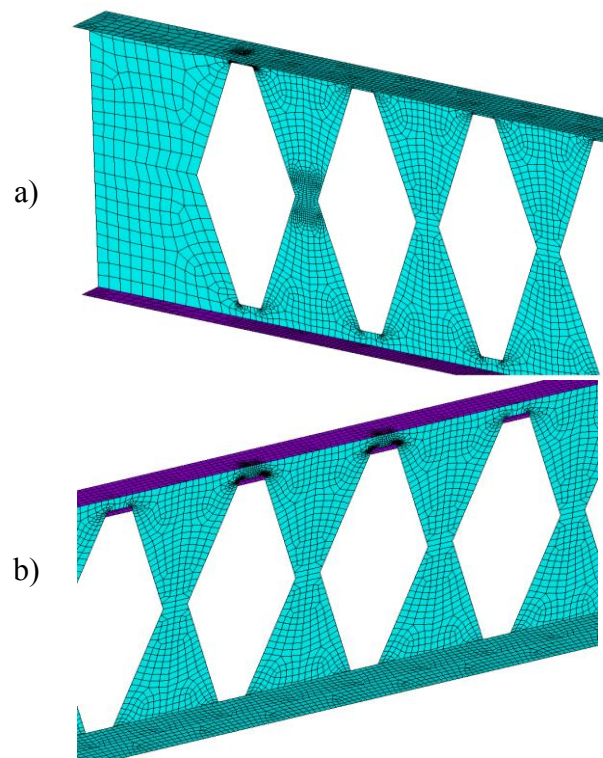

Figure 3. Fragments of the finite element model. Number of elements: ~15-40 thousands depending on the length of the beam and the number of holes.

3. DEPENDENCE OF F_b, F_f AND F_w ON GEOMETRUC PARAMETERS

The important points to make about F_w and F_f calculations:

- if $F_f < F_w$ (much), then F_w cannot be calculated correctly. The fact is that with the destruction of the flange of the castellated I-

beam, the entire structure suffers a collapse – a sharp increase in deflections and plastic strain. In such conditions, the solution loses stability and cannot give correct results;

- if weld length s_2 is large enough, then the weld failure does not occur under load $1.2F_p$. In this case, F_w is not calculated.
- if F_w and F_f changed less than 1%, when changing the argument, then curve F_w and F_f may contain some fluctuations, because load step is also 1%.

The geometric parameters of cutting significantly affect both the magnitude of the F_b, F_f and F_w and the types of failure. For example, large values of c and h_b increase stability (F_b), small values s_2 decreases weld strength (F_w). However, many other dependencies are not obvious and require preliminary study (figure 4).

Figure 4a shows the dependence of F_b, F_f and F_w on c. F_b is obviously increasing, F_f increasing as the failure of the flange at the support is moving away from the support, F_w decreases as reduced weld length s_2 (s_1 is fixed). The reasonable change c does not reveal the presence of extrema.

The number of holes N affects the weld length s_2 (if s_1 is fixed), so F_w decreases (figure 4b). Stability almost independent of hole count N. Flange strength decreases after weld failure – two holes merge into one and the span of the flange increases.

4. STATEMENT OF OPTIMIZATION PROBLEM OF THE CASTELLATED I-BEAM

The problem of maximizing the load carrying capacity can be defined as follow [5-6]:

$$F = \min\left(F_b, F_f, F_w\right) \to \max$$

Objective function F depends on the geometric parameters, type of load and support condition.
For the selected cutting method (Figure 1), the uniformly distributed load and supported at its ends:

$$F = F\left(I, L, c, r, N, s_1, s_2 = s_3, h_b = h_t\right),$$

where L – length of beam, I – number of profile, which defines a number of geometric parameters of section, r – radius of fillet in the corners due to cutting.
Parameter s_1 is not independent:

$$s_1 = \left(L - 2c - \left(2N - 1\right)s_2\right)/2N.$$

Parameters I and L do not change during of one optimization problem. $r=10mm$ for all schemes. In the article, we will determine c and N ourselves, and we will optimize only for two parameters s_2 and h_b. Thus, for each design scheme:

$$F = F\left(s_2, h_b\right).$$
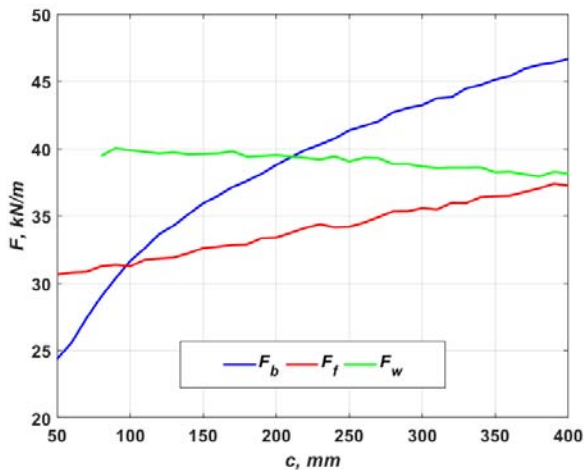
Only geometrically inequality constraints:

$$0 < s_2 < \left(L - 2c\right)/\left(2N - 1\right),$$
$$t < h_b < h/2,$$

where t – thickness of flange, h – height of original I-beam.
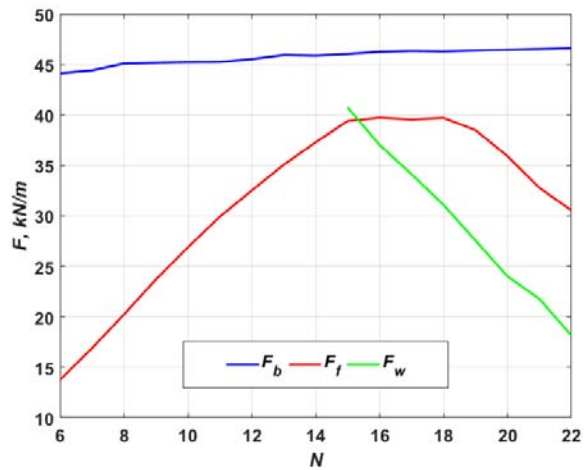From a technical point of view, we can narrow the conditions without risking losing the optimal point:

$$20mm \le s_2 \le \left(L - 2c\right)/\left(2N - 1\right) - 10mm,$$
$$2t \le h_b \le 2h/5$$

Computation the objective function using the finite element method has some problems. Non-linear material properties and a difference in the mesh at different points introduce a small noise into the objective function. The presence of noise does not allow the use of derivative-based optimization methods and impairs convergence [7-8].
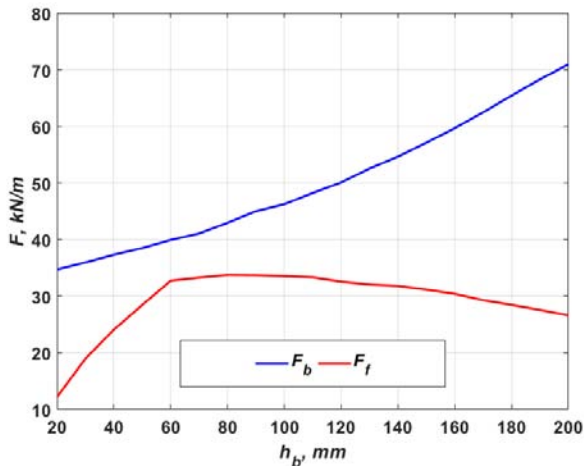
a) Dependence of F_b, F_f and F_w on the c

$c \in [50; 400]\, mm$, step $10\, mm$
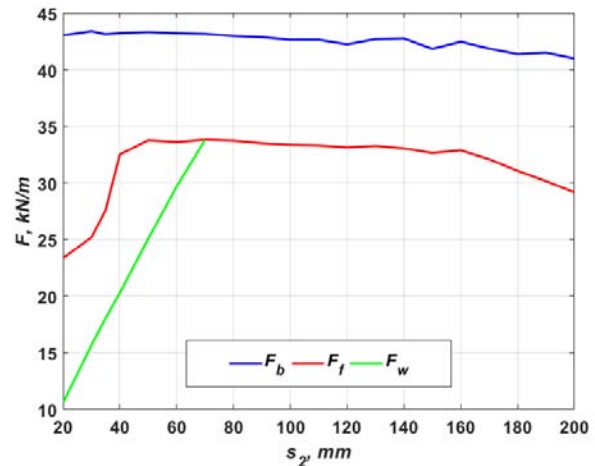
$I30$, $L = 6m$, $N = 16$, $s_1 = 100mm$, $h_b = 50mm$

b) Dependence of F_b, F_f and F_w on the N

$N \in [6; 22]$, step 1 pc

$I30$, $L = 6m$, $c = 300mm$, $s_1 = 100mm$, $h_b = 60mm$

c) Dependence of F_b, F_f on the h_b

$h_b \in [20; 200]\, mm$, step $10\, mm$

$I50$, $L = 12m$, $N = 24$, $c = 400mm$, $s_2 = 85.1mm$

d) Dependence of F_b, F_f and F_w on the s_2

$s_2 \in [20; 200]\, mm$, step $10\, mm$

$I50$, $L = 12m$, $N = 24$, $c = 400mm$, $h_b = 80mm$

Figure 4. Dependence of F_b, F_f and F_w for some configuration of castellated I-beam.

5. OPTIMIZATION RESULTS

To search the optimal point (s_2^*, h_b^*), the coordinate descent method was used. At each iteration, the golden-section search was used as the linear search method. The optimal point was calculated with sufficient accuracy after two iterations of the coordinate descent method.
Initial point:

$$\left(s_2^0, h_b^0 \right) = \left(\frac{s_2^{\min} + s_2^{\min}}{2}; \frac{h_b^{\min} + h_b^{\min}}{2} \right).$$

Tolerance of golden-section search

$$\Delta s_2 \leq 2.5mm; \quad \Delta h_b \leq 2.5mm .$$

The optimization results are presented in Table 1 and Figure 6.
An important non-dimensional parameter of castellated I-beams – height increase ratio (Table 2).

No.	I	L, m	c, mm	N	s2, mm			hb, mm			F, kN	
					min	max	opt	min	max	opt	min	max
1		4	300	10	20.0	168.9	99.6	20.4	120.0	77.5	59.73	70.70
2	30	6	300	16	20.0	164.2	79.1	20.4	120.0	67.4	28.61	39.33
3		8	300	20	20.0	179.7	93.3	20.4	120.0	57.4	19.10	22.53
4		5	350	12	20.0	177.0	107.2	26.0	160.0	117.2	78.04	98.90
5	40	8	350	16	20.0	225.5	94.9	26.0	160.0	80.9	32.57	44.96
6		10	350	20	20.0	228.5	80.0	26.0	160.0	78.1	26.25	29.19
7		6	400	12	20.0	216.1	122.7	30.4	200.0	156.4	89.59	115.83
8	50	9	400	20	20.0	200.3	92.7	30.4	200.0	99.9	46.35	59.11
9		12	400	20	20.0	277.2	89.6	30.4	200.0	96.3	19.74	33.78

No.	1	2	3	4	5	6	7	8	9
k	1.48	1.55	1.61	1.41	1.60	1.61	1.37	1.60	1.61

a) *I30, L=4m, c=300mm, N=10*
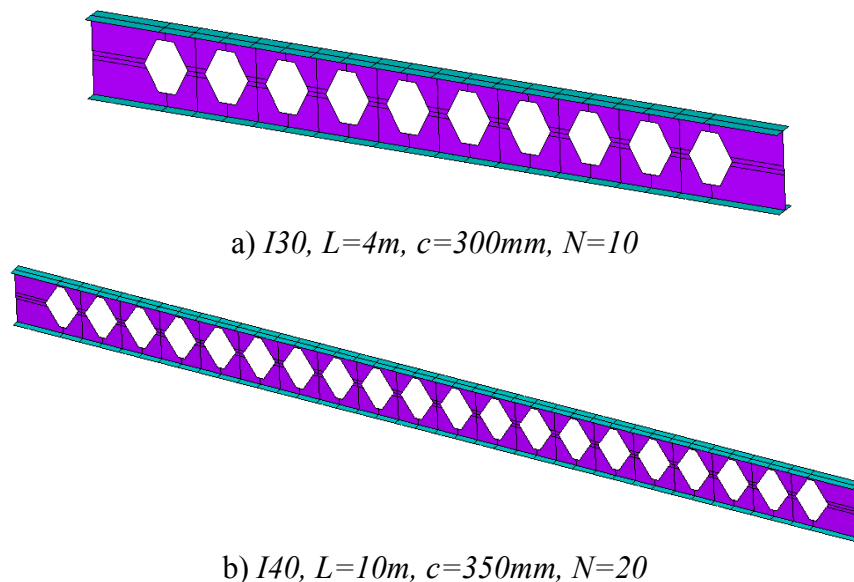

b) *I40, L=10m, c=350mm, N=20*

Figure 5. Several optimized castellated I-beams: No. 1 (a) and No. 6 (b) from Table 1.

$$k = 2(h - h_b)/h = 2(1 - h_b/h).$$

6. CONCLUSIONS

Castellated I-beam load carrying capacity has maximum for many design schemes. However, for some geometric parameters, not all schemes have extrema. Nevertheless, the task of increasing the carrying capacity remains relevant.

Section height and weld length always have a maximum point, therefore, in this work, the problem of optimizing these parameters was solved.
Based on the 9 considered design schemes (Table 1), some conclusions can be drawn:

- the optimal height increase ratio of castellated I-beams increases with increasing span (Table 2)
- the relative length of the weld decreases with increasing span.

The first conclusion is explained by the fact that in short beams, destruction occurs in the zone of

supports. Wall height is a determining factor in its stability. However, stability can be ensured by installing stiffeners. It can be expected that when using stiffeners, the optimal solution will change. In long beams, destruction occurs in the middle of the span from the action of a bending moment. In such type of failure, an increase in the inertia moment by increasing the section height gives the greatest increase in the carrying capacity.

The second conclusion is also related to the type of failure. Welds between end holes have the highest stresses. Therefore, the cause of the loss of load-carrying ability may be the failure of the weld. Increasing the weld length solves this problem.

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