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THE DISCRETE-LINEAR AND THE DISCRETE-CONTINUOUS MODELS OF THE NONLINEAR OBJECT MANAGING IN WIDE RANGE OF OPERATING MODES

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Abstract: The article deals with the features of calculating the vector and matrix parameters of a discrete-continuous model using the vector and matrix parameters of a discrete-linear model in a wide range of operating modes of a nonlinear object of management. The results of the application of the proposed approach for creating a discrete-continuous model of an aircraft gas-turbine engine are given.

Keywords: nonlinear object of managing, discrete-linear model, discrete-continuous model, wide range of operation modes

КУСОЧНО-ЛИНЕЙНАЯ И КУСОЧНО-НЕПРЕРЫВНАЯ МОДЕЛИ НЕЛИНЕЙНОГО ОБЪЕКТА УПРАВЛЕНИЯ В ШИРОКОМ ДИАПАЗОНЕ РЕЖИМОВ РАБОТЫ

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Аннотация: В настоящей статье рассмотрены особенности расчета векторных и матричных параметров кусочно-непрерывной модели с помощью векторных и матричных параметров кусочно-линейной модели в широком диапазоне режимов работы нелинейного объекта управления. Приведены результаты применения предлагаемого подхода при создании кусочно-непрерывной модели авиационного газотурбинного двигателя.

Ключевые слова: нелинейный объект управления, кусочно-линейная модель, кусочно-непрерывная модель, широкий диапазон режимов работы

1. INTRODUCTION

If we develop algorithms for detecting, localizing, specifying, and countering failures of digital automatic control systems (DACS), as well as for evaluating the robustness of these systems and algorithms, it is necessary to use linear models of nonlinear control objects [1]. The matrix parameters of linear and piecewise linear models are estimated by transient processes of nonlinear models of managing objects [1], [2]. The matrix parameters of the discrete-continuous model can also be evaluated by the transition processes of the nonlinear model [3]. In addition, the vector and matrix parameters of

the discrete-continuous model can be calculated using the vector and matrix parameters of the original discrete-linear model. The discrete-continuous model, in its turn, makes it possible to construct various new discrete-linear models consisting of an arbitrary number of linear models, which can alternate each other with arbitrary frequency, for example, at each step of DACS operation. This paper describes the features of calculating the vector and matrix parameters of a discrete-continuous model using vector and matrix parameters of the original discrete-linear model in a wide range of operating modes of a nonlinear object of managing.

2. NONLINEAR, LINEAR, DISCRETE-LINEAR AND DISCRETE-CONTINUOUS MODELS OF NONLINEAR OBJECT OF MANAGING

A nonlinear model of managed object takes the following form [1], [2], [3]

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}^p, \mathbf{u}^p, \mathbf{w}^p), \quad (1)$$

where $\mathbf{x}^p$ is vector of state, $\mathbf{u}^p$ is vector of managing, $\mathbf{w}^p$ is vector of external impacts, and $\mathbf{f}$ is nonlinear real vector function. The coordinates of the state vector (state variables) of such an object of managing, for example, an aviation gas-turbine engine (GTE), are rotor rotation frequencies, temperatures and pressures in different sections of the flow-through part of the engine. The coordinates of the control vector (control variables) are fuel consumption, variable geometrical characteristics of the flow part of the engine, in particular, the critical area of the jet nozzle and the angles of rotation of the fan directing equipment and the compressor. The coordinates of the vector of external impacts (variable external impacts) is the angle of deviations of the engine control lever (control action), as well as the height and speed of flight (variable external conditions). The constant control action and the external conditions included in the ${}_s\mathbf{w}^p$, vector determining the steady state of engine operation (operating point), which corresponds to the vector of steady-state values of control variables ${}_s\mathbf{u}^p$ and the steady-state vector of state variables ${}_s\mathbf{x}^p$.

In a small neighborhood of an arbitrary steady state, a nonlinear control object can be approximately described [1] using a linear model in the form of normalized deviations from steady values

$$\dot{\mathbf{x}}^n = \mathbf{A}(\mathbf{x}^n - {}_s\mathbf{x}^n) + \mathbf{B}(\mathbf{u}^n - {}_s\mathbf{u}^n), \quad (2)$$

where

$$x_i^n - {}_s x_i^n = \frac{x_i^p - {}_s x_i^p}{\max |{}_s x_i^p|}, \quad i = 1, \dots, n, \quad (3)$$

$$u_i^n - {}_s u_i^n = \frac{u_i^p - {}_s u_i^p}{\max |{}_s u_i^p|}, \quad i = 1, \dots, m, \quad (4)$$

$\mathbf{A}$ and $\mathbf{B}$ are matrixes of dynamic linear model for observed steady mode.

For steady values of normalized deviations it takes the following form

$$(\mathbf{x}^n - {}_s\mathbf{x}^n) = \mathbf{S}(\mathbf{u}^n - {}_s\mathbf{u}^n), \quad (5)$$

$$\mathbf{0} = \mathbf{A}(\mathbf{x}^n - {}_s\mathbf{x}^n) + \mathbf{B}(\mathbf{u}^n - {}_s\mathbf{u}^n). \quad (6)$$

From where the matrix of static linear model is

$$\mathbf{S} = -\mathbf{A}^{-1} \mathbf{B}, \quad (7)$$

Thus, we have

$$\mathbf{B} = -\mathbf{A}\mathbf{S}. \quad (8)$$

If the discreteness step is small enough, then at discrete points in time we can approximately assume that

$$\mathbf{x}^n(t_{k+1}) = \mathbf{x}^n(t_k) + \Delta t \mathbf{A}[\mathbf{x}^n(t_k) - {}_s\mathbf{x}^n] + \Delta t \mathbf{B}[\mathbf{u}^n(t_k) - {}_s\mathbf{u}^n], \quad (9)$$

$k = 0, 1, 2, \dots$ corresponds to discrete moments of time $t_0, t_1, t_2, \dots$, at that $t_{k+1} = t_k + \Delta t$, where Δt is discreteness step.

The set of linear models for all selected steady-state modes is a discrete-linear model

$$\dot{\mathbf{x}}^n = {}^{r_1 \dots r_m} \mathbf{A}(\mathbf{x}^n - {}^{r_1 \dots r_m} {}_s\mathbf{x}^n) + {}^{r_1 \dots r_m} \mathbf{B}(\mathbf{u}^n - {}^{r_1 \dots r_m} {}_s\mathbf{u}^n), \quad (10)$$

$$\mathbf{x}^n(t_{k+1}) = \mathbf{x}^n(t_k) + \Delta t {}^{r_1 \dots r_m} \mathbf{A}[\mathbf{x}^n(t_k) - {}_s\mathbf{x}^n] + \Delta t {}^{r_1 \dots r_m} \mathbf{B}[\mathbf{u}^n(t_k) - {}_s\mathbf{u}^n], \quad (11)$$

$${}^{r_1 \dots r_m} \mathbf{B} = {}^{r_1 \dots r_m} \mathbf{A} {}^{r_1 \dots r_m} \mathbf{S}, \quad (12)$$

where

$${}_{r_1 \dots r_m} \mathbf{u}^n = \begin{bmatrix} {}_{r_1} u_1^n \\ \dots \\ {}_{r_m} u_m^n \end{bmatrix}, \quad (13)$$

$$r_1 \in (1, \dots, R_1), \dots, r_m \in (1, \dots, R_m), \quad (14)$$

and $R_1, \dots, R_m$ are equal to quantity of choosen steady values of each from m variables of the equation.

Vectors of normalized steady values of variables of control ${}_s \mathbf{u}^n$ and state variables ${}_s \mathbf{x}^n$, as well as matrixes $\mathbf{A}$ and $\mathbf{B}$, involved into transition matrixes $\mathbf{I} + \Delta t \mathbf{A}$ and $\Delta t \mathbf{B}$ of discrete-continuous model [3], in this case can be determined as following

$${}_s \mathbf{u}^n = {}_{r_1 \dots r_m} \mathbf{u}^n + \theta \left({}_{\tilde{r}_1 \dots \tilde{r}_m} \mathbf{u}^n - {}_{r_1 \dots r_m} \mathbf{u}^n \right), \quad (15)$$

$${}_s \mathbf{x}^n = {}_{r_1 \dots r_m} \mathbf{x}^n + \theta \left({}_{\tilde{r}_1 \dots \tilde{r}_m} \mathbf{x}^n - {}_{r_1 \dots r_m} \mathbf{x}^n \right), \quad (16)$$

$$\mathbf{A} = {}_{r_1 \dots r_m} \mathbf{A} + \theta \left({}_{\tilde{r}_1 \dots \tilde{r}_m} \mathbf{A} - {}_{r_1 \dots r_m} \mathbf{A} \right), \quad (17)$$

$$\mathbf{B} = {}_{r_1 \dots r_m} \mathbf{B} + \theta \left({}_{\tilde{r}_1 \dots \tilde{r}_m} \mathbf{B} - {}_{r_1 \dots r_m} \mathbf{B} \right), \quad (18)$$

$$0 \leq \theta = \theta \left({}_{r_1 \dots r_m} \mathbf{u}^n, {}_{\tilde{r}_1 \dots \tilde{r}_m} \mathbf{u}^n \right) < 1 \quad \left| \begin{array}{l} \left\| {}_{r_1 \dots r_m} \mathbf{u}^n \right\| \leq \left\| \mathbf{u}^n \right\| < \left\| {}_{\tilde{r}_1 \dots \tilde{r}_m} \mathbf{u}^n \right\|, \\ \tilde{r}_1 \in (1, \dots, R_1), \dots, \tilde{r}_m \in (1, \dots, R_m). \end{array} \right. \quad (19)$$

3. CALCULATION EXAMPLE FOR VECTORS AND MATRIXES OF THE DISCRETE-CONTINUOUS MODEL

To determine the vectors and matrices of the discrete-continuous model, the vectors and matrixes of the discrete-linear model of the two-shaft double-circuit turbojet aviation GTE are used [1].

In this example (environmental variables), the altitude and speed (Mach number) of the flight of the aircraft (FA) are equal zero ($ALT = 0$, $MN = 0$), the values (control impact) of the deflection angle of the engine control leverage correspond to the operating modes of the GTE from the maximum ($\alpha_{lce} = 68^\circ$) to the minimum

($\alpha_{lce} = 15^\circ$). Four control variables ($m = 4$) correspond to the fuel consumption in the main combustion chamber G_{to} , the critical area of the jet nozzle F_{cr} , the angle of rotation of the fan directing equipment φ_{na} and the angle of rotation of the compressor directing equipment φ_{nak} . Five state variables of the control object ($n = 5$) correspond to the engine rotation frequency n_r , the engine compressor frequency n_c , the braking pressure behind the compressor P_c^* , the braking pressure behind the turbine P_t^* and the braking temperature behind the turbine T_t^* .

Let us assume [1] that the normalized control variables u_3^n and u_4^n change consistently with the control variable u_1^n , that is, approximately simultaneously these variables are on the boundaries of the neighboring acting areas of different linear models that are part of the discrete-linear model. Therefore, for all these three variables, let us to considering only $R_1 = 9$ selected steady-state values for the variable u_1^n . At the same time, the number of different considered steady-state values for the variables u_3^n and u_4^n equal to 5 and 7, respectively, since ones remain unchanged at the other working points.

Let us also assume that the control variable u_2^n changes in this way so that the number of selected steady-state values $R_2 = 9$.

Thus, the number of steady-state modes of operation of the considered discrete-linear model are $R_1 R_2 = 81$.

Vector and matrix parameters of the discrete-linear model, which is the source for creating a discrete-continuous model, are given in [1].

The vectors of steady-state values of control variables and state variables, as well as matrixes of linear models in the considered example, take the form

$${}_{r_1 r_2 r_3 r_4} \mathbf{u}^n = {}_{r_1 r_2} \mathbf{u}^n, \quad r_1 = r_3 = r_4 = 1, \dots, 9, \quad r_2 = 1, \dots, 9,$$

$${}_{r_1 r_2 r_3 r_4} \mathbf{x}^n = {}_{r_1 r_2} \mathbf{x}^n, \quad r_1 = r_3 = r_4 = 1, \dots, 9, \quad r_2 = 1, \dots, 9,$$

$${}_{r_1 r_2 r_3 r_4} \mathbf{A} = {}_{r_1 r_2} \mathbf{A}, \quad r_1 = r_3 = r_4 = 1, \dots, 9, \quad r_2 = 1, \dots, 9,$$

$${}_{r_1 r_2 r_3 r_4} \mathbf{B} = {}_{r_1 r_2} \mathbf{B}, \quad r_1 = r_3 = r_4 = 1, \dots, 9, \quad r_2 = 1, \dots, 9.$$

The acting areas of linear models, involved into discrete-linear model, is determined as following.

$$\begin{aligned} r_1 = 1 & : u_1^n \leq_s^1 u_1^n, \\ r_1 = 2, \dots, 9 & : r_1 u_1^n \geq u_1^n >_{s-1}^{r_1-1} u_1^n, \\ r_1 = 9 & : u_1^n >_s^9 u_1^n, \\ r_2 = 1 & : u_2^n \geq_s^1 u_2^n, \\ r_2 = 2, \dots, 9 & : r_2 u_2^n \leq u_2^n <_{s-1}^{r_2-1} u_2^n, \\ r_2 = 9 & : u_2^n <_s^9 u_2^n. \end{aligned}$$

If at some point in the transition process the control variables are outside the acting areas of the selected linear models, then the linear model, that has been used before, is used. At the same time, the matrixes of dynamic linear models for neighboring areas, where it is possible, are the same or close.

The vector of normalized steady-state values of the control variables ${}_s \mathbf{u}^n$ is assumed to be

$${}_s \mathbf{u}^n = \mathbf{u}^n(t_{k-1}).$$

The vectors of normalized steady state variables ${}_s \mathbf{x}^n$, as well as matrixes $\mathbf{A}$ and $\mathbf{B}$, included in the transition matrixes $\mathbf{I} + \Delta t \mathbf{A}$ and $\Delta t \mathbf{B}$ of the discrete-continuous model, at the time t_k , $k = 1, 2, \dots$ are calculated taking into account that

$$\mathbf{u}^n = \mathbf{u}^n(t_k).$$

Firstly we determine

$$\begin{aligned} r_1 = 1 & : u_1^n \leq_s^1 u_1^n, \\ {}_s x_i^n (u_1^n \leq_s^1 u_1^n, u_2^n =_{s-1}^{r_2-1} u_2^n) &= {}_s^{1,r_2} x_i^n, \quad i = 1, \dots, n, \\ \mathbf{A}(u_1^n \leq_s^1 u_1^n, u_2^n =_{s-1}^{r_2-1} u_2^n) &= {}_s^{1,r_2} \mathbf{A}, \\ \mathbf{B}(u_1^n \leq_s^1 u_1^n, u_2^n =_{s-1}^{r_2-1} u_2^n) &= {}_s^{1,r_2} \mathbf{B}, \\ r_1 = 2, \dots, 9 & : r_1 u_1^n \geq u_1^n >_{s-1}^{r_1-1} u_1^n, \\ {}_s x_i^n (r_1 u_1^n \geq u_1^n >_{s-1}^{r_1-1} u_1^n, u_2^n =_{s-1}^{r_2-1} u_2^n) &= {}_s^{r_1-1, r_2-1} x_i^n + \\ ({}_{s-1}^{r_1-1, r_2-1} x_i^n - {}_s^{r_1-1, r_2-1} x_i^n) / (r_1 u_1^n - {}_{s-1}^{r_1-1} u_1^n) \cdot (u_1^n - {}_{s-1}^{r_1-1} u_1^n), \\ i &= 1, \dots, n, \end{aligned}$$

$$\begin{aligned} \mathbf{A}(r_1 u_1^n \geq u_1^n >_{s-1}^{r_1-1} u_1^n, u_2^n =_{s-1}^{r_2-1} u_2^n) &= {}_s^{r_1-1, r_2-1} \mathbf{A} + \\ ({}_{s-1}^{r_1-1, r_2-1} \mathbf{A} - {}_s^{r_1-1, r_2-1} \mathbf{A}) / (r_1 u_1^n - {}_{s-1}^{r_1-1} u_1^n) \cdot (u_1^n - {}_{s-1}^{r_1-1} u_1^n), \\ \mathbf{B}(r_1 u_1^n \geq u_1^n >_{s-1}^{r_1-1} u_1^n, u_2^n =_{s-1}^{r_2-1} u_2^n) &= {}_s^{r_1-1, r_2-1} \mathbf{B} + \\ ({}_{s-1}^{r_1-1, r_2-1} \mathbf{B} - {}_s^{r_1-1, r_2-1} \mathbf{B}) / (r_1 u_1^n - {}_{s-1}^{r_1-1} u_1^n) \cdot (u_1^n - {}_{s-1}^{r_1-1} u_1^n), \\ r_1 = 9 & : u_1^n >_s^9 u_1^n, \\ {}_s x_i^n (u_1^n >_s^9 u_1^n, u_2^n =_{s-1}^{r_2-1} u_2^n) &= {}_s^{9, r_2-1} x_i^n, \quad i = 1, \dots, n, \\ \mathbf{A}(u_1^n >_s^9 u_1^n, u_2^n =_{s-1}^{r_2-1} u_2^n) &= {}_s^{9, r_2-1} \mathbf{A}, \\ \mathbf{B}(u_1^n >_s^9 u_1^n, u_2^n =_{s-1}^{r_2-1} u_2^n) &= {}_s^{9, r_2-1} \mathbf{B}, \\ r_2 &= 1, \dots, 9, \end{aligned}$$

Further we obtain

$$\begin{aligned} r_2 = 1 & : u_2^n \geq_s^1 u_2^n, \\ r_1 &= 1, \\ {}_s x_i^n = {}_s x_i^n (u_1^n \leq_s^1 u_1^n, u_2^n \geq_s^1 u_2^n) &= {}_s^{11} x_i^n, \quad i = 1, \dots, n, \\ \mathbf{A} &= \mathbf{A}(u_1^n \leq_s^1 u_1^n, u_2^n \geq_s^1 u_2^n) = {}_s^{11} \mathbf{A}, \\ \mathbf{B} &= \mathbf{B}(u_1^n \leq_s^1 u_1^n, u_2^n \geq_s^1 u_2^n) = {}_s^{11} \mathbf{B}, \\ r_1 &= 2, \dots, 9, \\ {}_s x_i^n = {}_s x_i^n (r_1 u_1^n \geq u_1^n >_{s-1}^{r_1-1} u_1^n, u_2^n \geq_s^1 u_2^n) &= {}_s^{1, r_2} x_i^n, \quad i = 1, \dots, n, \\ \mathbf{A} &= \mathbf{A}(r_1 u_1^n \geq u_1^n >_{s-1}^{r_1-1} u_1^n, u_2^n \geq_s^1 u_2^n), \\ \mathbf{B} &= \mathbf{B}(r_1 u_1^n \geq u_1^n >_{s-1}^{r_1-1} u_1^n, u_2^n \geq_s^1 u_2^n), \\ r_1 &= 9, \\ {}_s x_i^n = {}_s x_i^n (u_1^n >_s^9 u_1^n, u_2^n \geq_s^1 u_2^n) &= {}_s^{91} x_i^n, \quad i = 1, \dots, n, \\ \mathbf{A} &= \mathbf{A}(u_1^n >_s^9 u_1^n, u_2^n \geq_s^1 u_2^n) = {}_s^{91} \mathbf{A}, \\ \mathbf{B} &= \mathbf{B}(u_1^n >_s^9 u_1^n, u_2^n \geq_s^1 u_2^n) = {}_s^{91} \mathbf{B}, \\ r_2 = 2, \dots, 9 & : r_2 u_2^n \leq u_2^n <_{s-1}^{r_2-1} u_2^n, \\ r_1 &= 1, \\ {}_s x_i^n = {}_s x_i^n (u_1^n \leq_s^1 u_1^n, r_2 u_2^n \leq u_2^n <_{s-1}^{r_2-1} u_2^n) &= {}_s^{1, r_2-1} x_i^n + \\ ({}_{s-1}^{1, r_2-1} x_i^n - {}_s^{1, r_2-1} x_i^n) / (r_2 u_2^n - {}_{s-1}^{r_2-1} u_2^n) \cdot (u_2^n - {}_{s-1}^{r_2-1} u_2^n), \\ i &= 1, \dots, n, \\ \mathbf{A} &= \mathbf{A}(u_1^n \leq_s^1 u_1^n, r_2 u_2^n \leq u_2^n <_{s-1}^{r_2-1} u_2^n) = {}_s^{1, r_2-1} \mathbf{A} + \\ ({}_{s-1}^{1, r_2-1} \mathbf{A} - {}_s^{1, r_2-1} \mathbf{A}) / (r_2 u_2^n - {}_{s-1}^{r_2-1} u_2^n) \cdot (u_2^n - {}_{s-1}^{r_2-1} u_2^n), \\ \mathbf{B} &= \mathbf{B}(u_1^n \leq_s^1 u_1^n, r_2 u_2^n \leq u_2^n <_{s-1}^{r_2-1} u_2^n) = {}_s^{1, r_2-1} \mathbf{B} + \\ ({}_{s-1}^{1, r_2-1} \mathbf{B} - {}_s^{1, r_2-1} \mathbf{B}) / (r_2 u_2^n - {}_{s-1}^{r_2-1} u_2^n) \cdot (u_2^n - {}_{s-1}^{r_2-1} u_2^n), \\ r_1 &= 2, \dots, 9, \end{aligned}$$

$$\begin{aligned}
{}_s x_i^n &= {}_s x_i^n \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, {}_{r_2} u_2^n \leq u_2^n < {}_{r_2}^{-1} u_2^n \right) = \\
&{}_s x_i^n \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2}^{-1} u_2^n \right) + \\
&\left[{}_s x_i^n \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2} u_2^n \right) - \right. \\
&\left. {}_s x_i^n \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2}^{-1} u_2^n \right) \right] / \\
&\quad \left({}_{r_2} u_2^n - {}_{r_2}^{-1} u_2^n \right) \cdot \left(u_2^n - {}_{r_2}^{-1} u_2^n \right), \\
&\quad i = 1, \dots, n, \\
\mathbf{A} &= \mathbf{A} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, {}_{r_2} u_2^n \leq u_2^n < {}_{r_2}^{-1} u_2^n \right) = \\
&\quad \mathbf{A} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2}^{-1} u_2^n \right) + \\
&\quad \left[\mathbf{A} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2} u_2^n \right) - \right. \\
&\quad \left. \mathbf{A} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2}^{-1} u_2^n \right) \right] / \\
&\quad \left({}_{r_2} u_2^n - {}_{r_2}^{-1} u_2^n \right) \cdot \left(u_2^n - {}_{r_2}^{-1} u_2^n \right), \\
\mathbf{B} &= \mathbf{B} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, {}_{r_2} u_2^n \leq u_2^n < {}_{r_2}^{-1} u_2^n \right) = \\
&\quad \mathbf{B} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2}^{-1} u_2^n \right) + \\
&\quad \left[\mathbf{B} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2} u_2^n \right) - \right. \\
&\quad \left. \mathbf{B} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2}^{-1} u_2^n \right) \right] / \\
&\quad \left({}_{r_2} u_2^n - {}_{r_2}^{-1} u_2^n \right) \cdot \left(u_2^n - {}_{r_2}^{-1} u_2^n \right), \\
&\quad r_1 = 9, \\
{}_s x_i^n &= {}_s x_i^n \left(u_1^n > {}_{r_1} u_1^n, {}_{r_2} u_2^n \leq u_2^n < {}_{r_2}^{-1} u_2^n \right) = {}_{r_2} u_2^n \cdot {}_s x_i^n + \\
&\quad \left({}_{r_2} u_2^n - {}_{r_2}^{-1} u_2^n \right) \cdot \left(u_2^n - {}_{r_2}^{-1} u_2^n \right), \\
&\quad i = 1, \dots, n, \\
\mathbf{A} &= \mathbf{A} \left(u_1^n > {}_{r_1} u_1^n, {}_{r_2} u_2^n \leq u_2^n < {}_{r_2}^{-1} u_2^n \right) = {}_{r_2} u_2^n \cdot \mathbf{A} + \\
&\quad \left({}_{r_2} u_2^n - {}_{r_2}^{-1} u_2^n \right) \cdot \left(u_2^n - {}_{r_2}^{-1} u_2^n \right), \\
\mathbf{B} &= \mathbf{B} \left(u_1^n > {}_{r_1} u_1^n, {}_{r_2} u_2^n \leq u_2^n < {}_{r_2}^{-1} u_2^n \right) = {}_{r_2} u_2^n \cdot \mathbf{B} + \\
&\quad \left({}_{r_2} u_2^n - {}_{r_2}^{-1} u_2^n \right) \cdot \left(u_2^n - {}_{r_2}^{-1} u_2^n \right), \\
&\quad r_2 = 9 \quad : \quad u_2^n < {}_{r_2} u_2^n, \\
&\quad r_1 = 1, \\
{}_s x_i^n &= {}_s x_i^n \left(u_1^n \leq {}_{r_1} u_1^n, u_2^n < {}_{r_2} u_2^n \right) = {}_{r_1} u_1^n \cdot {}_s x_i^n, \quad i = 1, \dots, n, \\
\mathbf{A} &= \mathbf{A} \left(u_1^n \leq {}_{r_1} u_1^n, u_2^n < {}_{r_2} u_2^n \right) = {}_{r_1} u_1^n \cdot \mathbf{A}, \\
\mathbf{B} &= \mathbf{B} \left(u_1^n \leq {}_{r_1} u_1^n, u_2^n < {}_{r_2} u_2^n \right) = {}_{r_1} u_1^n \cdot \mathbf{B}, \\
&\quad r_1 = 2, \dots, 9,
\end{aligned}$$

$$\begin{aligned}
{}_s x_i^n &= {}_s x_i^n \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n < {}_{r_2} u_2^n \right) = \\
&{}_s x_i^n \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2} u_2^n \right), \quad i = 1, \dots, n, \\
\mathbf{A} &= \mathbf{A} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n < {}_{r_2} u_2^n \right) = \\
&\quad \mathbf{A} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2} u_2^n \right), \\
\mathbf{B} &= \mathbf{B} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n < {}_{r_2} u_2^n \right) = \\
&\quad \mathbf{B} \left({}_{r_1} u_1^n \geq u_1^n > {}_{r_1}^{-1} u_1^n, u_2^n = {}_{r_2} u_2^n \right), \\
&\quad r_1 = 9, \\
{}_s x_i^n &= {}_s x_i^n \left(u_1^n > {}_{r_1} u_1^n, u_2^n < {}_{r_2} u_2^n \right) = {}_{r_1} u_1^n \cdot {}_s x_i^n, \quad i = 1, \dots, n, \\
\mathbf{A} &= \mathbf{A} \left(u_1^n > {}_{r_1} u_1^n, u_2^n < {}_{r_2} u_2^n \right) = {}_{r_1} u_1^n \cdot \mathbf{A}, \\
\mathbf{B} &= \mathbf{B} \left(u_1^n > {}_{r_1} u_1^n, u_2^n < {}_{r_2} u_2^n \right) = {}_{r_1} u_1^n \cdot \mathbf{B}.
\end{aligned}$$

In figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, it is shown the transition processes for normalized control variables u_1^n , u_2^n , u_3^n , u_4^n , normalized state variables x_1^n , x_2^n , x_3^n , x_4^n , x_5^n nonlinear and discrete-continuous models of GTE as well as derivative of normalized state variables for transition processes $\dot{x}_5^n$ for nonlinear, discrete-linear and discrete-continuous models of GTE at changing of values of deviation angle of the engine control leverage from value corresponding to maximum operation mode ($\alpha_{\text{pyd}} = 68^\circ$) to value, corresponding to minimal operation mode ($\alpha_{\text{pyd}} = 15^\circ$), and further in contrary. That is, for example at falling or acceleration into closed-loop DACS. A number of the transition processes' point is $N = 601$, a discreteness step is $\Delta t = 0.025$ C. The deflections from real object (so called "errors") are calculated for each variable using either first or second technique

$$\begin{aligned}
&\sqrt{\frac{\sum_{k=1}^N [x_i^n(t_k) - {}^{\text{HJL}}x_i^n(t_k)]^2}{\sum_{k=1}^N [{}^{\text{HJL}}x_i^n(t_k) - {}^{\text{HJL}}x_i^n(t_0)]^2}}, \quad i = 1, \dots, n, \\
&\max_{k=1, \dots, N} |x_i^n(t_k) - {}^{\text{HJL}}x_i^n(t_k)|, \quad i = 1, \dots, n.
\end{aligned}$$

4. CONCLUSION

A technique for calculation the vector and matrix parameters of a discrete-continuous model using the vector and matrix parameters of a discrete-linear model of a nonlinear control object is carried out.

The basis of this technique is an approximate representation of the static characteristic of a nonlinear object in the form of a set of $n = 5$ three-dimensional surfaces (in fact, these surfaces are five-dimensional), or rather, a set of $n = 5$ three-dimensional variety $R_2 = 9$ discrete-linear broken lines, the each of this set corresponds to the selected constant value of one from two considered (from four available) control variables ($r_2 u_2^n$, $r_2 = 1, \dots, 9$).

The errors of the discrete-continuous model for some of the state variables are larger than the errors of the discrete-linear model, but at the boundaries of the acting areas of different linear models included in the discrete-linear model, the values of the first derivative of the state variable changes much smaller.

The results of this study can be applied to the development of control algorithms for nonlinear objects, which can be approximately described using linear, discrete-linear and discrete-continuous models, as well as algorithms for detecting failures of sensors and equipment of DACS.

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Figure 1. Transition process for normalized control variable u_1^n , corresponding to wide range of operation modes of nonlinear control object.

Figure 2. Transition process for normalized control variable u_2^n , corresponding to wide range of operation modes of nonlinear control object.

Figure 3. Transition process for normalized control variable u_3^n , corresponding to wide range of operation modes of nonlinear control object.

Figure 5. Transition process for normalized control variable x_1^n nonlinear (NL) model and discrete-continuous (DC) model, corresponding to wide range of operation modes of nonlinear control object.

Figure 4. Transition process for normalized control variable u_4^n , corresponding to wide range of operation modes of nonlinear control object.

Figure 6. Transition process for normalized control variable x_2^n nonlinear (NL) model and discrete-continuous (DC) model, corresponding to wide range of operation modes of nonlinear control object.

Figure 7. Transition process for normalized control variable x_3^n nonlinear (NL) model and discrete-continuous (DC) model, corresponding to wide range of operation modes of nonlinear control object.

Figure 9. Transition process for normalized control variable x_5^n nonlinear (NL) model and discrete-continuous (DC) model, corresponding to wide range of operation modes of nonlinear control object.

Figure 8. Transition process for normalized control variable x_4^n nonlinear (NL) model and discrete-continuous (DC) model, corresponding to wide range of operation modes of nonlinear control object.

Figure 10. Transition process for derivative of the normalized control variable x_5^n of nonlinear (NL) model, corresponding to wide range of operation modes of nonlinear control object.

Figure 11. Transition process for derivative of the normalized control variable x_5^n of discrete-linear (DL) model, corresponding to wide range of operation modes of nonlinear control object.

Figure 12. Transition process for derivative of the normalized control variable x_5^n discrete-continuous (DC) model, corresponding to wide range of operation modes of nonlinear control object.

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