DOI:10.22337/2587-9618-2017-13-4-140-148 МОДЕЛИРОВАНИЕ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ ТОЛСТЫХ ЖЕЛЕЗОБЕТОННЫХ ПЛИТ С УЧЕТОМ ПОЛЗУЧЕСТИ БЕТОНА

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Аннотация: В статье приводится вывод разрешающих уравнений для расчета с учетом ползучести толстых железобетонных плит. Используется гипотеза о параболическом законе распределения касательных напряжений по толщине плиты. Задача свелась к системе из двух дифференциальных уравнений относительно прогиба и функции сдвигов. Приведен пример расчета шарнирно опертой по контуру плиты, загруженной равномерно распределенной нагрузкой, с использованием вязкоупругой модели наследственного старения бетона. Решение производилось при помощи двойных тригонометрических рядов в сочетании с методом Эйлера для определения деформаций ползучести.

Ключевые слова: железобетонные плиты, теория толстых плит, ползучесть, вязкоупругость, численные методы

MODELING OF STRESS-STRAIN STATE OF THICK CONCRETE SLABS TAKING THE CREEP OF CONCRETE INTO ACCOUNT

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Abstract: In the article the derivation of the resolving equations for calculation taking into account creep of thick reinforced concrete plates is given. We use the hypothesis of a parabolic law for the distribution of tangential stresses over the thickness of a plate. The problem was reduced to a system of two differential equations with respect to deflection and the function of shear. An example is given of a calculation of a plate hinged on the contour loaded with a uniformly distributed load using a viscoelastic model of hereditary aging of concrete. The solution was carried out using double trigonometric series in combination with the Euler method for determining creep strains.

Keywords: reinforced concrete slabs, the theory of thick plates, creep, viscoelasticity, numerical methods

INTRODUCTION

At present, when calculating reinforced concrete slabs, the Kirchhoff-Love theory is widely used, based on the hypothesis of straight normals. This theory well describes the stress-strain state of thin plates, in which the shear deformations γ_{zx} , γ_{zy} are much smaller than the angles of rotation

$$\frac{\partial w}{\partial x}$$
 and $\frac{\partial w}{\partial y}$

Thin plates are those for which the following relation is valid [1]:

$$\left(\frac{1}{80} \div \frac{1}{100}\right) \le \frac{h}{a} \le \left(\frac{1}{5} \div \frac{1}{8}\right), \qquad (1)$$

where h – plate thickness, a – the smallest size in plane.

Calculations with allowance for the creep of thin plates are considered in papers [2-5].

There are many different versions of refined theories of thick plates, differing not only in the

sets of accepted hypotheses and the factors considered neglected in the classical theory, but also in methods of construction [6]. In dynamics and in the theory of stability, a refined theory generalizing Tymoshenko's theory of beam bending taking into account the shear has become widespread and allows us to accurately calculate the integral characteristics of the plate (critical loads, oscillation frequencies, etc.) However, in this theory, shear deformations, like tangential stresses τ_{zx} , τ_{zy} are assumed to be constant across the thickness of the plate, which contradicts the absence of tangential loads on the plate surface in the most common case of a uniformly distributed area load.

When calculating the stress-strain state of plates, theories are often used in which the quantities γ_{zx} and γ_{zy} as well as τ_{zx} and τ_{zy} in thickness are variable. The smallest errors are obtained by refined theories in which a parabolic change in stress τ_{zx} and τ_{zy} along the plate thickness is assumed. Among these theories, we can distinguish the theory of S.A. Ambartsumyan [7], also taking into account normal stresses σ_z .

1. DERIVATION OF RESOLVING EQUATIONS

In deriving the resolving equations, we will use the basic hypotheses adopted in [7]. The element of reinforced concrete slab considered is shown in Fig. 1.

For the tangential stresses τ_{bzx} and τ_{bzy} along the thickness of the plate, we take the distribution according to the law of a quadratic parabola:

$$\tau_{bzx} = \frac{\partial \varphi}{\partial x} \left(1 - 4 \frac{z^2}{h^2} \right); \tau_{bzy} = \frac{\partial \varphi}{\partial y} \left(1 - 4 \frac{z^2}{h^2} \right), \quad (2)$$

where $\varphi = \varphi(x, y)$ – shear function.



Figure 1. Element of reinforced concrete slab.

We assume that the tangent stresses τ_{zx} and τ_{zy} , as well as the normal stresses σ_z , the armature does not perceive, ie:

$$\tau_{zx} = \tau_{bzx}, \quad \tau_{zy} = \tau_{bzy}, \quad \sigma_z = \sigma_{bz}. \quad (13)$$

Stresses τ_{zx} , τ_{zy} and σ_z and are related by the equation of equilibrium:

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0.$$
 (4)

Substituting (2) into (4), we obtain:

$$\frac{\partial \sigma_z}{\partial z} = -\nabla^2 \varphi \left(1 - 4 \frac{z^2}{h^2} \right). \tag{5}$$

Integration of equation (5) will give:

where

$$\sigma_{z} = -\nabla^{2} \varphi \Phi(z) + C(x, y), \qquad (6)$$
$$\Phi(z) = z \left(1 - 4z^{2} / (3h^{2}) \right),$$

C(x, y) is an arbitrary integration function determined from the boundary conditions:

at
$$z = -h/2$$
: $\sigma_z = -q(x, y)$;
at $z = h/2$: $\sigma_z = 0$.

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Substituting the boundary conditions in (6), we obtain:

$$\frac{h}{3}\nabla^2 \varphi = C(x, y);$$

$$-q(x, y) = \frac{h}{3}\nabla^2 \varphi + C(x, y).$$
(7)

Summing the first and second equalities in (7), we find the function C(x, y):

$$C(x, y) = -q(x, y)/2.$$
 (8)

Substituting (8) in the first equation (7), we obtain the resolving equation for the shear function:

$$\nabla^2 \varphi = -\frac{3}{2} \frac{q(x, y)}{h}.$$
 (9)

We obtain an expression for σ_z by substituting (8) and (9) in (5):

$$\sigma_{z} = -\frac{q}{2} \left(1 - \frac{3z}{h} + 4\frac{z^{3}}{h^{3}} \right).$$
(10)

The Cauchy relations for shear strains have the form:

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}; \gamma_{zy} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}.$$
 (11)

From the physical point of view, the deformations γ_{zx} and γ_{zy} represent the sum of elastic deformations and creep strains:

$$\gamma_{zx} = \frac{\tau_{zx}}{G} + \gamma^*_{zx}; \gamma_{zy} = \frac{\tau_{zy}}{G} + \gamma^*_{zy}, \qquad (12)$$

where G – shear modulus of concrete. Substituting (2) into (12) and equating (12) to (11), we obtain:

$$\frac{\partial u}{\partial z} = \gamma_{xz} - \frac{\partial w}{\partial x} = \frac{1}{G} \frac{\partial \varphi}{\partial x} \left(1 - 4 \frac{z^2}{h^2} \right) + \gamma_{zx}^* - \frac{\partial w}{\partial x}.$$
 (13)

Integration of (13) with respect to z gives:

$$u = \frac{1}{G} \frac{\partial \varphi}{\partial x} \Phi(z) + \int_{0}^{z} \gamma_{zx}^{*} dz - z \frac{\partial w}{\partial x} + f_{1}(x, y). \quad (14)$$

We will consider the case of symmetric reinforcement, then the middle plane is not deformable, and when

$$z=0 \quad u=0,$$

from which it follows that

$$f_1(x,y)=0.$$

Similarly, for displacements v, we can write:

$$v = \frac{1}{G} \frac{\partial \varphi}{\partial y} \Phi(z) - z \frac{\partial w}{\partial y} + \int_{0}^{z} \gamma_{zy}^{*} dz.$$
(15)

Deformations ε_x , ε_y и γ_{xy} , and are defined as follows:

$$\varepsilon_{x} = \frac{\partial u}{\partial x} = -z \frac{\partial^{2} w}{\partial x^{2}} + \frac{\Phi}{G} \frac{\partial^{2} \varphi}{\partial x^{2}} + \Gamma_{x};$$

$$\varepsilon_{y} = \frac{\partial v}{\partial y} = -z \frac{\partial^{2} w}{\partial y^{2}} + \frac{\Phi}{G} \frac{\partial^{2} \varphi}{\partial y^{2}} + \Gamma_{y};$$
(16)
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{2\Phi}{G} \frac{\partial^{2} \varphi}{\partial x \partial y} - 2z \frac{\partial^{2} w}{\partial x \partial y} + \Gamma_{xy},$$

where
$$\Gamma_x = \frac{\partial}{\partial x} \int_0^z \gamma_{zx}^* dz; \Gamma_y = \frac{\partial}{\partial y} \int_0^z \gamma_{zy}^* dz;$$

 $\Gamma_{xy} = \frac{\partial}{\partial x} \int_0^z \gamma_{zy}^* dz + \frac{\partial}{\partial y} \int_0^z \gamma_{zx}^* dz.$

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It follows from (16) that the deformations, and consequently also the stresses in the plate along the thickness, are distributed nonlinearly.

The relationship between strains and stresses in concrete in the case of a volumetric stress state is written as:

$$\varepsilon_{x} = \frac{\sigma_{bx} - \nu \left(\sigma_{by} + \sigma_{bz}\right)}{E_{b}} + \varepsilon_{bx}^{*};$$

$$\varepsilon_{y} = \frac{\sigma_{by} - \nu \left(\sigma_{bx} + \sigma_{bz}\right)}{E_{b}} + \varepsilon_{by}^{*}; \quad (17)$$

$$\gamma_{xy} = \frac{\tau_{bxy}}{G} + \gamma_{bxy}^{*}.$$

We express from (17) the stresses through deformations:

$$\begin{split} \sigma_{bx} &= -\frac{E_b}{1-\nu^2} \left(z \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) + \varepsilon_{bx}^* + \nu \varepsilon_{by}^* + \right. \\ &\left. - \left(\Gamma_x + \nu \Gamma_y \right) \right) + \frac{2\Phi}{1-\nu} \left(\frac{\partial^2 \varphi}{\partial x^2} + \nu \frac{\partial^2 \varphi}{\partial y^2} \right) + \frac{\nu \sigma_{bz}}{1-\nu}; \\ \sigma_{by} &= -\frac{E_b}{1-\nu^2} \left(z \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) + \varepsilon_{by}^* + \nu \varepsilon_{bx}^* - \right. \\ &\left. - \left(\Gamma_y + \nu \Gamma_x \right) \right) + \frac{2\Phi}{1-\nu} \left(\frac{\partial^2 \varphi}{\partial y^2} + \nu \frac{\partial^2 \varphi}{\partial x^2} \right) + \frac{\nu \sigma_{bz}}{1-\nu}; \\ \tau_{bxy} &= \frac{E_b}{2(1+\nu)} \left(-2z \frac{\partial^2 w}{\partial x \partial y} + \Gamma_{xy} - \gamma_{bxy}^* \right) + 2\Phi \frac{\partial^2 \varphi}{\partial x \partial y}. \end{split}$$

Bending and twisting moments, perceived by concrete, are written in the form:

$$M_{bx} = \int_{-h/2}^{h/2} \sigma_{bx} z dz = -D_b \left(\frac{\partial^2 w}{\partial x^2} + v \frac{\partial^2 w}{\partial y^2} \right) - M_{bx}^* + \frac{2h^3}{15(1-v)} \left(\frac{\partial^2 \varphi}{\partial x^2} + v \frac{\partial^2 \varphi}{\partial y^2} \right) + \frac{v}{1-v} \frac{qh^2}{10};$$

$$M_{by} = \int_{-h/2}^{h/2} \sigma_{by} z dz = -D_b \left(\frac{\partial^2 w}{\partial y^2} + v \frac{\partial^2 w}{\partial x^2} \right) - M_{by}^* + \frac{2h^3}{15(1-v)} \left(\frac{\partial^2 \varphi}{\partial y^2} + v \frac{\partial^2 \varphi}{\partial x^2} \right) + \frac{vqh^2}{10(1-v)};$$

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$$H = \int_{-h/2}^{h/2} \tau_{bxy} z dz = -D_b \left(1 - \nu\right) \frac{\partial^2 w}{\partial x \partial y} + \frac{2h^3}{15} \frac{\partial^2 \varphi}{\partial x \partial y} - H_b^*,$$

where

$$\begin{split} D_b &= E_b h^3 / \left(12 \left(1 - \nu^2 \right) \right); \\ H_b^* &= G \int_{-h/2}^{h/2} \left(-\Gamma_{xy} + \gamma_{bxy}^* \right) z dz; \\ M_{bx}^* &= \frac{E_b}{1 - \nu^2} \int_{-h/2}^{h/2} \left(\varepsilon_{bx}^* + \nu \varepsilon_{by}^* - \left(\Gamma_x + \nu \Gamma_y \right) \right) z dz; \\ M_{by}^* &= \frac{E_b}{1 - \nu^2} \int_{-h/2}^{h/2} \left(\varepsilon_{by}^* + \nu \varepsilon_{bx}^* - \left(\Gamma_y + \nu \Gamma_x \right) \right) z dz. \end{split}$$

We define deformations of reinforcement from the condition of its joint work with concrete:

$$\varepsilon_{sx} = -z_{sx} \frac{\partial^2 w}{\partial x^2} + \frac{\Phi(z_{sx})}{G} \frac{\partial^2 \varphi}{\partial x^2} + \Gamma_x(z_{sx});$$

$$\varepsilon'_{sx} = z'_{sx} \frac{\partial^2 w}{\partial x^2} - \frac{\Phi(z_{sx})}{G} \frac{\partial^2 \varphi}{\partial x^2} + \Gamma_x(z_{sx});$$

$$\varepsilon_{sy} = -z_{sy} \frac{\partial^2 w}{\partial y^2} + \frac{\Phi(z_{sy})}{G} \frac{\partial^2 \varphi}{\partial y^2} + \Gamma_y(z_{sy});$$
(18)
$$\varepsilon'_{sy} = z'_{sy} \frac{\partial^2 w}{\partial y^2} - \frac{\Phi(z_{sy})}{G} \frac{\partial^2 \varphi}{\partial y^2} + \Gamma_y(z_{sy}).$$

The bending moments perceived by the armature are written as:

$$M_{sx} = \mu_{sx}hz_{sx}\sigma_{sx} - \mu'_{sx}hz'_{sx}\sigma'_{sx} =$$

$$= E_{s}h(\mu_{sx}z_{sx}\varepsilon_{sx} - \mu'_{sx}z'_{sx}\varepsilon'_{sx}) =$$

$$= -D_{sx}\frac{\partial^{2}w}{\partial x^{2}} + 2\mu_{sx}hz_{sx}E_{s}\frac{\Phi(z_{sx})}{G}\frac{\partial^{2}\varphi}{\partial x^{2}};$$

$$M_{sy} = \mu_{sy}hz_{sy}\sigma_{sy} - \mu'_{sy}hz'_{sy}\sigma'_{sy} = (19)$$

$$= E_{s}h(\mu_{sy}z_{sy}\varepsilon_{sy} - \mu'_{sy}z'_{sy}\varepsilon'_{sy}) =$$

$$= -D_{sy}\frac{\partial^{2}w}{\partial y^{2}} + 2\mu_{sy}hz_{sy}E_{s}\frac{\Phi(z_{sy})}{G}\frac{\partial^{2}\varphi}{\partial x^{2}},$$
where $D_{sx} = 2\mu_{sx}hz^{2}_{sx}E_{s}, D_{sy} = 2\mu_{sy}hz^{2}_{sy}E_{s}; \mu_{sx},$

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 $\mu'_{sx}, \mu_{sy}, \mu'_{sy}$ – reinforcement factors. The equation of equilibrium is written as:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q(x, y). \quad (20)$$

Substituting the expressions for the bending and torsional moments into the equilibrium equation (20), we obtain the second resolving equation:

$$D_{1}\frac{\partial^{4}w}{\partial x^{4}} + 2D_{b}\frac{\partial^{4}w}{\partial x^{2}\partial y^{2}} + D_{2}\frac{\partial^{4}w}{\partial y^{4}} = q + \frac{vh^{2}\nabla^{2}q}{(1-v)10}$$
$$-q^{*} + \frac{\partial^{4}\varphi}{\partial x^{4}}(\frac{2h^{3}}{15(1-v)} + \frac{2\mu_{sx}hz_{sx}E_{s}\Phi(z_{sx})}{G}) +$$
$$+ \frac{\partial^{4}\varphi}{\partial y^{4}}\left(\frac{2h^{3}}{15(1-v)} + \frac{2\mu_{sy}hz_{sy}E_{s}\Phi(z_{sy})}{G}\right) +$$
$$+ \frac{4h^{3}}{15(1-v)}\frac{\partial^{4}\varphi}{\partial x^{2}\partial y^{2}},$$
$$(21)$$

where

$$q^*(x, y) = -\left(\frac{\partial^2 M_{bx}^*}{\partial x^2} + 2\frac{\partial^2 H_b^*}{\partial x \partial y} + \frac{\partial^2 M_{by}^*}{\partial y^2}\right),$$
$$D_1 = D_b + D_{sx}, D_2 = D_b + D_{sy}.$$

Thus, the problem of calculating a thick reinforced concrete slab with allowance for creep was reduced to a system of two differential equations (9) and (21).

2. METHOD OF CALCULATION

For the hingedly supported contour of the plate, the solution is sought in the form:

$$\varphi(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \varphi_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b};$$

$$w(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}.$$
(22)

The functions q(x, y) and $q^*(x, y)$ can be expanded in a Fourier series:

$$q(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \sin \frac{m\pi x}{a} \sin \frac{m\pi x}{a};$$

$$q^{*}(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}.$$
(23)

The coefficients of expansion are determined by the formulas:

$$B_{mn} = \frac{4}{ab} \int_{0}^{a} \int_{0}^{b} q(x, y) \sin \frac{m\pi x}{a} \sin \frac{m\pi x}{a} dx dy;$$

$$C_{mn} = \frac{4}{ab} \int_{0}^{a} \int_{0}^{b} q^{*}(x, y) \sin \frac{m\pi x}{a} \sin \frac{m\pi x}{a} dx dy.$$

The coefficients B_{mn} for some loads can be determined analytically. The coefficients C_{mn} are determined by numerical integration. Substituting the expansions of the functions

q(x, y) and $\varphi(x, y)$ into (9), we obtain:

$$\varphi_{mn} = \frac{3B_{mn}}{2h\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}.$$
 (24)

The shear function does not depend on time, and therefore the coefficients φ_{mn} are determined only once.

Substituting (22) and (23) into (21), we obtain:

 $a_{11}w_{mn} = a_{1n}$,

where

$$a_{11} = D_1 \left(\frac{m\pi}{a}\right)^4 + 2D_b \left(\frac{m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2 + D_2 \left(\frac{n\pi}{b}\right)^4;$$

(25)

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$$a_{1p} = B_{mn} \left(1 - \frac{\nu h^2}{(1 - \nu)^{10}} \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right] \right) + C_{mn} + \varphi_{mn} \pi^4 \left(\frac{m^4}{a^4} \left\{\frac{2h^3}{15(1 - \nu)} + \frac{2\mu_{sx}hz_{sx}E_s}{G} \Phi(z_{sx})\right\} + \frac{2n^4}{b^4} \left\{\frac{h^3}{15(1 - \nu)} + \frac{\mu_{sy}hz_{sy}E_s \Phi(z_{sy})}{G} \right\} + \frac{2m^2n^2h^3}{15(1 - \nu)a^2b^2} \right).$$

As the law of creep, we will use the equation of the viscoelastic model of the hereditary aging of concrete, which under the uniaxial stress state has the form [8]:

$$\varepsilon(t) = \frac{\sigma(t)}{E(t)} - \int_{\tau_0}^t \sigma(\tau) \frac{\partial C(t,\tau)}{\partial \tau} d\tau;$$

$$C(t,\tau) = C \frac{e^{\alpha t} - e^{\alpha \tau}}{e^{\alpha t} - 1} + B(e^{-\gamma \tau} - e^{-\gamma t}),$$
(26)

where

$$C = 3.77 \cdot 10^{-6} \text{ m}^2 / \text{kN},$$

$$B = 5.68 \cdot 10^{-8} \text{ m}^2 / \text{kN},$$

$$\alpha = 0.032 \text{ days}^{-1}; \quad \gamma = 0.062 \text{ days}^{-1}$$

- rheological parameters of the material.

For a bulk stress state, the transition is performed on the basis of the superposition principle. The differential form of equation (26) is given in [9,10]. The calculation is carried out by the stepping method, the creep strains at the time $t + \Delta t$ are determined by strains and stresses at the time t using the Euler method.

3. RESULTS AND DISCUSSION

The calculation was carried out for a rectangular plate hinged by the contour at

$$q = 100 \text{ kPa}, a = 2.5 \text{ m}, b = 3 \text{ m},$$

$$E_b = 3 \cdot 10^4 \text{ MPa}, v = 0.2, E_s = 2 \cdot 10^5 \text{ MPa},$$

$$\mu_{sx} = \mu'_{sx} = 0.005, \ \mu_{sy} = \mu'_{sy} = 0.006,$$

$$h = 45 \text{ cm}, \ z_{sx} = z'_{sx} = 12 \text{ cm},$$

$$z_{sy} = z'_{sy} = 10.5 \text{ cm}.$$

Fig. 2 shows the growth curve of the deflection in the center of the plate. The dashed line corresponds to a solution based on the Kirchhoff-Love theory. The difference between displacements at $t = \tau_0$ is 13.6%, and at $t \rightarrow \infty$ is 5.11%.





over plate thickness.

The stress σ_{bx} distribution along the plate thickness at the end of the creep process is shown in Fig. 3. It can be seen from the presented graph that at a given ratio h/a the nonlinear character of the diagrams begins to appear.

Fig. 4 and Fig. 5 show variation of the maximum stresses in concrete and reinforcement. Stresses in concrete, obtained on the basis of the direct normal hypothesis and using a refined theory, differ by 4.1% at the beginning of the

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creep process, and by 9.6% at $t\rightarrow\infty$. For the stresses in the reinforcement the difference is 0.6% and 7.1%, respectively. When the shear strains are taken into account, the redistribution of stresses between the reinforcement and concrete is less pronounced.



<u>Figure 4.</u> The time variation of the maximum stresses σ_{bx} .



<u>Figure 5.</u> The time variation of the maximum stresses $\sigma_{sx.}$.

CONCLUSIONS

The equations obtained are universal and allow us to calculate thick reinforced concrete slabs under any creep law. As a result of the solution of the test problem, it is established that at a/h = 5.5 the nonlinear character of stress distribution in concrete over the plate thickness begins to appear. Also, at this ratio between the thickness of the slab and its smallest size in the plan, the discrepancy between the displacements obtained on the basis of the Kirchhoff-Love theory and taking into account the shear deformations reaches 13.6%, despite the fact that, according to the generally accepted classification, the slab is thin.

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