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WAVELET-BASED DISCRETE-CONTINUAL FINITE ELEMENT PLATE ANALYSIS WITH THE USE OF DAUBECHIES SCALING FUNCTIONS

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Abstract: The distinctive paper is detoded to special version of wavelet-based discrete-continual finite element method of plate analysis. Daubechies scaling functions are used within this version. Its field of application comprises plates with constant (generally piecewise constant) physical and geometrical parameters along one direction (so-called “basic” direction). Modified continual operational formulation of the problem with the use of the method of extended domain (proposed by A.B. Zolotov) is presented. Corresponding discrete-continual formulation is given as well. Brief information about computer implementation of the method with the use of MATLAB software is provided. Besides numerical sample of analysis of thin plate is considered.

Keywords: boundary problem, structural analysis, plate analysis, thin plate, numerical solution, wavelet-based discrete-continual finite element method, wavelet analysis, Daubechies scaling function, Daubechies wavelet

ВЕЙВЛЕТ-РЕАЛИЗАЦИЯ ДИСКРЕТНО-КОНТИНУАЛЬНОГО МЕТОДА КОНЕЧНЫХ ЭЛЕМЕНТОВ ДЛЯ РАСЧЕТА ПЛИТ С ИСПОЛЬЗОВАНИЕМ МАСШТАБИРУЮЩИХ ФУНКЦИЙ ДОБЕШИ

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Аннотация: Настоящая статья посвящена одной специальной версии вейвлет-реализации дискретно-континуального метода конечных элементов для расчета плитных конструкций. В рамках указанной версии используются масштабирующие функции Добеши. Область применения данного метода составляют пластины с постоянными (в общем случае кусочно-постоянными) физико-геометрическими параметрами по одному из направлений (это так называемое «основное» направление). В статье приведена преобразованная континуальная постановка задачи с использованием аппарата метода расширенной области, предложенного А.Б. Золотовым. Кроме того, здесь представлены соответствующая дискретно-континуальная постановка, краткие сведения о компьютерной реализации метода с использованием системы MATLAB, а также пример расчета.

Ключевые слова: краевая задача, расчеты строительных конструкций, расчет пластин, тонкая пластина, численное решение, вейвлет-реализация дискретно-континуального метода конечных элементов, вейвлет-анализ, масштабирующая функция Добеши, вейвлет Добеши

As is known wavelet analysis has the desirable advantages of multi-resolution properties and various basis functions, which fulfill an enormous potential fo rsolving partial differential equations. The distinctive paper is devoted to further development of wavelet-based discrete-continual finite element method of structural analysis. Particularly problems of plate analysis with the use of Daubechies scaling functions [1-29] are under consideration.

1. CONTINUAL FORMULATION OF PROBLEM

Let x_1, x_2 are cartesian coordinates. Besides, let x_2 be coordimate corresponding to "basic" direction of plate (i.e. direction along which physical and geometrical parameters of plate are constant). It is is necessary to note that physical and geometrical parameters of plate can be changed arbitrarily along x_1. Let us consider the following domain occupied by plate:

$$\Omega = \{ (x_1, x_1): \ 0 < x_1 < \ell_1, \ \ 0 < x_2 < \ell_2 \}. \quad (1.1)$$

Operational formulation of coorresponding boundary problem of plate analysis (Kirghoff model) at extended domain [30], embordering considering structure has the form:

$$L\,y = \tilde{F}, \quad 0 \le x_1 \le \ell_1, \quad 0 \le x_2 \le \ell_2, \quad (1.2)$$

where y is plate deflection in domain Ω; L is the operator of the considering problem; $\tilde{F}$ is the corresponding right-side function;

$$L = -L_4\,\partial_2^4 + L_2\,\partial_2^2 + L_0; \quad (1.3)$$

$$L_4 = \theta D; \quad (1.4)$$

$$L_2 = -[\partial_1^2 \theta D\nu + 2\partial_1 D(1-\nu)\partial_1 + \theta D\nu\partial_1^2]; \quad (1.5)$$

$$L_0 = -\partial_1^2 \theta D \partial_1^2; \quad (1.6)$$

$$\tilde{F} = \theta F + \delta_\Gamma f; \quad (1.7)$$

$$\partial_j = \partial / \partial x_j, \quad \partial_j^* = -\partial / \partial x_j, \quad j = 1, 2; \quad (1.8)$$

$\bar{F}$ is the force in domain Ω; $\bar{f}_k$ is the boundary force at $\Gamma = \partial\Omega$; D, ν are plate modulus and Poison's ratio in domain Ω;

$$D = Eh^3 / [12(1-\nu^2)]; \quad (1.9)$$

h is plate thickness in domain Ω;

$$\theta = \theta(x_1, x_2) = \begin{cases} 1, & (x_1, x_2) \in \Omega \\ 0, & (x_1, x_2) \notin \Omega; \end{cases} \quad (1.10)$$

is the characteristic function of domain Ω [30];

$$\delta_\Gamma = \delta_\Gamma(x_1, x_2) = \frac{\partial \theta}{\partial \bar{n}}; \quad (1.11)$$

is the delta-function of border $\Gamma = \partial\Omega_k$; $\bar{n} = [\,n_1 \ \ n_2\,]^T$ is unit normal vector of domain boundary $\Gamma = \partial\Omega$ [30];
Let us introduce the following notation [4,7-15]:

$$y_1 = y, \quad y_2 = \partial_2 y = y_1', \quad y_3 = \partial_2^2 y = y_2', \quad y_4 = \partial_2^3 y = y_3'. \quad (1.12)$$

Therefor we can rewrite (1.2) in the form

$$-L_4 y_4' + L_2 y_3 + L_0 y_1 = \tilde{F}. \quad (1.13)$$

Taking into account (1.12) and (1.13), we get

$$\begin{bmatrix} y_1' \\ y_2' \\ y_3' \\ y_4' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ L_4^{-1}L_0 & 0 & L_4^{-1}L_2 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -L_4^{-1}\tilde{F} \end{bmatrix}. \quad (1.14)$$

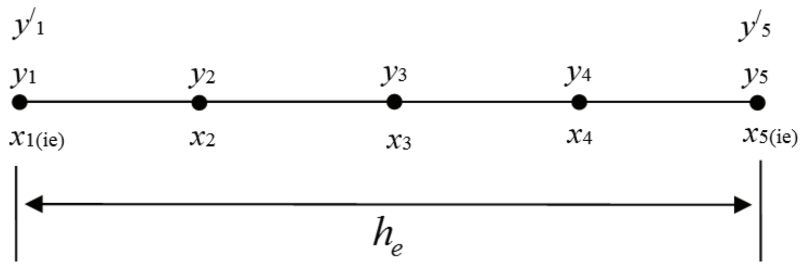


Figure 1. *Approximation on the element.*

We can also rewrite (1.14) in matrix form

$$\bar{U}' = \tilde{L}\bar{U} + \tilde{\tilde{F}}, \quad (1.15)$$

where

$$\tilde{L} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ L_4^{-1}L_0 & 0 & L_4^{-1}L_2 & 0 \end{bmatrix}; \quad (1.16)$$

$$\tilde{\tilde{F}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -L_4^{-1}\tilde{F} \end{bmatrix}; \quad \bar{U} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}; \quad (1.17)$$

Besides, the system of equations (1.15) is supplemented by the boundary conditions that are specified in sections with coordinates (coordinates of boundary points)

$$x_{2,1}^b = 0, \quad x_{2,2}^b = \ell_2. \quad (1.18)$$

2. DISCRETE-CONTINUAL FORMULATION OF PROBLEM

The discrete component of the numerical solution is represented by the direction along the axis x_1. Fulfillment on an element (interval) for all components of the vector-function $\bar{U}$ is the same. Therefore, for simplicity let us denote locally in this paragraph

$$x = x_1, \quad \ell = \ell_1, \quad y = y_j \quad (2.1)$$

for all $j = 1,2,3,4$. We can divide the interval $[0,\ell]$ into N_e parts (elements) and $h_e = \ell / N_e$ is the length of each element. Besides, we can divide each element into N_k parts (for instance, sample with $N_k = 4$ is presented in Fig. 1. Let us use the following notation: i_e is the number of the element; $x_1(i_e)$ is the coordinate of the starting point; $x_5(i_e)$ is the coordinate of the end point of the ith element. Let y_i and $y_i' = \partial_1 y(x_i)$ be unknowns at boundary points ($i = 1,5$). Let y_i be unknown internal points ($i = 2,3,4$). Thus, the total number of unknowns on an element with such discretization is equal to $N = N_k - 1 + 2 \cdot 2 = N_k + 3 = 7$.

Let us introduce the local coordinate within the element:

$$t = \frac{x - x_{1(i_e)}}{h_e}, \quad x_{1(i_e)} \le x \le x_{5(i_e)}. \quad (2.2)$$

Thus, we have the following formulas:

$$\begin{cases} x = x_{1(i_e)} \Rightarrow t = 0 \\ x = x_2 \Rightarrow t = 0.25 \\ x = x_3 \Rightarrow t = 0.5 \\ x = x_4 \Rightarrow t = 0.75 \\ x = x_{5(i_e)} \Rightarrow t = 1; \end{cases} \quad (2.3)$$

$$\frac{d}{dx} = \frac{d}{dt} \cdot \frac{dt}{dx} = \frac{1}{h_e} \frac{d}{dt}; \quad \frac{d^p}{dx^p} = \frac{1}{h_e^p} \frac{d^p}{dt^p}; \quad (2.4)$$

$$dx = h_e \cdot dt. \tag{2.5}$$

In order to construct local stiffness matrices corresponding to continual operators (1.6), (1.5) and (1.4) let us consider bilinear forms taking into account relations (2.3)-(2.5):

$$B_0(y,z) =< L_0 y, z >=$$
$$= -\int_{x_{1(i_e)}}^{x_{5(i_e)}} \frac{d^2}{dx^2}\theta D \frac{d^2 y}{dx^2} \cdot z\,dx =$$
$$= -\theta_{i_e} D_{i_e} \int_{x_{1(i_e)}}^{x_{5(i_e)}} \frac{d^2 y}{dx^2} \cdot \frac{d^2 z}{dx^2}\,dx = \tag{2.6}$$
$$= -\frac{1}{h_e^3}\theta_{i_e} D_{i_e} \int_0^1 \frac{d^2 w}{dt^2} \cdot \frac{d^2 v}{dt^2}\,dt =$$
$$= B_0(w,v);$$

$$B_4(y,z) =< L_4 y, z >=$$
$$= \theta_{i_e} D_{i_e} \int_{x_{1(i_e)}}^{x_{5(i_e)}} y \cdot z\,dx = \tag{2.7}$$
$$= h_e \theta_{i_e} D_{i_e} \int_0^1 w \cdot v\,dt = B_4(w,v);$$

$$B_2(y,z) =< L_2 y, z >=$$
$$=< L_{21} y, z > + < L_{22} y, z > + \tag{2.8}$$
$$+ < L_{23} y, z >,$$

where

$$< L_{21} y, z >= -\theta_{i_e} D_{i_e} \nu_{i_e} \int_{x_{1(i_e)}}^{x_{5(i_e)}} \frac{d^2 y}{dx^2} z\,dx =$$
$$= -\frac{1}{h_e}\theta_{i_e} D_{i_e} \nu_{i_e} \int_0^1 \frac{d^2 w}{dt^2} \cdot v\,dt = \tag{2.9}$$
$$= B_{21}(w,v);$$

$$< L_{23} y, z >= -\theta_{i_e} D_{i_e} \nu_{i_e} \int_{x_{1(i_e)}}^{x_{5(i_e)}} y \cdot \frac{d^2 z}{dx^2}\,dx =$$
$$= -\frac{1}{h_e}\theta_{i_e} D_{i_e} \nu_{i_e} \int_0^1 w \cdot \frac{d^2 v}{dt^2}\,dt = \tag{2.10}$$
$$= B_{23}(w,v);$$

$$< L_{22} y, z >= -2\int_{x_{1(i_e)}}^{x_{5(i_e)}} \frac{d}{dx}\theta D(1-\nu)\frac{dy}{dx} \cdot z\,dx =$$
$$= 2\theta_{i_e} D_{i_e}(1-\nu_{i_e}) \int_{x_{1(i_e)}}^{x_{5(i_e)}} \frac{dy}{dx} \cdot \frac{dz}{dx}\,dx = \tag{2.11}$$
$$= \frac{1}{h_e} 2\theta_{i_e} D_{i_e}(1-\nu_{i_e}) \int_0^1 \frac{dw}{dt} \cdot \frac{dv}{dt}\,dt =$$
$$= B_{22}(w,v)$$

$$y(x) = w(t) = \sum_{k=0}^{N-1} \alpha_k \varphi(t+k),$$
$$x_{1(i_e)} \le x \le x_{5(i_e)}, \quad 0 \le t \le 1; \tag{2.12}$$

$$z(x) = v(t) = \sum_{k=0}^{N-1} \beta_k \varphi(t+k),$$
$$x_{1(i_e)} \le x \le x_{5(i_e)}, \quad 0 \le t \le 1; \tag{2.13}$$

$\varphi(s)$ is Daubechies scaling function,

$$[0,\ N] \subseteq \sup \mathrm{p}\varphi.$$

Thus, we can substitute (2.12), (2.13) sequentially in (2.6)-(2.11):

$$B_0(w,v) = -\frac{1}{h_e^3}\theta_{i_e} D_{i_e} \int_0^1 \frac{d^2 w}{dt^2} \cdot \frac{d^2 v}{dt^2}\,dt =$$
$$-\frac{\theta_{i_e} D_{i_e}}{h_e^3} \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} \alpha_i \beta_j \int_0^1 \varphi''(t+i)\varphi''(t+j)\,dt =$$
$$= -\frac{\theta_{i_e} D_{i_e}}{h_e^3}(K^0_{\alpha\beta}\overline{\alpha}, \overline{\beta}), \tag{2.14}$$

where

$$K^0_{\alpha\beta}(i,j) = \int_0^1 \varphi''(t+i)\varphi''(t+j)\,dt, \quad \varphi'' = \frac{d^2\varphi}{dt^2}; \tag{2.15}$$

$$B_4(w,v) = h_e \theta_{i_e} D_{i_e} \int_0^1 w \cdot v\,dt =$$
$$= \theta_{i_e} D_{i_e} h_e \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} \alpha_i \beta_j \int_0^1 \varphi(t+i)\varphi(t+j)\,dt =$$
$$= h_e \theta_{i_e} D_{i_e}(K^4_{\alpha\beta}\overline{\alpha}, \overline{\beta}); \tag{2.16}$$

$$\theta_{i_e} D_{i_e} h_e \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\alpha_i\beta_j\int_0^1 \varphi(t+i)\varphi(t+j)\,dt = \tag{2.17}$$
$$= h_e\theta_{i_e} D_{i_e}(K^4_{\alpha\beta}\overline{\alpha},\overline{\beta}),$$

where

$$K^4_{\alpha\beta}(i,j) = K^4_{\alpha\beta}(i,j) = \int_0^1 \varphi(t+i)\varphi(t+j)\,dt\,; \tag{2.18}$$

$$B_{21}(w,v) = -\frac{1}{h_e}\theta_{i_e} D_{i_e}\nu_{i_e}\int_0^1 \frac{d^2w}{dt^2}\cdot v\,dt =$$
$$= -\frac{\theta_{i_e} D_{i_e}\nu_{i_e}}{h_e}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\alpha_i\beta_j\int_0^1 \varphi''(t+i)\varphi(t+j)\,dt =$$
$$= -\frac{\theta_{i_e} D_{i_e}\nu_{i_e}}{h_e}(K^{21}_{\alpha\beta}\overline{\alpha},\overline{\beta}), \tag{2.19}$$

where

$$K^{21}_{\alpha\beta}(i,j) = \int_0^1 \varphi''(t+i)\varphi(t+j)\,dt\,; \tag{2.20}$$

$$B_{23}(w,v) = -\frac{1}{h_e}\theta_{i_e} D_{i_e}\nu_{i_e}\int_0^1 w\cdot\frac{d^2v}{dt^2}\,dt =$$
$$= -\frac{\theta_{i_e} D_{i_e}\nu_{i_e}}{h_e}\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\alpha_i\beta_j\int_0^1 \varphi(t+i)\varphi''(t+j)\,dt =$$
$$= -\frac{\theta_{i_e} D_{i_e}\nu_{i_e}}{h_e}(K^{23}_{\alpha\beta}\overline{\alpha},\overline{\beta}), \tag{2.21}$$

where

$$K^{23}_{\alpha\beta}(i,j) = \int_0^1 \varphi(t+i)\varphi''(t+j)\,dt = K^{21}_{\alpha\beta}(j,i)\,; \tag{2.22}$$

$$B_{22}(w,v) = \frac{1}{h_e}2\theta_{i_e} D_{i_e}(1-\nu_{i_e})\int_0^1 \frac{dw}{dt}\cdot\frac{dv}{dt}\,dt =$$
$$= 2\frac{\theta_{i_e} D_{i_e}(1-\nu_{i_e})}{h_e}\times$$
$$\times\sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\alpha_i\beta_j\int_0^1 \varphi'(t+i)\varphi'(t+j)\,dt =$$
$$= 2\frac{\theta_{i_e} D_{i_e}(1-\nu_{i_e})}{h_e}(K^{22}_{\alpha\beta}\overline{\alpha},\overline{\beta}); \tag{2.23}$$

$$K^{22}_{\alpha\beta}(i,j) = \int_0^1 \varphi'(t+i)\varphi'(t+j)\,dt\,,\quad \varphi' = \frac{d\varphi}{dt}. \tag{2.24}$$

We define the parameters α_k and β_k through the node unknowns on the element:

$$\begin{cases} y_1 = w(0) = \sum_{k=0}^{N-1}\alpha_k\varphi(k) \\ \dfrac{dy_1}{dx} = \dfrac{1}{h_e}w'(0) = \dfrac{1}{h_e}\sum_{k=0}^{N-1}\alpha_k\varphi'(k) \\ y_2 = w(0.25) = \sum_{k=0}^{N-1}\alpha_k\varphi(k+0.25) \\ y_3 = w(0.5) = \sum_{k=0}^{N-1}\alpha_k\varphi(k+0.5) \\ y_4 = w(0.75) = \sum_{k=0}^{N-1}\alpha_k\varphi(k+0.75) \\ y_5 = w(1) = \sum_{k=0}^{N-1}\alpha_k\varphi(k+1) \\ \dfrac{dy_5}{dx} = \dfrac{1}{h_e}w'(1) = \dfrac{1}{h_e}\sum_{k=0}^{N-1}\alpha_k\varphi'(k+1) \end{cases} \tag{2.25}$$

Therefor we have

$$\overline{y}^{i_e} = T\overline{\alpha}\,, \tag{2.26}$$

where

$$\overline{y}^{i_e} = [y_1\ \ \frac{dy_1}{dx}\ \ y_2\ \ y_3\ \ y_4\ \ y_5\ \ \frac{dy_5}{dx}]^T; \tag{2.27}$$
$$\overline{\alpha} = [\alpha_0\ \ \alpha_1\ \ \alpha_2\ \ \alpha_3\ \ \alpha_4\ \ \alpha_5\ \ \alpha_6]^T; \tag{2.28}$$

$$T = D\begin{bmatrix} \varphi(0) & \varphi(1) & \varphi(2) & \varphi(3) & \varphi(4) & \varphi(5) & \varphi(6) \\ \varphi'(0) & \varphi'(1) & \varphi'(2) & \varphi'(3) & \varphi'(4) & \varphi'(5) & \varphi'(6) \\ \varphi(0.25) & \varphi(1.25) & \varphi(2.25) & \varphi(3.25) & \varphi(4.25) & \varphi(5.25) & \varphi(6.25) \\ \varphi(0.5) & \varphi(1.5) & \varphi(2.5) & \varphi(3.5) & \varphi(4.5) & \varphi(5.5) & \varphi(6.5) \\ \varphi(0.75) & \varphi(1.75) & \varphi(2.75) & \varphi(3.75) & \varphi(4.75) & \varphi(5.75) & \varphi(6.75) \\ \varphi(1) & \varphi(2) & \varphi(3) & \varphi(4) & \varphi(5) & \varphi(6) & \varphi(7) \\ \varphi'(1) & \varphi'(2) & \varphi'(3) & \varphi'(4) & \varphi'(5) & \varphi'(6) & \varphi'(7) \end{bmatrix}; \quad (2.29)$$

$$D = diag(1 \quad 1/h_e \quad 1 \quad 1 \quad 1 \quad 1 \quad 1/h_e). \quad (2.30)$$

Similarly, we get

$$\bar{z}^{i_e} = T\bar{\beta}. \quad (2.31)$$

Taking into account (2.26) and (2.31) we get

$$\bar{\alpha} = T^{-1}\bar{y}^{i_e}; \quad \bar{\beta} = T^{-1}\bar{z}^{i_e}. \quad (2.32)$$

In the general case, the following chain of equalities holds:

$$\begin{aligned}(K_{\alpha\beta}\bar{\alpha}, \bar{\beta}) &= (K_{\alpha\beta}T^{-1}\bar{y}^{i_e}, T^{-1}\bar{z}^{i_e}) = \\ &= ((T^{-1})^{\mathrm{T}} K_{\alpha\beta}T^{-1}\bar{y}^{i_e}, \bar{z}^{i_e}).\end{aligned} \quad (2.33)$$

Therefore, substituting (2.32) sequentially into (2.14), (2.16), (2.19), (2.21), (2.23), we obtain local stiffness matrices $K_0^{i_e}$, $K_4^{i_e}$, $K_{21}^{i_e}$, $K_{23}^{i_e}$, $K_{22}^{i_e}$, $K_2^{i_e}$, corresponding to the operators L_0, L_4, L_{21}, L_{23}, L_{22}, L_2,

$$K_0^{i_e} = -\frac{\theta_{i_e} D_{i_e}}{h_e^3}(T^{-1})^{\mathrm{T}} K_{\alpha\beta}^0 T^{-1}; \quad (2.34)$$

$$K_4^{i_e} = h_e \theta_{i_e} D_{i_e} (T^{-1})^{\mathrm{T}} K_{\alpha\beta}^4 T^{-1}; \quad (2.35)$$

$$K_{21}^{i_e} = -\frac{\theta_{i_e} D_{i_e} \nu_{i_e}}{h_e}(T^{-1})^{\mathrm{T}} K_{\alpha\beta}^{21} T^{-1}; \quad (2.36)$$

$$K_{23}^{i_e} = -\frac{\theta_{i_e} D_{i_e} \nu_{i_e}}{h_e}(T^{-1})^{\mathrm{T}} K_{\alpha\beta}^{23} T^{-1} = (K_{21}^{i_e})^T; \quad (2.37)$$

$$K_{22}^{i_e} = 2\frac{\theta_{i_e} D_{i_e}(1-\nu_{i_e})}{h_e}(T^{-1})^{\mathrm{T}} K_{\alpha\beta}^{22} T^{-1}. \quad (2.38)$$

Due to the fact that

$$L_2 = L_{21} + L_{22} + L_{23}, \quad (2.39)$$

we have the following local stiffness matrix

$$K_2^{i_e} = K_{21}^{i_e} + K_{22}^{i_e} + K_{23}^{i_e}. \quad (2.40)$$

3. NUMERICAL IMPLEMENTATION

The presented algorithm can be implemented using the tools of MATLAB. In particular, the reference to the standard function

```
wavefun('db14',0)
```

allows researcher to get the values of the scaling Daubeshi function φ on the interval $[0, 27] = \mathrm{supp}(\varphi)$ with steps $h_t = 1/256 = 2^{-8}$. Let us denote $N_t = 256 = 2^8$. For the considering value $N = 7$ we can use the first $N_l = N_t \cdot N + 1$ values of φ, defined on the segment $[0, N] = [0, 7]$. With such a small step, it will be natural to compute the derivatives in the form of finite differences:

$$\varphi'(t_k) \approx d\varphi_k = \frac{\varphi_{k+1} - \varphi_{k-1}}{2h_t}, \quad k = 1, 2, \ldots, N_l; \quad (3.1)$$

$$\varphi''(t_k) \approx d2\varphi_k = \frac{\varphi_{k+1} - 2\varphi_k + \varphi_{k-1}}{h_t^2}, \quad k = 1, 2, \ldots, N_l, \quad (3.2)$$

where $\varphi_k = \varphi(t_k); \quad t_k = k \cdot h_t$. (3.3)

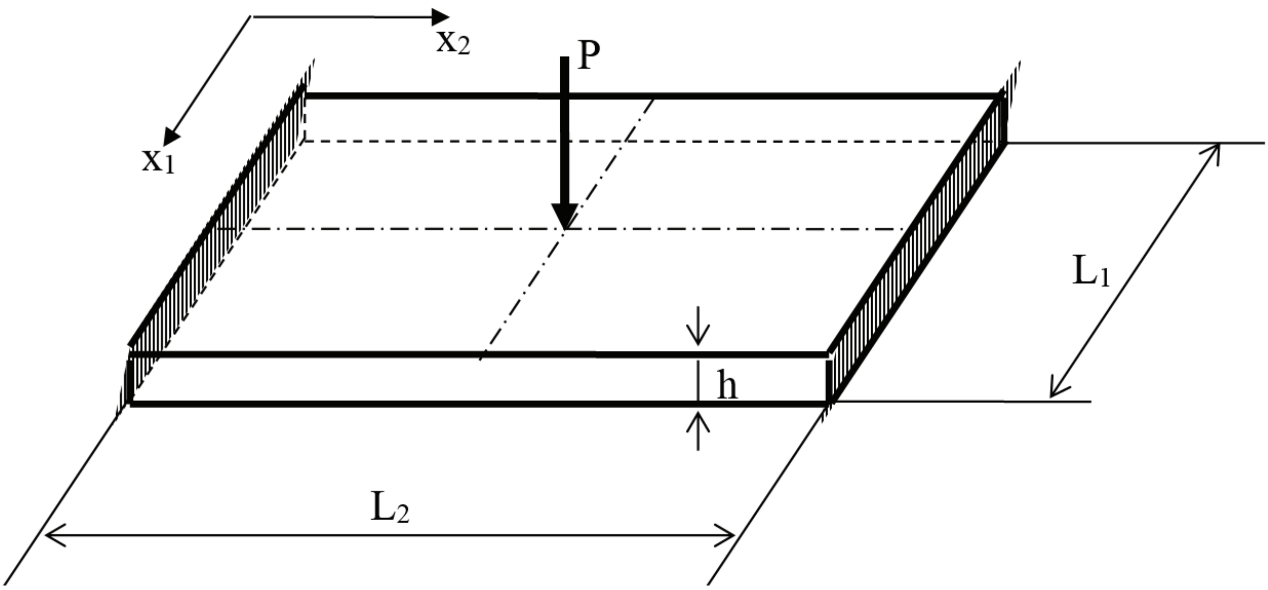


Figure 2. About formulation of the problem.

If $t_k \notin [0,27]$ then $\varphi_k = \varphi(t_k) = 0$.

When computing the coefficients of the local stiffness matrix (formulas (2.14), (2.16), (2.19), (2.21), (2.23)), one can use the simplest quadrature formulas for numerical integration, in particular, midpoint quadrature rule with step $2h_t$.

As an example, we consider the problem of bending a thin plate rigidly fixed along the lateral faces under the influence of a load concentrated in the center (Figure 2).

Geometric parameters: $L_1 = 0.9\,\text{m}$, $L_2 = 1.0\,\text{m}$, $h = 0.05\,\text{m}$ (thickness). The calculated parameters of the plate material: coefficient of elasticity $E = 3000 \cdot 10^4\,\text{kN / m}^2$, $\nu = 0.16$ is Poisson's ratio. External load parameter: $P = 1\,\text{kN}$.

Let $N_e = 3$ be the number of elements.

The length of the element

$$h_e = L_1 / N_e = 0.9/3 = 0.3.$$

Distance between coordinates of nodes

$$h_p = h_e / 4 = 0.3/4 = 0.075\ .$$

The number of nodal unknowns for each component of the vector function y_j, $j = 1,2,3,4$:

$$N_g = N_p + 2N_b = 3\cdot 3 + 2\cdot(3+1) = 17\,,$$

where $N_p = N_e(N_k - 1)$ is the total number of internal nodes for all elements; $N_b = N_e + 1$ is the total number of boundary nodes for all elements. Total number of unknowns is equal to

$$N_U = 4N_g = 4\cdot 17 = 68\ .$$

For comparison, we use the traditional finite element method, where unknown functions on an element are represented as a cubic parabola and in the node each unknown function is represented by two unknown nodal quantities: the nodal value of the unknown function itself and its first derivative in the discrete direction.

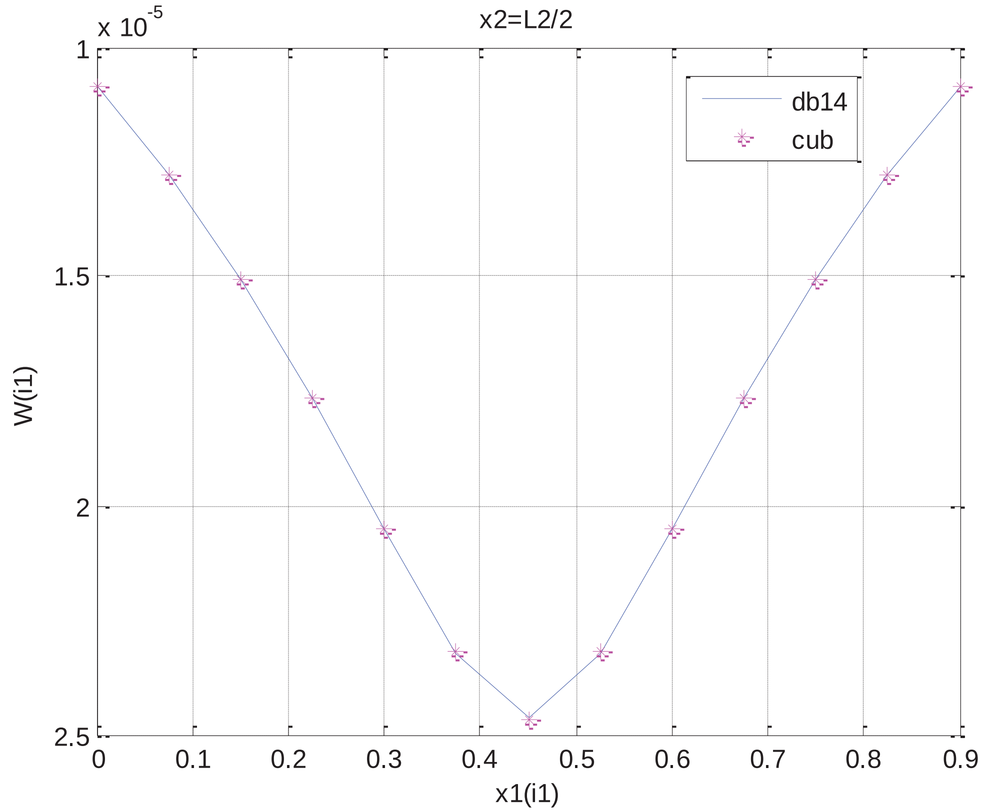
The total number of nodal points in a discrete direction x_1 is equal to

$$N_1 = L_1 / h_p + 1 = 0.9/0.075 + 1 = 13\,.$$

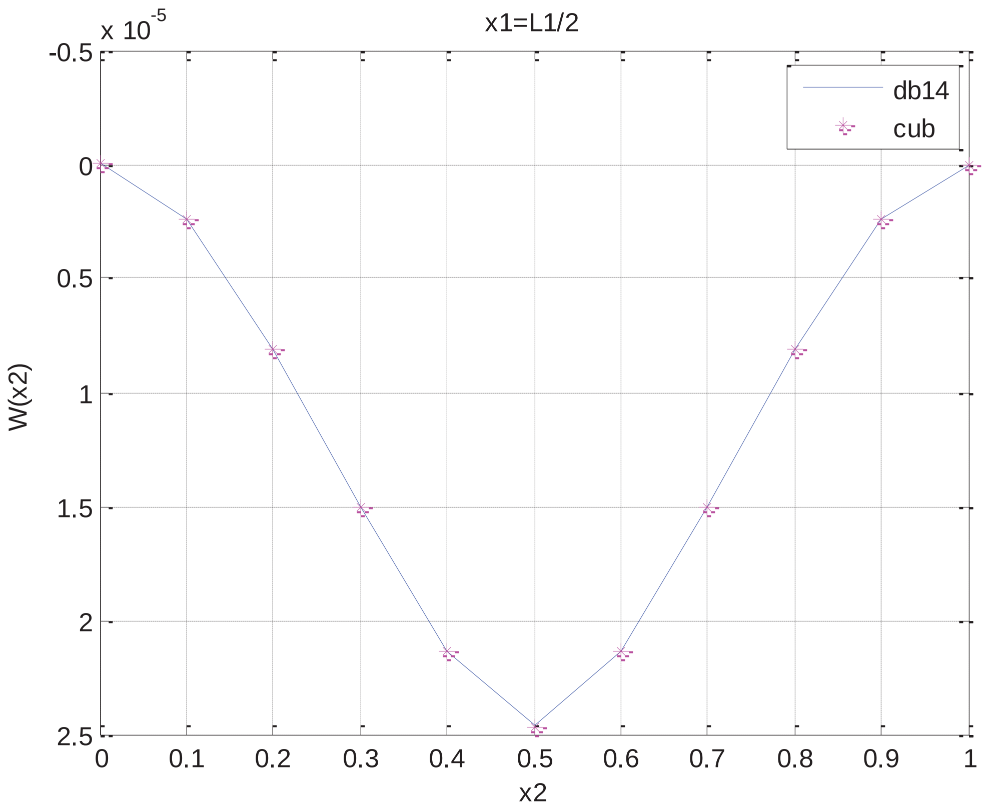
The number of nodal unknowns for each component of the vector function y_j, $j = 1,2,3,4$:

$$N_g = 2N_1 = 26\,.$$

Total number of unknowns is equal to



a)



b)

Figure 3. Comparison of results in mid sections in each direction.

$$N_U = 4N_g = 4 \cdot 26 = 104 \ .$$

Graphical comparison of the results of analysis is presented at Figure 3 (`db14` is deflection values obtained using the Daubechies scaling function; `cub` is deflection values obtained, obtained on the basis of the traditional finite element method; $h_1 = 0.075$, $h_2 = 0.1$ are steps of issuing results in the directions x_1 and x_2, respectively.

As can be seen, the results obtained practically coincide. Moreover, the finite element method algorithm based on the Daubechies scaling function leads to a significant decrease in the number of unknowns. The difference is equal to $4N_p$.

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