DOI:10.22337/2587-9618-2019-15-2-159-164

NON-CONSERVATIVENESS OF FORMULAS AS AN OBSTACLE TO THE MOTION SIMULATION

Vladimir B. Zylev, Alexey V. Steyn, Nikita A. Grigoryev
Russian University of Transport (MIIT), Moscow, RUSSIA

Abstract: The paper considers the specifics of connection setting between deformations and stresses as applied to the problem of large non-linear oscillations, when deformations are considered arbitrarily large. It is noted that the introduction of some arbitrary physical relations in the computational model can lead to the establishment of a non-conservative system. It is shown, for example, that distribution of the ordinary Hooke's law formulas to the area of large deformations leads to the establishment of material with the non-conservative properties. The examples are given for the numerical solution of problems with the nonlinear oscillations, where an increase in the oscillation amplitudes or occurrence of unauthorized internal friction is shown. The simplest version of the material properties free from the indicated deficiencies is given. One of the paper conclusions is that when specifying the elastic properties of the material, it is necessary to ensure that the resulting system is conservative.

Keywords: membranous systems, n-step numerical method, equations of motion, large deformations, non-conservativeness

ФОРМУЛЬНАЯ НЕКОНСЕРВАТИВНОСТЬ КАК ПОМЕХА МОДЕЛИРОВАНИЯ ДВИЖЕНИЯ

В.Б. Зылев, А.В. Штейн, Н.А. Григорьев
Российский университет транспорта (МИИТ), г. Москва, РОССИЯ

Аннотация: В работе рассматриваются особенности задания связи между деформациями и напряжениями применительно к задаче больших нелинейных колебаний, когда и деформации рассматриваются произвольно большими. Отмечается, что введение некоторых произвольных физических соотношений в расчетную модель может приводить к созданию неконсервативной системы. Показывается, например, что распространение обычных формул закона Гука в область больших деформаций ведет к созданию материала с неконсервативными свойствами. Приводятся примеры численного решения задач о нелинейных колебаниях, где демонстрируется нарастание амплитуд колебаний или появление несанкционированного внутреннего трения. Приводится простейший вариант свойств материала, свободного от указанных недостатков. Один из выводов работы заключается в том, что при задании упругих свойств материала нужно следить за тем, чтобы полученная система была консервативной.

Ключевые слова: Мембранные системы, шаговый численный метод, уравнения движения, большие деформации, неконсервативность

The paper relates to the field of structural mechanics that considers the dynamics of deformable systems without any restrictions on the magnitude of displacements and deformations. In order to reduce the scope of work, it shall be limited to non-linear elasticity, isotropic material, and the plane stress condition. From the geometrical point of view, it shall be assumed that the system can be considered using a triangular finite element, the displacement field in which is given by a linear function. The object of practical application here may be the dynamics of inflatable capsules when they are filled with air [1]. The dynamic solution method in the examples given below is based on an explicit computational scheme with extrapolation according to Adams [2,3,4]. The article, however, is devoted not to the integration method of the motion equations, but to the influence on the resulting motion of the selected physical relation system, that allow one to determine stresses and then the nodal reactions in the element using the well-known deformations. Figure 1 shows a triangular finite element before deformations (solid line) and the same element after deformations (dashed line) at some current moment of the dynamic solution. The deformed element can be combined with the initial one to calculate reactions in the local axes (Figure 1).

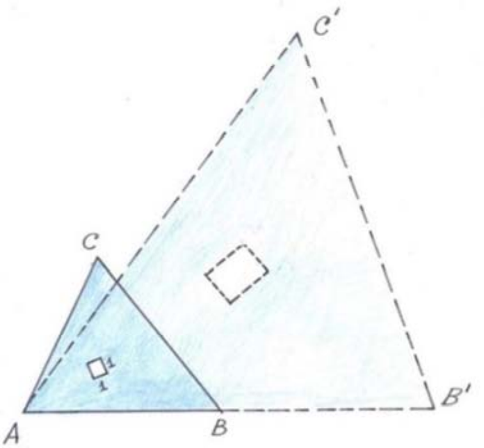

Figure 1. Trianglular finite element before and after deformations.

In the original triangular element, it is possible to select a square with ordinary sides that correspond to the main directions. After the deformations, this square will turn into a rectangle. The problem of determining the main directions with the well-known position of the element points is a fundamentally non-complicated purely geometrical problem and its solution will not be highlighted. It only shall be noted that the main directions in the original and deformed elements do not coincide, since the elementary square undergoes not only a deformation, but also a hard rotation. The current values of the main deformations ε_1 and ε_2 shall be determined since the displacement field inside the element is given. The values ε_1 and ε_2 can be determined in different ways; here the relative elongations shall mean the ratio of the segment elongation to its original length. Figure 2 shows the unit cube before and after deformations.

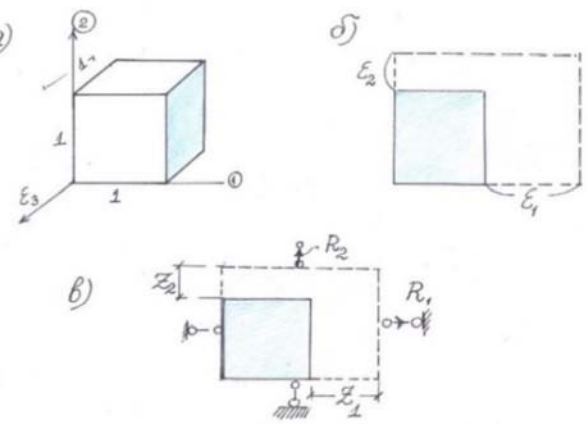

Figure 2. The unit cube before and after deformations.

Now we will discuss the issue of calculating the reactions R_1 and R_2 in the system with two displacements $z_1 = \varepsilon_1$ and $z_2 = \varepsilon_2$. We will first consider the possibility of using the usual Hooke's law formulas, in the form in which it is applied for small deformations. For the main stresses and for the main deformation in the direction of the plate thickness the formula shall be as follows:

$$
\begin{aligned}
\sigma_1 &= \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2) \\
&= \frac{E}{1-\mu^2}(z_1 + \mu z_2) \\
\sigma_2 &= \frac{E}{1-\mu^2}(\varepsilon_2 + \mu\varepsilon_1) \\
&= \frac{E}{1-\mu^2}(z_2 + \mu z_1) \\
\varepsilon_3 &= -\frac{\mu}{E}(\sigma_1 + \sigma_2) \\
&= -\frac{\mu}{1-\mu}(z_1 + z_2)
\end{aligned}
\tag{1}
$$

where E is a modulus of elasticity, μ is a Poisson's ratio.

Further, it can be assumed that the obtained stresses refer to the deformed dimensions of the element, then the required reactions shall be as follows:

$$R_1 = \sigma_1(1+\varepsilon_2)\cdot(1+\varepsilon_3) = \frac{E}{1-\mu^2}(z_1+\mu z_2)\cdot(1+z_2)\cdot\left[1-\frac{\mu}{1-\mu}(z_1+z_2)\right]$$

$$R_2 = \sigma_2(1+\varepsilon_1)(1+\varepsilon_3) = \frac{E}{1-\mu^2}(z_2+\mu z_1)(1+z_1)\left[1-\frac{\mu}{1-\mu}(z_1+z_2)\right] \qquad (2)$$

Having determined the partial derivatives of the reactions by displacements, we shall obtain the elements of the tangent stiffness matrix for the system shown in Figure 2.

$$r_{12} = \frac{\partial R_1}{\partial z_2} = \frac{E}{1-\mu^2}\left\{\mu + z_1 + 2\mu z_2 - \frac{\mu}{1-\mu}\left[(1+\mu)z_1 + {z_1}^2 + 2(1+\mu)z_1 z_2 + 2\mu z_2 + 3\mu z_2^2\right]\right\}$$

$$r_{21} = \frac{\partial R_2}{\partial z_1} = \frac{E}{1-\mu^2}\left\{\mu + z_2 + 2\mu z_1 - \frac{\mu}{1-\mu}\left[(1+\mu)z_2 + {z_2}^2 + 2(1+\mu)z_1 z_2 + 2\mu z_1 + 3\mu z_1^2\right]\right\} \qquad (3)$$

The formulas (3) show that the tangent stiffness matrix in this case is not symmetrical to $r_{12} \neq r_{21}$ that means that the system is not conservative [4]. The special particular case is possible when $\mu = \frac{1}{3}$ but this is exactly the exception. Non-conservativeness in this case means that the use of this material model shall lead to an increase in oscillations or to occurrence of an unauthorized internal friction, that is there shall be not just an inaccurate reflection of the material properties, but a qualitatively incorrect model behavior.

We shall consider a test case. Figure 3 shows a plane computational scheme. The triangular finite element with two fixed vertices is connected to a much more massive and rigid rod element ***A B***.

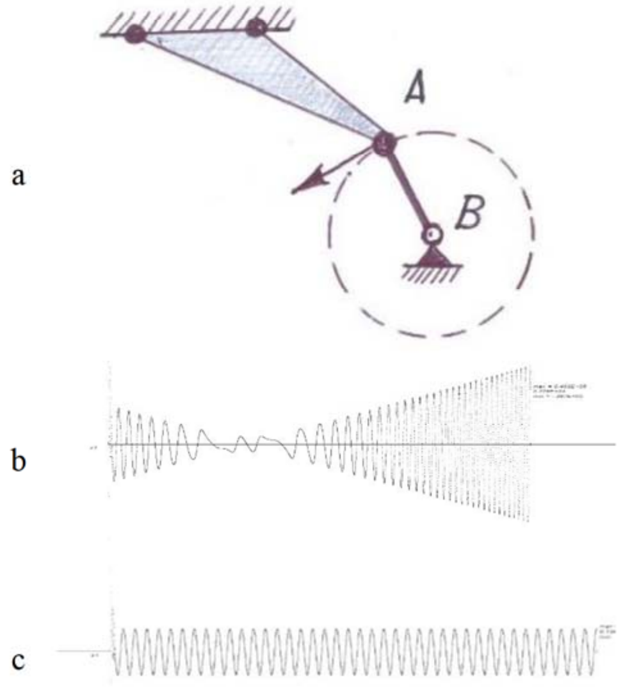

Figure 3. Computing solution of the problem of the rod rotation connected by a triangular element, the diagrams show the time dependence of the horizontal velocity component at point A: a) the computational scheme; b) an incorrect model behavior with non-conservative properties; c) the conservative model.

Point A is given with an initial velocity corresponding to its counterclockwise rotation. The rod material has a moderate Voigt-type internal friction that provides a quick transition to the motion that can be approximately described as rotation around point ***B***.

The diagram Figure 3b shows a case where the triangle material has the non-conservative properties described above. The specified initial counter-clockwise rotation of the rod ***A B*** initially slows down rapidly; the non-conservativeness is evident as an unauthorized internal friction. When the rod stops, it begins to unwind in a clockwise direction. Its angular velocity and rotation frequency are increased. The system works like a perpetual motion machine that means, of course, a qualitative error in the physical relations of the model. This article is

devoted to this negative specification. The well-formed material properties are certainly known [5]. However, the process of forming new material properties cannot be fully completed. Sometimes, the quite complex computational algorithms can be used to obtain stress values, and during their development it is possible to obtain negative results similar to those just considered.
Having returned to our formulas, it shall be noted that they can be greatly simplified and the non-conservativeness defect can be removed by assuming that the strain energy of a single element (Figure 2) is expressed through displacements as follows:

$$U = \frac{E}{2(1-\mu^2)}(z_1^2 + 2\mu z_1 z_2 + z_2^2)$$

and then the reactions shall be determined by formulas

$$R_1 = \frac{\partial U}{\partial z_1}; \quad R_2 = \frac{\partial U}{\partial z_2}.$$

In this case, the matrix symmetry shall be guaranteed. If these simplifications are introduced into the computer program, then instead of the incorrect diagram in Figure 3, b we will receive the diagram with stable rotation in Figure 3, c.
We will demonstrate the considered specifications with a more complicated example. The thin membrane in a stress-free state has a square form with the sides of 2×2 meters, thickness of 10^{-4} meters, the density of the sheath material of 10^3 kg/m^3, modulus of elasticity E=120 MPa, the Poisson's ratio of $\mu = 0{,}2$. The membrane is affected by its own weights and nine equal, suddenly exerted and single forces of 1444 N directed upwards. The forces are applied to the central points of the sheath model. The system nodes are influenced by the external resistance forces proportional to the node velocities and their weight with a friction coefficient of 5 s^{-1}. The expected motion scenario is that the membrane is elevated from a horizontal position, the damped oscillations are available for some time, after which the equilibrium position shall be established.

If we take the material properties as non-conservative, then the expected scenario shall not be implemented (see Figure 4). At first, the sheath takes a position close to the equilibrium position that can also be judged by the vertical displacement diagram in Fig. 4. Next, there are increasing fluctuations, like a flutter that lead to the time moment $t \approx 0{,}5c$ and to the computational chaos (see Figure 4).

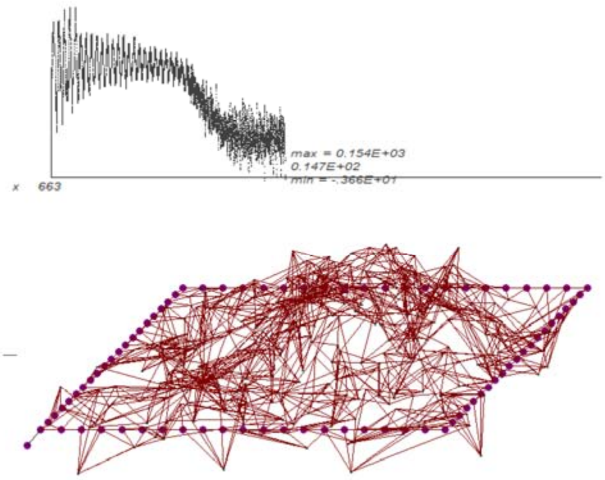

Figure 4. Attempt to calculate the membrane using the non-conservative material that leads to the computational chaos. The diagram shows the change of the center point vertical coordinate in time.

If we take the material that corresponds to the conservativeness properties, then the solution shall be relevant to the scenario described above, its result is shown in Figure 5, the system is almost in the equilibrium position by the time moment $t \approx 1c$.
It shall be noted that with a decrease in load, when the deformations are actually small, both the material models considered behave practically consistently, and the non-conservative properties are weakened so that they can be ignored. An increase in the external resistance forces also leads to the fact that the system is out of the equilibrium position, but this transition shall be surely simulated incorrectly.

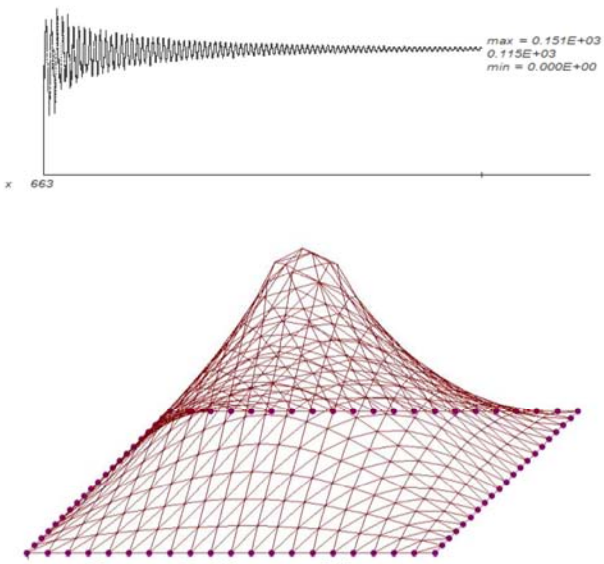

Figure 5. The equilibrium position of the system, obtained by using a material with conservative properties. The diagram shows the change of the central point vertical coordinate in time.

CONCLUSIONS

1. When the material is simulated, it is necessary to ensure that the material has conservative properties.
2. The extention of the traditional Hooke's law formulas for small deformations to the large deformations leads not only to the inaccurate physical relations, but to the qualitatively incorrect material description that leads to the energy release during nonlinear motion of the system.
3. The symmetry of tangent stiffness matrix can be used to control the non-conservativeness properties of the computational models.

REFERENCES

1. **Zylev V.B., Steyn A.V., Grigoryev N.A.** Dinamicheskoe modelirovanie vozdukhonesomoi pnevmokonstruktsii v protsesse napolneniia [Dynamic modeling of airborne pneumatic construction during filling]. *Collection of conference articles. Actual problems and prospects for the development of building structures: innovation, modernization and energy efficiency in construction*. Almaty, 2018, pp. 140-145 (in Russian).
2. **Zylev V.B.** Vychislitel'nye metody v nelineinoi mekhanike konstruktsii [Computational methods in the non-linear structural mechanics]. Moscow, Research Center “Engineer”, 144 pages (in Russian).
3. **Aleksandrov A.V., Potapov V.D., Zylev V.B.** Stroitel'naia mekhanika. Kniga 2. Dinamika i ustoichivost' uprugikh sistem [Structural mechanics. Vol.2. Dynamics and stability of elastic systems]. Moscow, Higher School, 2008, 384 pages (in Russian).
4. **Bolotin V.V.** Nekonservativnye zadachi teorii uprugoi ustoichivosti [Non-conservative problems of the elastic stability theory]. Moscow, State Publishing House of Physical and Mathematical Literature, 1961, 340 pages (in Russian).
5. **Oden J.** Konechnye elementy v nelineinoi mekhanike sploshnykh sred [Finite elements of nonlinear continua]. Moscow, Mir, 1976, 464 pages (in Russian).

СПИСОК ЛИТЕРАТУРЫ

1. **Зылев В.Б., Штейн А.В., Григорьев Н.А.** Динамическое моделирование воздухонесомой пневмоконструкции в процессе наполнения. // *Сборник статей конференции «Актуальные проблемы и перспективы развития строительных конструкций: инновации, модернизация и энергоэффективность в строительстве»*. – Алматы, 2018, с. 140-145.
2. **Зылев В.Б.** Вычислительные методы в нелинейной механике конструкций. – М.: НИЦ Инженер, 1999. – 144 с.
3. **Александров А.В., Потапов В.Д., Зылев В.Б.** Строительная механика. Книга

2. Динамика и устойчивость упругих систем. – М.: Высшая школа, 2008. – 384 с.
4. **Болотин В.В.** Неконсервативные задачи теории упругой устойчивости. – М.: «Государственное издательство физико-математической литературы», 1961. – 340 с.
5. **Оден Дж.** Конечные элементы в нелинейной механике сплошных сред. – М.: Мир, 1976. – 464 с.

Vladimir B. Zylev, Advisor of the Russian Academy of Architecture and Construction Sciences, Professor, Dr.Sc, Head of Department of Structure mechanics, Russian University of Transport (MIIT); 9, Obraztsova Street, 127994, Moscow, Russia;
E-mail: Zylevvb@yandex.ru.

Alexey V. Steyn, PhD, Assistant professor of "Department of Structure mechanics", Russian University of Transport (MIIT); 9, Obraztsova Street, 127994, Moscow, Russia; e-mail: avsh7714@yandex.ru.

Nikita A. Grigoryev, PhD, Assistant professor of "Department of Structure mechanics", Russian University of Transport (MIIT); Build. 9, 9 Obraztsova St., Moscow, Russia 127994; e-mail: Gr_Nik2003@mail.ru.

Зылев Владимир Борисович, советник Российской академии архитектуры и строительных наук, профессор, доктор технических наук, заведующий кафедрой «Строительная механика», Российский университет транспорта (МИИТ); 127994, Россия, г. Москва, ул. Образцова, д 9, стр. 9; e-mail: Zylevvb@yandex.ru.

Штейн Алексей Владимирович, кандидат технических наук; доцент кафедры «Строительная механика», Российский университет транспорта (МИИТ); 127994, Россия, г. Москва, ул Образцова, д 9, стр. 9;
E-mail: avsh7714@yandex.ru.

Григорьев Никита Алексеевич, кандидат технических наук; кандидат технических наук; доцент кафедры «Строительная механика», Российский университет транспорта (МИИТ); 127994, Россия, г. Москва, ул. Образцова, д 9, стр. 9; e-mail: Gr_Nik2003@mail.ru.