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COMPUTATIONAL RHEOLOGICAL MODEL OF CONCRETE

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Abstract: The substantiation of the computational rheological model of the material for use in computer calculations of concrete and reinforced concrete structures of arbitrary complexity, taking into account the creep of concrete, with possible changes in the intensity of the current load over time, is presented. The computational model is based on the Maxwell-Weichert generalized model of the viscoelastic material. The model was verified on the basis of experimental data and requirements of regulatory documents for the calculations of concrete and reinforced concrete structures, taking into account the creep of the material. Verification of the computational model and numerical calculations using the selected computational model were performed in the SIMULIA Abaqus software environment.

Keywords: creep, viscoelasticity, computational model, generalized Maxwell-Weichert model, calculation on deformed scheme

ВЫЧИСЛИТЕЛЬНАЯ РЕОЛОГИЧЕСКАЯ МОДЕЛЬ БЕТОНА

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Аннотация: Представлено обоснование вычислительной реологической модели материала для использования в компьютерных расчётах бетонных и железобетонных конструкций произвольной сложности с учётом ползучести бетона, с возможным изменением интенсивности действующей нагрузки во времени. Вычислительная модель построена на основе обобщенной модели вязкоупругого материала Максвелла-Вайхерта. Модель верифицировалась на основе данных экспериментальных исследований и требований нормативных документов к расчётам бетонных и железобетонных конструкций с учётом ползучести материала. Верификация вычислительной модели и численные расчёты с использованием выбранной вычислительной модели выполнялись в программной среде SIMULIA Abaqus.

Ключевые слова: ползучесть, вязкоупругость, вычислительная модель, обобщенная модель Максвелла-Вайхерта, расчёт по деформированной схеме

1. INTRODUCTION

The most visible manifestations of concrete over time are creep, relaxation and shrinkage. These rheological properties of concrete as a building material are sufficiently fully investigated and formulated by thorough theories [1–5], etc., and their accounting in the calculations of building structures is quite clearly regulated in national and international normative documents [6-9]. However, the availability of proven theories and sound regulatory procedures, unfortunately, does not mean the existence of appropriate computational models that are conveniently calibrated based on the results of experimental studies and regulatory approaches, while of quite versatile, stable and flexible in calculating of complex building structures. The presented computational rheological model of concrete was verified on the basis of data from experimental studies of concrete creep, and it was also substantiated and calibrated in accordance with the requirements of regulatory documents for calculations of concrete and reinforced concrete structures, taking into account the material creep. When calculating structures that noticeably change their configuration in time due to material creep, such as flat arches and shells, attention is paid to the effect of calculating them by deformed scheme, which significantly specifies the change in the stress-strain state in time.

2. THEORETICAL FOUNDATIONS OF THE COMPUTATIONAL CREEP MODEL

The complete deformation of concrete ε is usually represented by its three components:

$$\varepsilon = \varepsilon_0 + \varepsilon_{cr} + \varepsilon_{shr}, \tag{1}$$

where ε_0 is the so-called "instantaneous" deformation which manifests itself at the moment of loading the structure, ε_{cr} is the creep deformation that develops over time, ε_{shr} is the shrinkage deformation. The key parameter characterizing the creep of concrete in EC2 [6] and PN [7] is the creep coefficient (creep factor) $\varphi(t,t_0)$ established between the moment of application of the load t_0 and the age of concrete t:

$$\varphi(t,t_0) = \frac{\varepsilon_{cr}}{\varepsilon_0}. \tag{2}$$

At the same time, the concept of creep measure $C(t,t_0)$ is also used in these regulatory documents:

$$\varphi(t,t_0) = \mathrm{E_b}(t_0) \cdot C(t,t_0), \tag{3}$$

where $\mathrm{E_b}(t_0)$ is the elasticity modulus of concrete at the time of loading t_0.

In turn, the creep measure is the initial key parameter in the NIIZhB Recommendations [8]:

$$C(t,t_0) = \frac{1}{\mathrm{E_b}(t_0)} - \frac{1}{\mathrm{E_b}(t)} + C(\infty,t_0), \tag{4}$$

where $C(\infty,t_0 = 28)$ is the limit creep measure.

In this case the concept of creep characteristic (creep coefficient) $\varphi(t,t_0)$ is also used:

$$C(t,t_0) = \frac{\varphi(t,t_0)}{E(t_0)}. \tag{3a}$$

The rheological model based on the generalized model of a viscoelastic material — the generalized Maxwell model (Wiechert model) [10], [11] was chosen to study and apply to the modeling of concrete creep. Elements of the mechanical interpretation of this model is a parallel combination of n springs ($i = 1 \div n$) with "temporary" elastic moduli E_i, and n dampers with viscosity coefficients η_i connected in series with each other in pairs, as well as of an elastic element with "long-term" stiffness E_∞.

The basis of the generalized Maxwell model is the elementary model of the viscoelastic Maxwell material. Its mechanical representation is a spring with stiffness (modulus of elasticity) E, consistently coaxed with a damper having a viscosity coefficient η. According to this model, the material resistance is proportional to the speed of its strain

$$\dot{\varepsilon} = \dot{\varepsilon}_y + \dot{\varepsilon}_в, \tag{10}$$

folding from the rate of elastic strain $\dot{\varepsilon}_y$ and the rate of viscous strain $\dot{\varepsilon}_в$. In turn, the elastic strain of the material is interpreted by the strain of the elastic spring with the stiffness E:

$$\varepsilon_y = \frac{\sigma_y}{E}, \quad \text{from where} \quad \dot{\varepsilon}_y = \frac{\dot{\sigma}_y}{E}. \tag{11}$$

And the rate of viscous strain in this elementary model is represented by the properties of a damper with the coefficient of viscosity η:

$$\dot{\varepsilon}_в = \frac{\sigma_в}{\eta}. \tag{12}$$

The serial connection of the spring and damper in the Maxwell model means that the stresses σ in both its elements are the same, i.e. $\sigma = \sigma_y = \sigma_в$, but the strains in the elastic and viscous elements are different, i.e.

$$\varepsilon_y \neq \varepsilon_в \, (\varepsilon = \varepsilon_y + \varepsilon_в).$$

Thus, according to (10)-(12), the strain rate of the material, represented by the Maxwell elementary model, will vary in proportion to time t:

$$\dot{\varepsilon}(t) = \frac{\sigma(t)}{\eta} + \frac{\dot{\sigma}(t)}{E}, \tag{10a}$$

herewith at a constant stress σ_0 the strain ε will increase linearly.
In turn, the solution (10a) with respect to $\sigma(t)$:

$$\sigma(t) = \sigma_0 \cdot e^{\left(\frac{-t}{\tau}\right)} = E \cdot \varepsilon_0 \cdot e^{\left(\frac{-t}{\tau}\right)} \tag{13}$$

shows that according to the elementary Maxwell model, when the strain ε_0 is fixed, the stress $\sigma(t)$ decreases in time t according to the exponential law. It in general corresponds to the nature of the manifestation of relaxation in a deformable material, such as for example concrete. In (13)

$$\tau = \frac{\eta}{E}$$

is the so-called relaxation time.
If you move from the elementary Maxwell model (one pair of series-connected spring and a damper) to the Maxwell generalized model (n parallel-connected the pairs with an additional spring of "long-term" stiffness E_∞), dependence (13) takes the form:

$$\sigma(t) = E_\infty \cdot \varepsilon_0 + \sum_{i=1}^{n} E_i \cdot \varepsilon_0 \cdot e^{\left(\frac{-t}{\tau_i}\right)},$$

or

$$\sigma(t) = \varepsilon_0 \cdot \left[E_\infty + \sum_{i=1}^{n} E_i \cdot e^{\left(\frac{-t}{\tau_i}\right)} \right], \tag{14}$$

Where

$$\tau_i = \frac{\eta_i}{E_i}$$

is the relaxation time of the i -th damper ($i = 1 \div n$) in the generalized Maxwell model. Thus, in the generalized Maxwell model, the viscoelasticity of a material is characterized by the generalized relaxation modulus depending on time (the so-called relaxation function):

$$E_R(t) = \frac{\sigma(t)}{\varepsilon_0} = E_\infty + \sum_{i=1}^{n} E_i \cdot e^{\left(-\frac{t}{\tau_i}\right)}. \tag{15}$$

It can be seen that the relaxation function $E_R(t)$ of a material is actually represented in (14), (15) in the form of an exponential series (Prony series).

3. COMPUTATIONAL IMPLEMENTATION OF THE RHEOLOGICAL MODEL

The generalized Maxwell model of viscoelastic properties of a material is implemented in the SIMULIA Abaqus software environment with the "viscoelastic" option [12]. It is represented here by two functions, having the form of an exponential Proni series. This is the s so-called shear relaxation function:

$$G(t) = G_\infty + \sum_{i=1}^{n} G_i \cdot e^{\left(-\frac{t}{\tau_i}\right)}, \qquad (16)$$

where

$$G = \frac{E}{2 \cdot (1 + \nu_0)}$$

is the modulus of elasticity in shear, ν_0 is the Poisson coefficient, and also the function of bulk relaxation:

$$K(t) = K_\infty + \sum_{i=1}^{n} K_i \cdot e^{\left(-\frac{t}{\tau_i}\right)}, \qquad (17)$$

where

$$K = \frac{E}{3 \cdot (1 - 2 \cdot \nu_0)}$$

is the coefficient of volume stiffness.

In the SIMULIA Abaqus program, the parameters of the "viscoelastic" material model are set by the dimensionless shear relaxation modulus:

$$g_R(t) = \frac{G(t)}{G_0} = 1 - \sum_{i=1}^{n} g_i \cdot \left(1 - e^{\left(-\frac{t}{\tau_i}\right)}\right) \qquad (16a)$$

where

$$G_0 = E_0/(2 \cdot (1 + \nu_0))$$

is the instantaneous modulus of elasticity in shear, as well as the dimensionless modulus of bulk relaxation:

$$k_R(t) = \frac{K(t)}{K_0} = 1 - \sum_{i=1}^{n} k_i \cdot \left(1 - e^{\left(-\frac{t}{\tau_i}\right)}\right), \qquad (17a)$$

where

$$K_0 = E_0/ \cdot (1 - 2 \cdot \nu_0)$$

is the coefficient of instantaneous volume stiffness.

Obvious complexity here can have the assignment of values of the parameters of the material G_i, K_i, τ_i, which can be obtained by static and dynamic tests of the material for creep and relaxation. However, under the condition of relatively small changes in stresses over time in the simulated structure, you can use the simple case of specifying the modules $g_r(t)$ (16*a*) and $k_r(t)$ (17*a*) with the introduction of the concept of effective modulus of elasticity [6], [8]:

$$E_{c,eff} = \frac{E_0}{1 + \varphi(t, t_0)}. \qquad (18)$$

Then the expression of the dimensionless shear relaxation modulus is considerably simplified:

$$g_r(t) = \frac{G_R(t)}{G_0} = \frac{\dfrac{E_0}{2 \cdot (1 + \nu_0) \cdot (1 + \ (t, t_0))}}{\dfrac{E_0}{2(1 + \nu_0)}},$$

or

$$g_r(t) = \frac{1}{1 + \varphi(t, t_0)}, \qquad (16b)$$

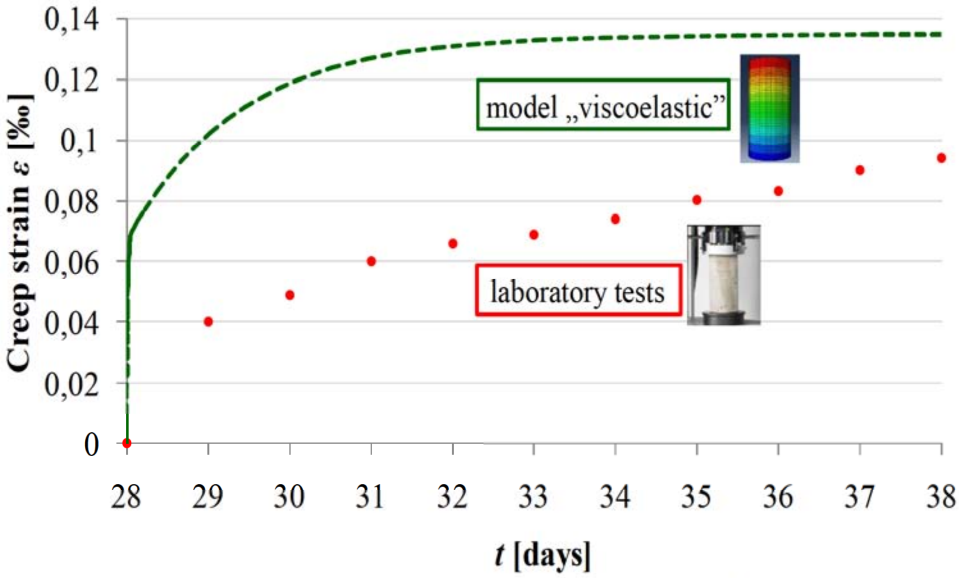

Figure 1. Comparison of the dependences of creep strain in samples of the 1-st group (t_0 = 28 days), obtained in the experiment and in numerical simulation.

where $\varphi(t,t_0)$ is the creep coefficient of the material (2). In this case, the values of the time-varying coefficient $\varphi(t,t_0)$ in the computational model of creep can be set both on the basis of the material test results, and in accordance with the regulatory recommendations [6] - [8] and others. Similarly, the view of the dimensionless bulk relaxation modulus is simplified:

$$k_r(t) = \frac{K_R(t)}{K_0} = \frac{1}{1+\varphi(t,t_0)} \qquad (17б)$$

4. VERIFICATION OF THE COMPUTATIONAL CREEP MODEL

To test the “viscoelastic” model, we used the results of experimental studies of two groups of concrete samples [11]. Each sample had the shape of a cylinder with a height of 0.3m and a diameter of 0.15m. Cylindrical samples subjected to prolonged axial compression. Samples of the 1-st group were loaded (σ_I = 10.67[MPa]) at the age of t_0 = 28 days, having on this day the modulus of elasticity $E_{(28)}$ = 36200[MPa]. Samples of the 2-nd group were loaded (σ_{II} = 2.47[MPa]) 1 day after their manufacture (t_0 = 1), having the elastic modulus $E_{(1)}$ = 22201[MPa]. When specifying the dimensionless relaxation moduli of the material using formulas (16*b*) and (17*b*) in the “viscoelastic” model, the concrete creep factor $\varphi(t, t_0)$ was calculated based on the experimental results. The figures 1 and 2 show the results of a comparison of the dependences of the creep strain in the samples obtained in the experiment and in numerical simulation using the “viscoelastic” model.

Testing the performance of the “viscoelastic” model with setting the values of the dimensionless moduli of material relaxation using formulas (16*b*) and (17*b*) with an abrupt change in load value in time gave quite satisfactory results (figure 3).

5. CONCLUSION

Accounting for material creep in the calculation of reinforced concrete structures of some types, such as flat arches and shells, can reveal a significant change in their stress-strain state over a long time. Below the difference in the results of the calculation in the traditional way and by deformed scheme of the flat reinforced concrete arch with a span of 20 m and a height of 2 m under a permanent load is shown.

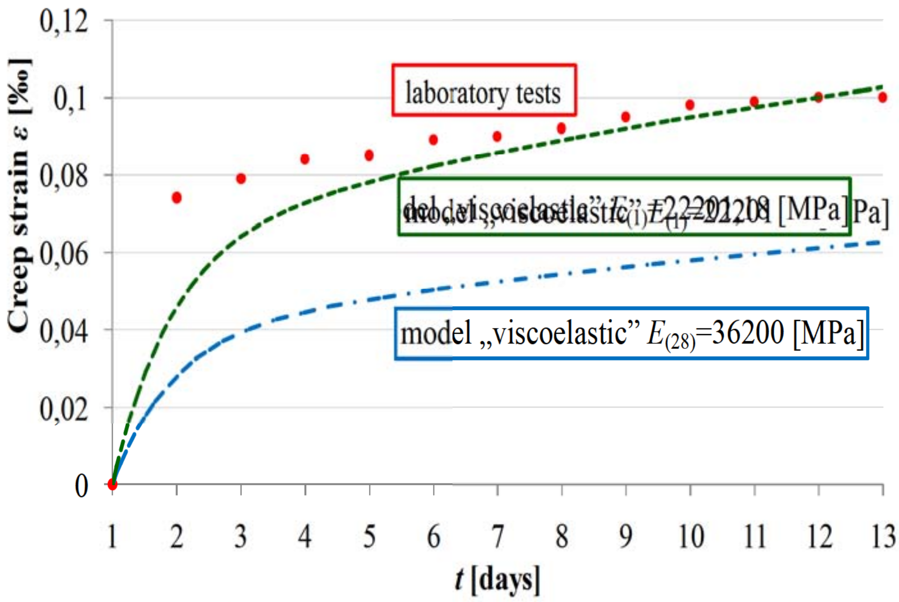

Figure 2. Comparison of the dependences of creep strain in samples of the 2-st group (t_0 = 1 day), obtained in the experiment and in numerical simulation.

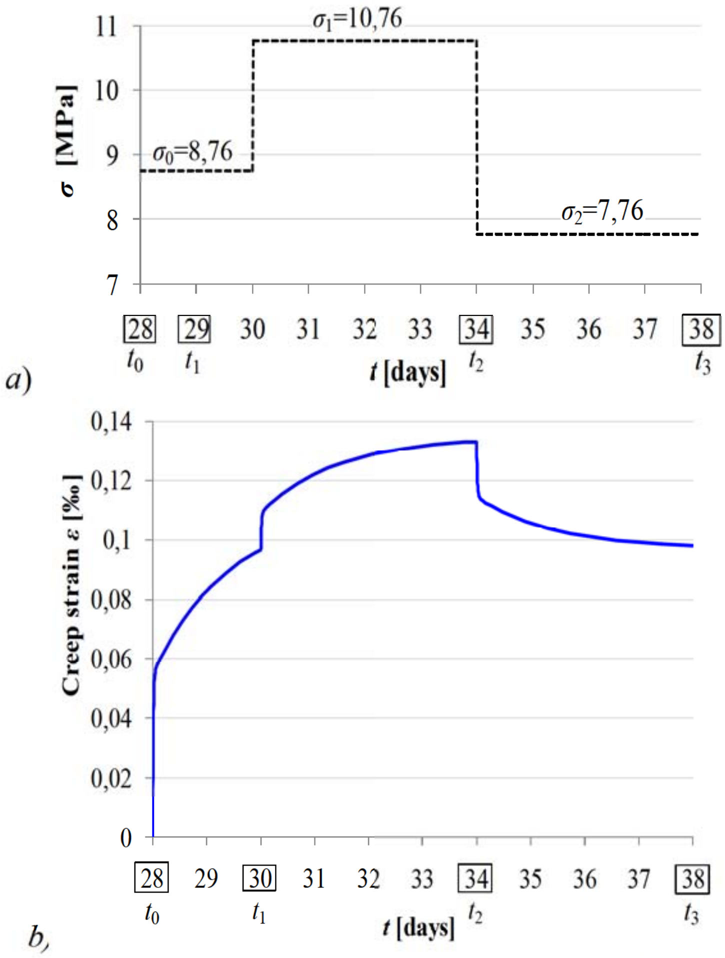

Figure 3. The change in creep strain in time with an abrupt change in load value: a) change in stress over time; b) change in creep strain in time.

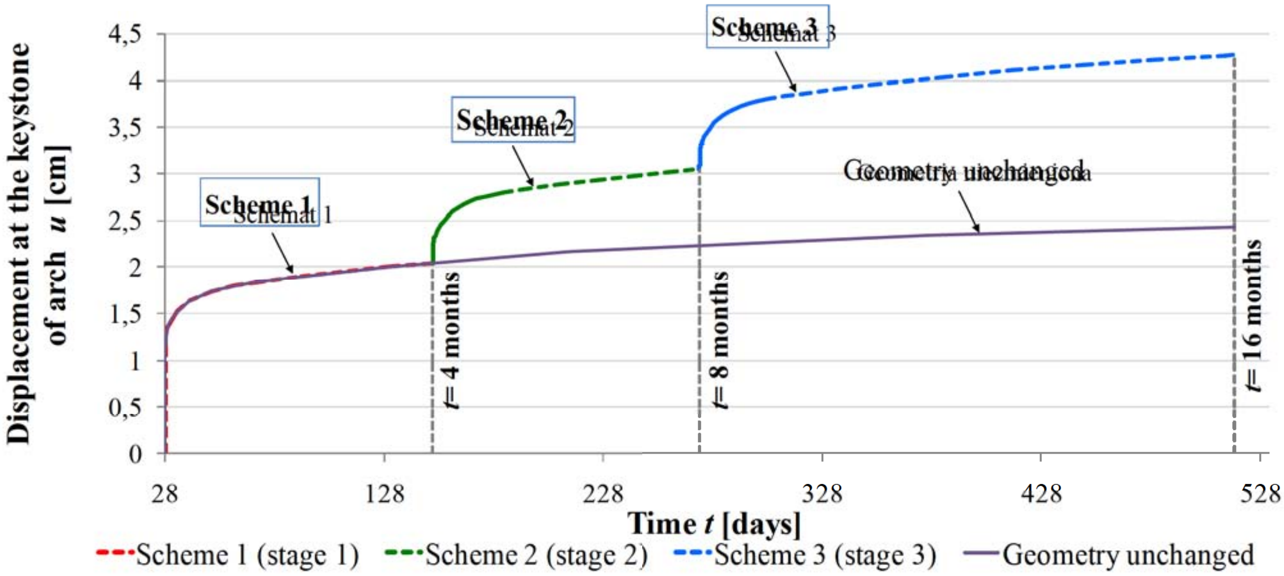

*Figure 4. The increase in the deflection of a gentle arch in the castle section **u** in time **t***
– at traditional time calculation (bottom line);
– when calculating by the deformed scheme (upper lines).

The arch was simulated in a time period of 16 months, taking into account the creep of the material. When calculating this construction, the "viscoelastic" concrete creep model calibrated in accordance with the requirements of [6] was implemented. When calculating by deformed scheme, the arch configuration was recalculated on the 4-th and 8-th month of loading. The difference in the deflections of the castle arch section obtained on the 516th day in two ways was 58 percent (figure 4).

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REFERENCES

1. **Arutjunjan N.H.**, **Zevin A.A.** Rastchjot Elementov Stroitelnych Konstrukcji s Utchjotom Polzutchesti [Raschet Jelementov Stroitel'nyh Konstrukcij s Uchetom Polzuchesti]. Moscow, Strojizdat, 1988, 256 pages (in Russian).
2. **Aleksandrovsky S.V.** Calculation of Concrete and Reinforced Concrete Structures for Changes in Temperature and Humidity Over Time [Raschet Betonnyh i Zhelezobetonnyh Konstrukcij na Izmenenija Temperatury i Vlazhnosti vo Vremeni]. Moscow, Strojizdat, 1973 (in Russian).
3. **Bondarenko V.M.**, **Bondarenko S.V.** Engineering Methods of the Nonlinear Theory

of Reinforced Concrete [Inzhenernye Metody Nelinejnoj Teorii Zhelezobetona]. Moscow, Stroiizdat, 1982, 287 pages. (in Russian).
4. **Prokopovich I.Ye.**, **Zedgenidze V.A.** Prikladnaja Teorija Polzuchesti [Applied Theory of Creep]. Moscow, Strojizdat, 1980, 240 pages (in Russian).
5. **Bazant Z.P.** Scaling of Structural Strength. London, Elsevier, 2005.
6. EN1992-1-1. Eurocode 2: Design of concrete structures, 2008.
7. PN-B-03264: Pre-norma fib Model Code. Concrete, Reinforced Concrete and Prestressed structures. Static Calculations and Design, 2010.
8. NIIZhB. Recommendations on Concrete Creep and Shrinkage Consideration when Calculating Concrete and Reinforced Concrete Structures [Rekomendacii po Uchetu Polzuchesti i Usadki Betona pri Raschete Betonnyh i Zhelezobetonnyh Konstrukcij]. Moscow, Stroiizdat, 1988, 124 pages (in Russian).
9. SNiP 52-01-2003. Construction norms and rules of the Russian Federation. Concrete and reinforced concrete structures. Principal rules [Betonnye i Zhelezobetonnye Konstrukcii. Osnovnye Polozhenija]. Moscow, Stroyizdat, 1988 (in Russian).
10. **Schiessel H.**, **Metzler R.**, **Blumen A.**, **Nonnenmacher T.F.** Generalized Viscoelastic Models: Their Fractional Equations with Solutions. // *Journal of Physics A: Mathematical and General*, 1995, No. 28, pp. 6567-6584
11. **Sidorov V.N.**, **Nowak K.** Substantiation of the Use of Viscoelastic Material Model in Numerical Analysis the Creep of Concrete Structures. // *IDT 2017 – Proceedings of the 2017 International Conference on Information and Digital Technologies*, 2017, pp. 339-343.
12. ABAQUS – Standard, User's Manual, Theory Manual and Keywords Manual, 19.7.1. Time domain viscoelasticity. USA, Boston: Hibbitt, Karlsson and Sorensen, Inc., 1306 pages.

СПИСОК ЛИТЕРАТУРЫ

1. **Аратюнян Н.Х.**, **Зевин А.А.** Расчет элементов строительных конструкций с учетом ползучести. – М.: Стройиздат, 1988. – 256 с.
2. **Александровский С.В.** Расчет бетонных и железобетонных конструкций на изменения температуры и влажности во времени. – М.: Стройиздат, 1973.
3. **Бондаренко В.М.**, **Бондаренко С.В.** Инженерные методы нелинейной теории железобетона. – М.: Стройиздат, 1982. – 287 с.
4. **Прокопович И.Е.**, **Зедгенидзе В.А.** Прикладная теория ползучести. – М.: Стройиздат, 1980. – 240 pages.
5. **Bazant Z.P.** Scaling of Structural Strength. London, Elsevier, 2005.
6. EN1992-1-1. Eurocode 2: Design of concrete structures, 2008.
7. PN-B-03264: Pre-norma fib Model Code. Concrete, Reinforced Concrete and Prestressed structures. Static Calculations and Design, 2010.
8. НИИЖБ. Рекомендации по учету ползучести и усадки бетона при расчете бетонных и железобетонных конструкций. М., Стройиздат, 1988. – 124 с.
9. Строительные нормы и правила РФ. СНиП 52-01-2003. Бетонные и железобетонные конструкции. Основные положения. М., 2004.
10. **Schiessel H.**, **Metzler R.**, **Blumen A.**, **Nonnenmacher T.F.** Generalized Viscoelastic Models: Their Fractional Equations with Solutions. // *Journal of Physics A: Mathematical and General*, 1995, No. 28, pp. 6567-6584
11. **Sidorov V.N.**, **Nowak K.** Substantiation of the Use of Viscoelastic Material Model in Numerical Analysis the Creep of Concrete Structures. // *IDT 2017 – Proceedings of*

the 2017 International Conference on Information and Digital Technologies, 2017, pp. 339-343.

12. ABAQUS – Standard, User's Manual, Theory Manual and Keywords Manual, 19.7.1. Time domain viscoelasticity. USA, Boston: Hibbitt, Karlsson and Sorensen, Inc., 1306 pages.

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