DOI:10.22337/2587-9618-2019-15-2-95-110

# ABOUT WAVELET-BASED COMPUTATIONAL BEAM ANALYSIS WITH THE USE OF DAUBECHIES SCALING FUNCTIONS

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**Abstract:** The first part of the distinctive paper contains brief review of wavelet-based numerical and semianalytical analysis, particularly with the use of Daubechies scaling functions. The second part of the paper is devoted to numerical solution of the problem of static analysis of beam on elastic foundation within Winkler model. Finite element method (FEM) and wavelet analysis (Daubechies scaling functions) are used. Variational formulation and approximation of the problem are under consideration. Numerical sample is presented as well. The third part of the paper is dedicated to wavelet based discrete-continual finite element method of beam analysis with allowance for impulse load. Daubechies scaling functions are used as well.

**Keywords:** boundary problem, structural analysis, static analysis, dynamic analysis, beam analysis, numerical solution, finite element method, wavelet analysis, Daubechies scaling function, Daubechies wavelet, impulse load, review

# О ВЕЙВЛЕТ-РЕАЛИЗАЦИЯХ ВЫЧИСЛИТЕЛЬНЫХ МЕТОДОВ РАСЧЕТА БАЛОЧНЫХ КОНСТРУКЦИЙ С ИСПОЛЬЗОВАНИЕМ МАСШТАБИРУЮЩИХ ФУНКЦИЙ ДОБЕШИ

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**Аннотация:** Настоящая статья в своей первой части содержит относительно краткий обзор вейвлет-реализаций численных и численно-аналитических методов решения краевых задач, в частности с использованием масштабирующих функций Добеши. Вторая часть статьи посвящена численному решению статической задачи об изгибе балки на упругом основании (в рамках модели Винклера). Для построения решения используются метод конечных элементов (МКЭ) в сочетании с аппаратом вейвлет-анализа (масштабирующие функции Добеши). Приведена вариационная (континуальная) формулировка задачи, описаны ее аппроксимация и построение численного решения, представлен пример расчета. В третьей части статьи описан вейвлет-реализация дискретно-континуального метода конечных элементов для расчета балки при ударе на основе использования масштабирующих функций Добеши.

**Ключевые слова:** краевая задача, расчеты строительных конструкций, статический расчет, динамический расчет, расчет балки, численное решение, метод конечных элементов, вейвлет-анализ, масштабирующая функция Добеши, вейвлет Добеши, ударная нагрузка, обзор

## 1. INTRODUCTION

As is known, the numerical analysis with wavelet (wavelet-based numerical analysis) received its first attention in 1992, since then (particularly since the corresponding basic work of I. Daubechies [1]) researchers have shown growing interest in it. Various methods including so-called wavelet weighted residual method, wavelet finite element method, wavelet-based numerical methods of local structural analysis [2-6], wavelet boundary method, wavelet meshless method, wavelet-optimized finite difference method, wavelet-based discrete-continual methods of local structural analysis [7-15], wavelet-based multigrid method [16] etc. have acquired an important role in recent years.

First of all, it should be noted that Daubechies scaling functions can be effectively employed within wavelet finite element method or within wavelet-based numerical and semianalytical (discrete-continual) methods of local structural analysis as asapproximate functions in the procedure of construction of so-called wavelet finite element [17].

Corresponding compact support property proves to be more effective in using minimum degrees of freedom over an element to approximate displacement functions. Moreover, sparseness of the matrix is a result of the scaling functions, which have the compactly supported property. Cancellation property allows one to perfectly interpolate polynomials of degree up to $N$ by the scaling function with order $N$. Corresponding experiments gathered in the wavelet-Galerkin context indicate that orthogonal property satisfies the condition that the matrix is sparse as well as banded if the global nodes are numbered sequentially. Dur to corresponding main properties, Daubechies wavelets can describe the details of the problem conveniently and accurately, and the corresponding Daubechies wavelets-based element has an enormous potential in the analysis of the singularity problem [17].

J. Ko, A.J. Kurdila and M.S. Pillant [18] developed special finite element method based on application of Daubehies wavelet. Corresponding algebraic eigenvalue problem derived from the dyadic refinement equation can be solved by this method. The resulting finite elements could be considered as several generalizations of the connection coefficients employed in the corresponding Daubechies wavelet expansion of periodic differential operators [17].

R.D. Patton and P.C. Marks [19] utilized a Daubechies scaling function as interpolation function of one-dimensional finite element. This element can reduce the computation time and reduce the number of degrees of freedom, which is normally needed for correct solution of vibration and wave propagation problems [17].

J.X. Ma and J.J. Xuee in paper [20] constructed one-dimensional Daubechies wavelet beam element [17].

X.F. Chen, S.J. Yang and J.X. Ma [21] extended such elements to higher dimensions, constructed two-dimensional Daubechies wavelet element, derived the corresponding bending equations for the thin plate based on wavelet finite element, and solved the L-shape plate stress problems. Their results show that wavelet finite element can be effectively used for solution of singularity problems [17].

However, the tensor product space should be constructed firstly [22], which decreases the computational effectiveness [17].

J.M. Jin, P.X. Xue, Y.X. Xu and Y.L. Shu [23] built a two-dimensional Daubechies wavelet directly without tensor product computation and developed corresponding two-dimensional plate element [17].

Nevertheless, the wavelet deflection formulation depends on specific boundary conditions. Besides it is effective only for homogeneous boundary conditions for square plates. In addition, only simple boundary conditions were considered in above mentioned works. Triggered by this motivation, a modified form of wavelet approximation of deflection solution was proposed for solution of bending problems of beams and square thin plates by Y.H. Zhou and J. Zhou [24]. Boundary rotational degrees of freedom for beams and square plates were explicitly introduced as Daubechies wavelet coefficients in this paper. Thus variation equations were established with the use of corresponding modified approximations and variation principles. Homogeneous and non-homogeneous boundary conditions can betreated in the same way (by analogy with corresponding versions of conventional FEM [17].

M. Mitra and S. Gopalakrishnan presented so-called Daubechies wavelet-based spectral finite element method (WSFEM) for analysis of elastic wave propagation in one-dimensional and two-dimensional connected wave guides [25-27]. First of all, this method transforms the initial partial differential wave equation to corresponding ordinary differential equations (ODEs) with the use of Daubechies wavelet approximation in time domain. Then these ODEs are solved within FEM by deriving the exact interpolating function in the transformed domain. Spectral element can capture the exact mass distribution. Therefor the system size required is very much smaller than the corresponding system size within the conventional FEM. Besides, due to the localized nature of Daubechies wavelet basis functions, the WSFEM proves to be more efficient as it removes the wrap around problem associated with spectral finite element method for time domain analysis.

M. Mitra and S. Gopalakrishnan [28] later extended the method and considered problems of analysis of composite beam with embedded delamination. However, the real-scale structural wave propagation problem requires more different complex spectral elements, interconnections and flexible in flatable components. In accordance with assessments from paper [17], future research work will focus on extending the spectral element method for analysis of damaged structures with more complex geometry [17].

Thus, Daubechies wavelets are used to approximate the displacement and force in the domain, where unknown wavelet coefficients can be determined through imposing the essential boundary condition. The Daubechies wavelet finite elements embodies the properties of locality and adaptivity. However, because Daubechies wavelets lack the explicit function expression, traditional numerical integrals such as Gaussian integrals cannot provide desirable precision. Therefor, the applications of Daubechies wavelets are limited by the this weakness [17].

# 1. WAVELET-BASED NUMERICAL BEAM ANALYSIS WITH THE USE OF DAUBECHIES SCALING FUNCTIONS

## 1.1. Mathematical (continual) formulation of the problem of static beam analysis.

The essence of the Winkler model is the assumption that the reaction of the foundation $r(x)$ at an arbitrary point of the beam $x$ is proportional to deflection at this point $r(x) = \beta y$. Therefore, graphically, such a model can be represented by springs that are not connected to each other, each of which has a stiffness proportional to the deflection of the beam at this point (Figure 1.1).

The stress-strain state of such a beam corresponds to the solution of the problem of the minimum of the following functional (energy functional) [29]:

$$\Phi(y) = \frac{1}{2}\int_0^l (EJ(y'')^2 + \beta y^2)dx - \int_0^l q(x)y dx, \quad (1.1)$$

where $EJ(x)$ is bending stiffness of beam; $\beta(x)$ is Winkler coefficient; $q(x)$ is applied load.

## 1.2. Wavelet-based finite element approximation of the problem of static beam analysis.

Let us divide domain (one-dimensional interval $(0,\ l)$), occupied by the beam into $N_e$ parts (finite elements);

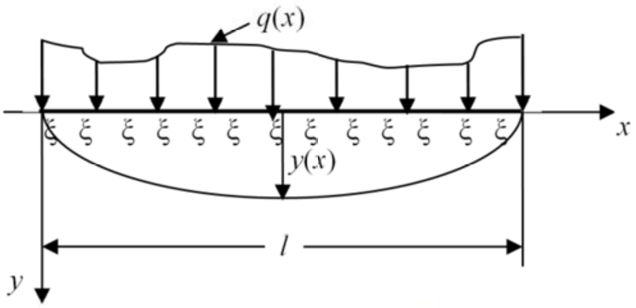


*Figure 1.1. Computational scheme of beam.*

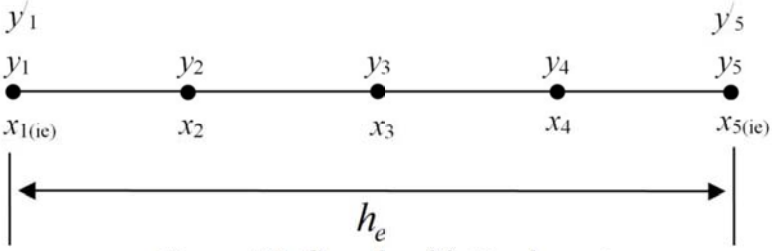


*Figure 1.2. Sample of finite element.*

$$h_e = l / N_e$$

is the length of the element.

Each element is also divided into $N_k$ parts, for example, $N_k = 4$ (Figure 1.2).

Let us introduce the following notation: $i_e$ is element number; $x_1(i_e)$ is coordinate of the starting point of the $i_e$-th element; $x_5(i_e)$ is coordinate of the end point of the $i_e$-th element. At the boundary points (nodes), we can choose unknowns $y_i$ and $y_i'$; at the inner points we can choose unknowns $y_i$, $i = 2,3,4$. Thus, the total number of unknowns on an element is equal to

$$N = N_k - 1 + 2 \cdot 2 = N_k + 3 = 7 .$$

The number of boundary points for all elements is equal to

$$N_b = N_e + 1 .$$

Besides, the number of interior points for all elements is equal to

$$N_p = N_e(N_k - 1) .$$

Thus, the total (global) number of unknowns is equal to

$$N_g = N_p + 2N_b .$$

We have

$$\Phi(y) = \sum_{i_e=1}^{N_e} \Phi_{i_e}(y) , \tag{1.2}$$

where

$$\Phi_{i_e}(y) = \frac{1}{2} \int_{x_1(i_e)}^{x_5(i_e)} (EJ(y'')^2 + \beta y^2) dx - \int_{x_1(i_e)}^{x_5(i_e)} qy dx . \tag{1.3}$$

Let us introduce the local coordinates:

$$t = (x - x_{1(i_e)}) / h_e , \quad x_{1(i_e)} \le x \le x_{5(i_e)} . \tag{1.4}$$

In this case, we have the following relations:

$$\begin{cases} x = x_{1(i_e)} \Rightarrow t = 0 \\ x = x_2 \Rightarrow t = 0.25 \\ x = x_3 \Rightarrow t = 0.5 \\ x = x_4 \Rightarrow t = 0.75 \\ x = x_{5(i_e)} \Rightarrow t = 1; \end{cases} \tag{1.5}$$

$$\frac{d}{dx}=\frac{d}{dt}\cdot\frac{dt}{dx}=\frac{1}{h_e}\frac{d}{dt};\qquad \frac{d^p}{dx^p}=\frac{1}{h_e^p}\frac{d^p}{dt^p}; \tag{1.6}$$

$$dx=h_e\cdot dt. \tag{1.7}$$

We can represent displacement (deflection) of beam $y(x)$ in the form

$$y(x)=w(t)=\sum_{k=0}^{N}\alpha_k\varphi(t+k),\ x_{1(i_e)}\le x\le x_{5(i_e)}, \tag{1.8}$$

where $\varphi(s)$ is Daubechies scaling function, $[0, N]\subseteq \mathrm{supp}(\varphi)$.

We substitute (1.8) into (1.2), taking into account relations (1.5)-(1.7).

$$\begin{aligned}\Phi_{i_e}(y)&=\frac{1}{2}\int_{x_1(i_e)}^{x_5(i_e)}\left(EJ\left(\frac{d^2y}{dx^2}\right)^2+\beta y^2\right)dx-\\&-\int_{x_1(i_e)}^{x_5(i_e)}qydx=\frac{1}{2}\int_0^1\left(\frac{EJ}{h_e^3}(w'')^2+\beta h_e w^2\right)dt-\\&-\int_0^1 h_e qwdt=\frac{1}{2}\sum_{i=0}^{N}\sum_{j=0}^{N}\alpha_i\alpha_j\times\\&\times\int_0^1\left(\frac{EJ}{h_e^3}(\varphi''(t+i)\varphi''(t+j))+\beta h_e(\varphi(t+i)\varphi(t+j))\right)dt-\\&-\sum_{i=0}^{N}\alpha_i\int_0^1 h_e q\varphi(t+i)dt=\\&=\frac{1}{2}(K_\alpha^{i_e}\overline{\alpha},\overline{\alpha})-(\overline{R}_\alpha^{i_e},\overline{\alpha})=\Phi_\alpha(\overline{\alpha})\end{aligned}$$

and finally

$$\Phi_{i_e}(y)=\Phi_\alpha(\overline{\alpha})=\frac{1}{2}(K_\alpha^{i_e}\overline{\alpha},\overline{\alpha})-(\overline{R}_\alpha^{i_e},\overline{\alpha}), \tag{1.9}$$

where

$$K_\alpha^{i_e}(i,j)=\\=\int_0^1\left(\frac{EJ}{h_e^3}(\varphi''(t+i)\varphi''(t+j))+\beta h_e(\varphi(t+i)\varphi(t+j))\right)dt; \tag{1.10}$$

$$R_\alpha^{i_e}(i)=\int_0^1(h_e q(t)\varphi(t+i))dt. \tag{1.11}$$

We can define the parameters $\alpha_k$ through the nodal unknowns on the element:

$$\begin{cases} y_1=w(0)=\sum_{k=0}^{N}\alpha_k\varphi(k)\\ \frac{dy_1}{dx}=\frac{1}{h_e}w'(0)=\frac{1}{h_e}\sum_{k=0}^{N}\alpha_k\varphi'(k)\\ y_2=w(0.25)=\sum_{k=0}^{N}\alpha_k\varphi(k+0.25)\\ y_3=w(0.5)=\sum_{k=0}^{N}\alpha_k\varphi(k+0.5)\\ y_4=w(0.75)=\sum_{k=0}^{N}\alpha_k\varphi(k+0.75)\\ y_5=w(1)=\sum_{k=0}^{N}\alpha_k\varphi(k+1)\\ \frac{dy_5}{dx}=\frac{1}{h_e}w'(1)=\frac{1}{h_e}\sum_{k=0}^{N}\alpha_k\varphi'(k+1).\end{cases} \tag{1.12}$$

Thus, we have

$$\overline{y}^{i_e}=T\overline{\alpha}, \tag{1.13}$$

where

$$\overline{y}^{i_e}=[y_1\ \frac{dy_1}{dx}\ y_2\ y_3\ y_4\ y_5\ \frac{dy_5}{dx}]^T; \tag{1.14}$$

$$\overline{\alpha}=[\alpha_0\ \alpha_1\ \alpha_2\ \alpha_3\ \alpha_4\ \alpha_5\ \alpha_6]^T; \tag{1.15}$$

$$D=diag(1\quad 1/h_e\quad 1\quad 1\quad 1\quad 1\quad 1/h_e); \tag{1.16}$$

$T$ is matrix, which is defined in accordance with the following formula:

$$T = D\begin{bmatrix} \varphi(0) & \varphi(1) & \varphi(2) & \varphi(3) & \varphi(4) & \varphi(5) & \varphi(6) \\ \varphi'(0) & \varphi'(1) & \varphi'(2) & \varphi'(3) & \varphi'(4) & \varphi'(5) & \varphi'(6) \\ \varphi(0.25) & \varphi(1.25) & \varphi(2.25) & \varphi(3.25) & \varphi(4.25) & \varphi(5.25) & \varphi(6.25) \\ \varphi(0.5) & \varphi(1.5) & \varphi(2.5) & \varphi(3.5) & \varphi(4.5) & \varphi(5.5) & \varphi(6.5) \\ \varphi(0.75) & \varphi(1.75) & \varphi(2.75) & \varphi(3.75) & \varphi(4.75) & \varphi(5.75) & \varphi(6.75) \\ \varphi(1) & \varphi(2) & \varphi(3) & \varphi(4) & \varphi(5) & \varphi(6) & \varphi(7) \\ \varphi'(1) & \varphi'(2) & \varphi'(3) & \varphi'(4) & \varphi'(5) & \varphi'(6) & \varphi'(7) \end{bmatrix}; \tag{1.17}$$

Then we have

$$\overline{\alpha} = T^{-1}\overline{y}^{i_e}. \tag{1.18}$$

Taking into account (1.9) and (1.18) we get

$$\begin{aligned}\Phi_\alpha(\overline{\alpha}) &= \frac{1}{2}(K_\alpha^{i_e}T^{-1}\overline{y}^{i_e}, T^{-1}\overline{y}^{i_e}) - (\overline{R}_\alpha^{i_e}, T^{-1}\overline{y}^{i_e}) = \\ &= \frac{1}{2}((T^{-1})^{\mathrm{T}}K_\alpha^{i_e}T^{-1}\overline{y}^{i_e}, \overline{y}^{i_e}) - ((T^{-1})^{\mathrm{T}}\overline{R}_\alpha^{i_e}, \overline{y}^{i_e}) = \\ &= \frac{1}{2}(K^{i_e}\overline{y}^{i_e}, \overline{y}^{i_e}) - (\overline{R}^{i_e}, \overline{y}^{i_e}) = \Phi_{i_e}(\overline{y}^{i_e})\end{aligned}$$

and finally

$$\begin{aligned}\Phi_\alpha(\overline{\alpha}) &= \Phi_{i_e}(\overline{y}^{i_e}) = \\ &= \frac{1}{2}(K^{i_e}\overline{y}^{i_e}, \overline{y}^{i_e}) - (\overline{R}^{i_e}, \overline{y}^{i_e}),\end{aligned} \tag{1.19}$$

where $K^{i_e}$ is local stiffness matrix; $\overline{R}^{i_e}$ is local load vector;

$$K^{i_e} = (T^{-1})^{\mathrm{T}}K_\alpha^{i_e}T^{-1}; \quad \overline{R}^{i_e} = (T^{-1})^{T}\overline{R}_\alpha^{i_e}. \tag{1.20}$$

**1.3. Numerical implementation and sample of static beam analysis.**

The presented algorithm can be implemented using the tools of MATLAB. In particular, the reference to the standard function

```
wavefun('db10',0)
```

allows researcher to get the values of the scaling Daubeshi function $\varphi$ on the interval $[0,19] = \mathrm{supp}(\varphi)$ with steps $h_t = 1/256 = 2^{-8}$. Let us denote $N_t = 256 = 2^8$. For the considering value $N = 7$ we can use the first $N_l = N_t \cdot N + 1$ values of $\varphi$, defined on the segment $[0, N] = [0, 7]$. With such a small step, it will be natural to compute the derivatives in the form of finite differences:

$$\varphi'(t_k) \approx d\varphi_k = \frac{\varphi_{k+1} - \varphi_{k-1}}{2h_t}, \quad k = 1, 2, \ldots, N_l; \tag{1.21}$$

$$\varphi''(t_k) \approx d2\varphi_k = \frac{\varphi_{k+1} - 2\varphi_k + \varphi_{k-1}}{h_t^2}, \quad k = 1, 2, \ldots, N_l, \tag{1.22}$$

where $\varphi_k = \varphi(t_k); \quad t_k = k \cdot h_t.$ (1.23)

If $t_k \notin [0, 19]$ then $\varphi_k = \varphi(t_k) = 0$.

When computing the coefficients of the local stiffness matrix (formulas (10) and (20)), one can use the simplest quadrature formulas for numerical integration, in particular, midpoint quadrature rule with step $2h_t$.

Let us consider (as a model sample) analysis of beam on an elastic foundation with the following parameters: $q(x) = P\delta(x - L/2)$, $P = 100$ kN is the load specified at the midpoint (Figure 6); $L = 4$ m; $h_b = 1.3$ m; $b_b = 1$ m; $E = 2560 \cdot 10^4$ kN/m$^2$; $k = 75 \cdot 10^3$ kN/m$^3$.

In this case, we can consider the following boundary conditions:

$$\begin{cases} y(0) = y''(0) = 0 \\ y(L) = y''(L) = 0. \end{cases} \tag{1.24}$$

Thus, we consider beam hinged on both sides (simply supported beam).

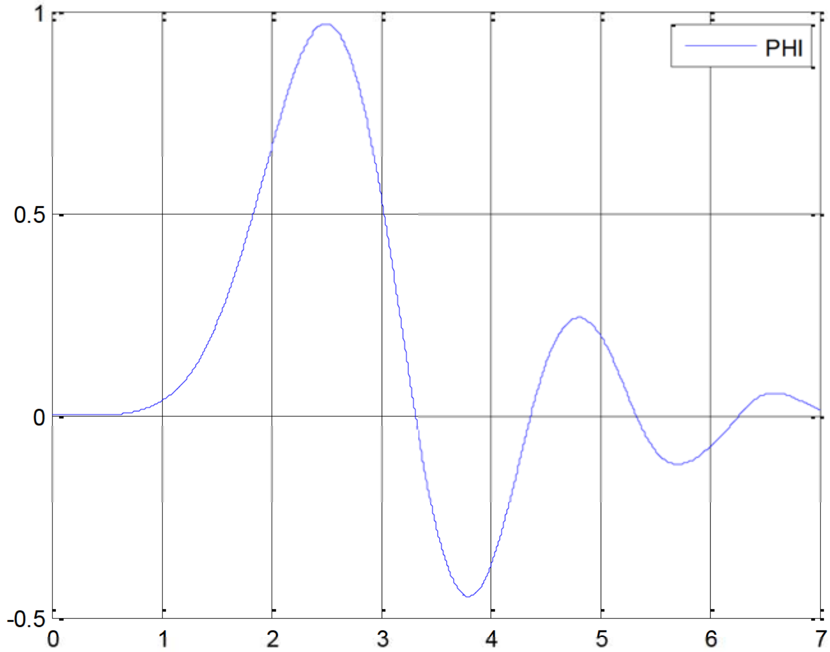


*Figure 1.3. Daubechies scaling function.*

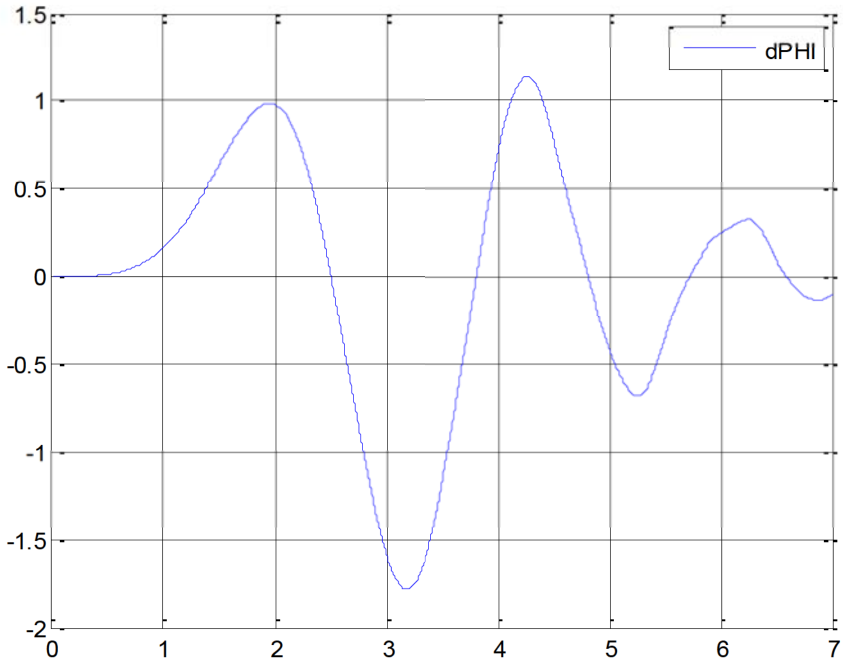


*Figure 1.4. The first-order difference derivative of Daubechies scaling function.*

Let $N_e = 8$ be a number of elements. Then the total number of unknowns is equal to

$$N_g = N_p + 2N_b = 3 \cdot 8 + 2 \cdot (8+1) = 42 .$$

The length of the element is defined by formula

$$h_e = L / N_e = 4/8 = 0.5 .$$

Distance between coordinates of nodes (step) is equal to

$$h_p = h_e / 4 = 1/8 = 0.125 .$$

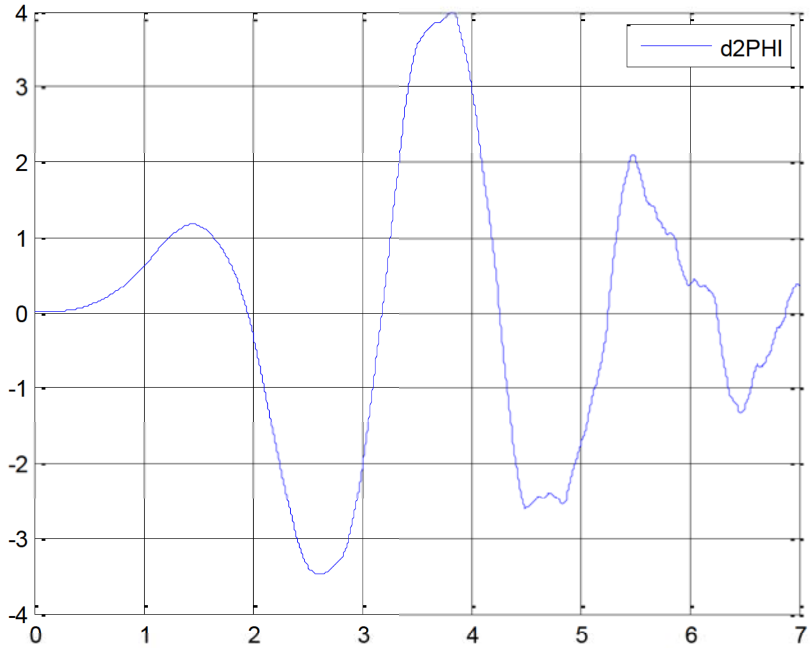


*Figure 1.5. The second-order difference derivative of Daubechies scaling function.*

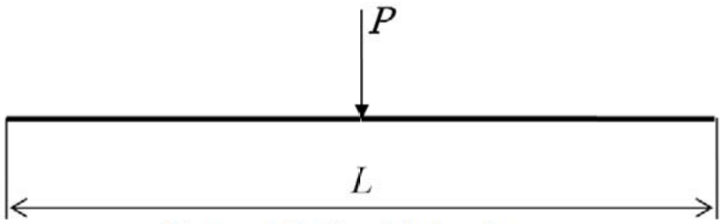


*Figure 1.6. Considering beam.*

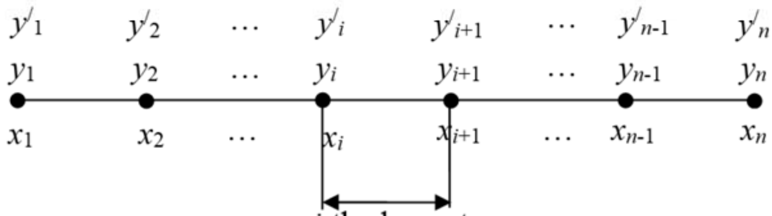


*Figure 1.7. Finite element discretization of beam (based on cubic parabola).*

As is known, for comparison, we can use the conventional finite element method, where the unknown function of the deflection on the element is represented as a cubic parabola. In this case, the finite element discretization is shown in Figure 7.

Adequate beam approximation requires 32 elements. In this case, the total number of unknowns is equal to

$$N_g = 2 \cdot (32 + 1) = 66 \, .$$

Graphical comparison of results of analysis is shown in Figure 8. The following notation is used: `Ydb` is the result obtained using Daubechies scaling function; `Yfem` is the result obtained on the basis of a cubic parabola.

As is obvious, the results obtained are almost the same. However, the wavelet-based algorithm of the finite element method based on the Daubechies scaling function leads to a decrease in the number of unknowns.

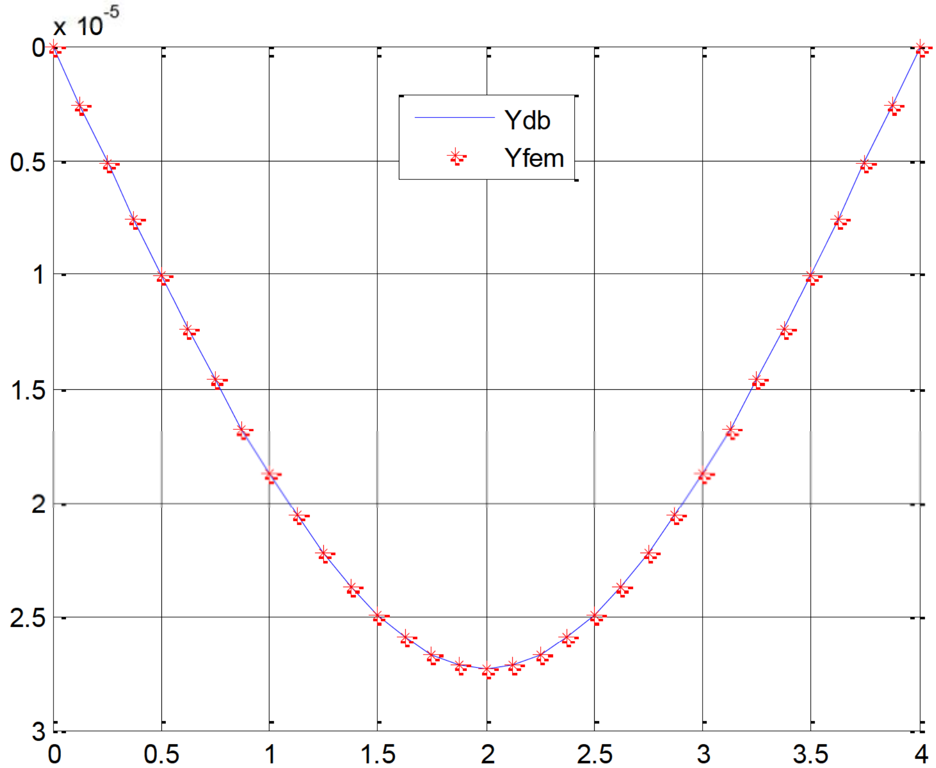


*Figure 1.8. Comparison of results of analysis.*

## 2. WAVELET-BASED SEMIANALYTICAL BEAM ANALYSIS WITH THE USE OF DAUBECHIES SCALING FUNCTIONS

### 2.1. Mathematical (continual) formulation of the problem of dynamic beam analysis.

Let us consider the problem of dynamic beam analysis (Figure 2.1). Corresponding impulse load is applied in the middle of the beam. Mathematical formulation of the problem has the form:

$$\begin{cases} \dfrac{\partial^2 y}{\partial t^2} = -\beta_0 \dfrac{\partial^4 y}{\partial x^4} + F(x,t), \\ 0 < x < \ell, \quad t > 0, \\ \left.\begin{array}{l} y(0,t) = y(\ell,t) = 0 \\ y''(0,t) = y''(\ell,t) = 0 \end{array}\right\}, \ t \geq 0 \\ \left.\begin{array}{l} y(x,0) = y_0(x) = 0 \\ \dfrac{\partial y}{\partial t}(x,0) = y_t'(x) = 0 \end{array}\right\}, \ 0 \leq x \leq \ell, \end{cases} \qquad (2.1)$$

where $y(x,t)$ is the deflection of a beam at a point $x$ at a time $t$; $x$ is the coordinate along the length of the beam, $0 \leq x \leq \ell$; $t$ is time coordinate, $t \geq 0$; $\beta_0 = EJ/\rho$; $EJ$ is bending stiffness of beam; $\rho$ is density of the beam material; $F(x,t) = P \cdot \delta(x-\ell/2)\delta(t)$ is function simulating the transverse impact of the impact on the beam at a point; $\delta(x-\ell/2)$ and $\delta(t)$ are Dirac delta functions.

### 2.2. Wavelet-based discrete-continual approximation of the problem of dynamic beam analysis.

Discere-continual finite element method (discrete-analytical approach) is used for solution of the considering problem. Within this method we use finite element approximation along the $x$ axis, and a continual problem is considered along the time axis $t$.

Let us divide domain (one-dimensional interval $(0,\ l)$), occupied by the beam into $N_e$ parts (finite elements);

$$h_e = l / N_e$$

is the length of the element.
Each element is also divided into $N_k$ parts, for example, $N_k = 4$.
Once again we can use the following notation: $i_e$ is element number; $x_1(i_e)$ is coordinate of the starting point of the $i_e$-th element; $x_5(i_e)$ is coordinate of the end point of the $i_e$-th element. At the boundary points (nodes), we can choose unknowns $y_i$ and $y_i'$; at the inner points we can choose unknowns $y_i$, $i = 2,3,4$. Thus, the total number of unknowns on an element is equal to

$$N = N_k - 1 + 2 \cdot 2 = N_k + 3 = 7 .$$

The number of boundary points for all elements, the number of interior points for all elements and the total number of unknowns are equal to

$$N_b = N_e + 1; \quad N_p = N_e(N_k - 1); \\ N_g = N_p + 2N_b .$$

We can also introduce the local coordinates

$$t = \frac{x - x_{1(i_e)}}{h_e}, \quad x_{1(i_e)} \le x \le x_{5(i_e)} \tag{2.2}$$

with corresoinding relations

$$\begin{cases} x = x_{1(i_e)} \Rightarrow t = 0 \\ x = x_2 \Rightarrow t = 0.25 \\ x = x_3 \Rightarrow t = 0.5 \\ x = x_4 \Rightarrow t = 0.75 \\ x = x_{5(i_e)} \Rightarrow t = 1; \end{cases} \tag{2.3}$$

$$\begin{aligned} &\frac{d}{dx} = \frac{d}{dt} \cdot \frac{dt}{dx} = \frac{1}{h_e} \frac{d}{dt}; \\ &\frac{d^p}{dx^p} = \frac{1}{h_e^p} \frac{d^p}{dt^p}; \end{aligned} \tag{2.4}$$

$$dx = h_e \cdot dt . \tag{2.5}$$

We can represent displacement (deflection) of beam $y(x,t)$ for a given $t$ in the form

$$y(x,\cdot) = w(q) = \sum_{k=0}^{N-1} \alpha_k \varphi(q + k), \; x_{1(i_e)} \le x \le x_{5(i_e)}, \tag{2.6}$$

where $\varphi(s)$ is Daubechies scaling function, $[0, N] \subseteq \mathrm{supp}(\varphi)$.
We substitute (2.6) into the quadratic part of the corresponding energy functional, taking into account relations (2.3)-(2.5). Then we have

$$\begin{aligned} \int_{x_1(i_e)}^{x_5(i_e)} \left( \frac{d^2 y}{dx^2} \right)^2 dx &= \frac{1}{h_e^3} \int_0^1 (w'')^2 dq = \\ &= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{1}{h_e^3} \alpha_i \alpha_j \int_0^1 (\varphi''(q+i)\varphi''(q+j)) dq = \\ &= (K_\alpha^{i_e} \overline{\alpha}, \overline{\alpha}) \end{aligned} \tag{2.7}$$

We can define the parameters $\alpha_k$ through the corresponding nodal unknowns on the element $y_i = y_i(t) = y(x_i, t)$:

$$\begin{cases} y_1 = w(0) = \sum_{k=0}^{N} \alpha_k \varphi(k) \\ \dfrac{dy_1}{dx} = \dfrac{1}{h_e} w'(0) = \dfrac{1}{h_e} \sum_{k=0}^{N} \alpha_k \varphi'(k) \\ y_2 = w(0.25) = \sum_{k=0}^{N} \alpha_k \varphi(k + 0.25) \\ y_3 = w(0.5) = \sum_{k=0}^{N} \alpha_k \varphi(k + 0.5) \\ y_4 = w(0.75) = \sum_{k=0}^{N} \alpha_k \varphi(k + 0.75) \\ y_5 = w(1) = \sum_{k=0}^{N} \alpha_k \varphi(k + 1) \\ \dfrac{dy_5}{dx} = \dfrac{1}{h_e} w'(1) = \dfrac{1}{h_e} \sum_{k=0}^{N} \alpha_k \varphi'(k + 1). \end{cases} \tag{2.8}$$

Thus, we have

$$\overline{y}^{i_e} = T\overline{\alpha} , \tag{2.9}$$

$$T = D\begin{bmatrix} \varphi(0) & \varphi(1) & \varphi(2) & \varphi(3) & \varphi(4) & \varphi(5) & \varphi(6) \\ \varphi'(0) & \varphi'(1) & \varphi'(2) & \varphi'(3) & \varphi'(4) & \varphi'(5) & \varphi'(6) \\ \varphi(0.25) & \varphi(1.25) & \varphi(2.25) & \varphi(3.25) & \varphi(4.25) & \varphi(5.25) & \varphi(6.25) \\ \varphi(0.5) & \varphi(1.5) & \varphi(2.5) & \varphi(3.5) & \varphi(4.5) & \varphi(5.5) & \varphi(6.5) \\ \varphi(0.75) & \varphi(1.75) & \varphi(2.75) & \varphi(3.75) & \varphi(4.75) & \varphi(5.75) & \varphi(6.75) \\ \varphi(1) & \varphi(2) & \varphi(3) & \varphi(4) & \varphi(5) & \varphi(6) & \varphi(7) \\ \varphi'(1) & \varphi'(2) & \varphi'(3) & \varphi'(4) & \varphi'(5) & \varphi'(6) & \varphi'(7) \end{bmatrix}; \tag{2.10}$$

$$\bar{y}^{i_e} = [y_1 \ \frac{dy_1}{dx} \ y_2 \ y_3 \ y_4 \ y_5 \ \frac{dy_5}{dx}]^T; \tag{2.11}$$

$$\bar{\alpha} = [\alpha_0 \ \alpha_1 \ \alpha_2 \ \alpha_3 \ \alpha_4 \ \alpha_5 \ \alpha_6]^T; \tag{2.12}$$

$$D = diag(1 \quad 1/h_e \quad 1 \quad 1 \quad 1 \quad 1 \quad 1/h_e). \tag{2.13}$$

Then we have

$$\bar{\alpha} = T^{-1}\bar{y}^{i_e}. \tag{2.14}$$

Substituting (2.14) into (2.9), we get

$$(K_\alpha^{i_e} T^{-1}\bar{y}^{i_e}, T^{-1}\bar{y}^{i_e}) = (K^{i_e}\bar{y}^{i_e}, \bar{y}^{i_e}), \tag{2.15}$$

where $$K^{i_e} = (T^{-1})^{\mathrm{T}} K_\alpha^{i_e} T^{-1} \tag{2.16}$$

is local stiffness matrix.

We can use the simplest quadrature formulas of numerical integration for computing of coefficients of the local stiffness matrix.

Let us denote

$$\bar{y}(t) = [\,y_1(t) \quad y_2(t) \quad \ldots \quad y_{N_g}(t)]^T. \tag{2.17}$$

We can obtain the resultant system of finite element equations in the matrix form

$$\begin{cases} \bar{y}''(t) = -A\bar{y} + \bar{F} \\ \left.\begin{aligned} \bar{y}(0) = \bar{y}_0 \\ y'(0) = \bar{y}'_0 \end{aligned}\right\}, \end{cases} \tag{2.18}$$

where $A$ is global stiffness matrix.

The matrix $A$ is positive definite. The general solution of the problem (2.18) has the form:

$$\begin{aligned} \bar{y}(t) &= \cos(\sqrt{A}\,t)\bar{y}_0 + \\ &+ \sqrt{A^{-1}}\sin(\sqrt{A}\,t)\bar{y}'_0 + \\ &+ \sqrt{A^{-1}}\int_0^t \sin\sqrt{A}(t-\tau)\bar{F}(\tau)d\tau. \end{aligned} \tag{2.19}$$

In accordance with formulation of the considering problem we have

$$F(x,t) = P\cdot\delta(x-\ell/2)\delta(t) \tag{2.20}$$

and consequently

$$\bar{F}(t) = \delta(t)\cdot\bar{F}_0; \tag{2.21}$$

$$\bar{F}_0(i) = P\cdot\begin{cases} 1, & i = (N_g+1)/2 \\ 0, & i \neq (N_g+1)/2. \end{cases} \tag{2.22}$$

Substituting (2.22) into (2.19) and taking into account the initial conditions, we obtain the final form of the solution of the problem:

$$\bar{y}(t) = \sqrt{A^{-1}}\sin(\sqrt{A}\,t)\bar{F}_0. \tag{2.23}$$

## ACKNOWLEDGMENTS

The distinctive research work was carried out at the expense of the State program of the Russian Federation “Scientific and technological development of the Russian Federation”, State program of the Russian Federation “Development of science and technology” for 2013-2020 and the Program for Fundamental Research of State Academies of Science for 2013–2020, as part of the Plan for Fundamental Scientific Research of the Ministry of Construction and Housing and Communal Services of the Russian Federation and the Russian Academy of Architecture and Construction Sciences for 2018, within science topic 7.4.2 "Development and numerical implementation of methods for determining the stress-strain state of spatial slab-shell reinforced concrete structures, taking into account physical non-linearity, cracking and acquired anisotropy" and science topic 7.4.17 "Research and development of adaptive mathematical models, numerical and numerical-analytical methods as the basis and component of monitoring systems of bearing structures of unique buildings and constructions".

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