DOI:10.22337/2587-9618-2019-15-2-14-23

NUMERICAL SIMULATION OF LAMB WAVE PROPOGATION IN ISOTROPIC LAYER

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Abstract: Propagation of Lamb waves in an elastic isotropic layer is studied by analytical and numerical methods. The influence of numerical simulation parameters on the solution stability is analyzed. The results of the finite element modeling and the analytical solution are compared.

Keywords: Numerical simulation, finite element model, Lamb waves, isotropic layer, polarization

ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ РАСПРОСТРАНЕНИЯ ВОЛН ЛЭМБА В ИЗОТРОПНОМ СЛОЕ

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Аннотация: С помощью численных методов исследуется распространение волн Лэмба в упругом изотропном слое. Исследуется влияние параметров численного моделирования на устойчивость решения. Сравниваются результаты конечно-элементного моделирования и аналитического решения.

Ключевые слова: численное моделирование, конечно-элементная модель, волны Лэмба, изотропный слой, поляризация

1. INTRODUCTION

Acoustic methods are widely used in the NDT for determining the physical and mechanical properties of materials in aerospace, civil and echanical engineering, and geophysics, to ensure safety, reliability, and precision. These methods allow constructing the dispersion relations that connect the phase velocity of the wave with frequency.

The first theoretical studies described by Rayleigh in [1-4], by Lamb in [5-7]. Later on, theoretical and experimental studies are described in [8-21].

Lamb waves are particular interest. They have an elliptical polarization in the sagittal plane and can penetrate on all thickness of the layer. The three-dimensional formalism [22] and six-dimensional formalisms [13, 23-26] were developed for analysing Lamb waves propagating in anisotropic plates. Experimental studies of Lamb waves propagation are very expensive and it requires the participation of high quality experts. Available theoretical methods for analysis the propagation of Lamb waves in the media are limited. Solution to the problem of Lamb waves propagation with using finite-element modeling is necessary to study.

The finite element method (FEM) used in the algorithms of numerical complexes, such as ABAQUS, ANSYS, NASTRAN and it used in various branches of science and technology.

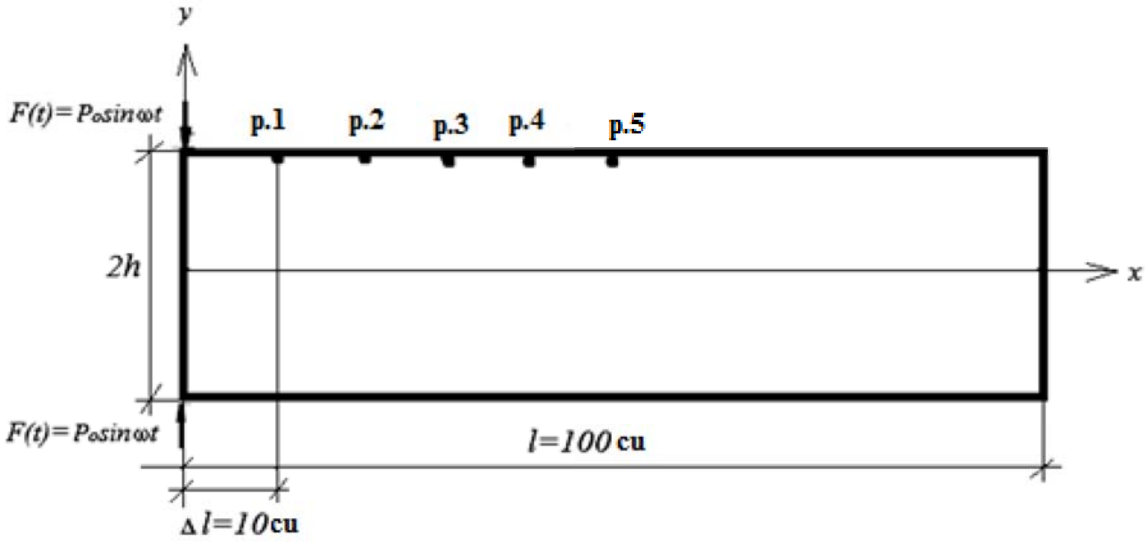

Figure 1. Schematic representation of the problem.

The main ideas of the FEM are given in [27]. FEM in problems of solid mechanics is described in [28, 29].

The use of various finite element complexes for solution problems of Lamb waves propagation will facilitate experimental diagnostics and processing of results. A comparison of the experimental dispersion relations with those found numerically will make to determine the properties of any layer. Of course, numerical modeling cannot replace experimental studies, but it can to help them. FEM integration in experimental diagnostics is an actual scientific technical problem.

Also the problem of Lamb waves polarization is interest since it was not studied in detail before.

The basic concepts of harmonic waves polarization are discussed in [30]. The results of analytical studies of surface waves polarization are given in [31]. In [32] the elastic waves polarization in the layer-elastic half-space system were considered. In this paper, we study the Lamb waves polarization in a layer by the finite element method.

2. FORMULATION OF THE PROBLEM

The isotropic elastic layer thickness in this study is $2h$ with boundaries $\mathbf{x} = \pm h$ (Figure 1). Harmonic in time concentrate force is applied to the layer, as a results longitudinal and transverse waves radiate out from the point of load application. Displacements in the x-direction correspond to longitudinal waves with velocity c_p, and the displacements in the y-direction correspond to vertical shear waves with velocity c_s. Movements in the z-direction is not included.

Finite element modeling and subsequent calculation has been conducted in finite element complex Abaqus®.

Analytical and finite element calculations were performed at density $\rho = 1$, $c_p = 1$, $h = 1$ cu, when the Poisson ratio ranges from 0 to 0.5, where cu is conventional unit (here in after it is assumed that all physical quantities are dimensionless).

The finite element model consists of rectangular 4-node linear elements. Results were obtained for 5 observation points (p.1, p.2, ..., p.5) located at intervals of 10 cu.

3. MESH CONVERGENCE

Finite element modeling of Lamb waves propagation has certain difficulties associated with the stability of difference schemes. So it is a very small amount of work on finite-element modeling of Lamb wave propagation in layer.Finite-element programs use one of two difference schemes: explicit

$$x_{i+1} = x_i + \Delta t \cdot F(x_i, t), \quad (3.1)$$

or implicit

$$x_{i+1} = x_i + \Delta t \cdot F(x_{i+1}, t), \quad (3.2)$$

where x is unknown variable value, Δt is time step, i is integration step number l.

Explicit finite difference schemes used in this study, this scheme allows to analyze all the main effects that occur during the propagation of acoustic waves. For this purpose the module Abaqus/Explicit is used in Abaqus®. To obtain a stable solution we used an approximate method, known as the Courant Criterion [33].

For this criterion time step must satisfy the condition

$$\Delta t < \Delta x_{\min} / c_p, \quad (3.3)$$

where Δt is the time step, Δx is the smallest element dimension, c_p is longitudinal wave velocity.

The magnitude of the amplitude of the impact P_0 is important. For the condition of small deformations it is necessary that the value P_0 satisfies the condition

$$P_0 < 1 \cdot 10^{-4} \cdot E \cdot \Delta x, \quad (3.4)$$

where E – dimensionless modulus of elasticity.

The study of the influence of P_0 and FEM element dimension Δx on the accuracy of the solution was carried out for two values of the dimensionless circular frequency:

$$\omega = 0.397657 \text{ and } \omega = 1.5016813 .$$

Poisson's ratio of the layer was taken to be $\nu = 0.35$.

Results were obtained for 2 observation points spaced 10 cu and 20 cu from the point of load application. The element dimension ranges from 0.005 to 0.5. Results for

$$P_0 > 1 \cdot 10^{-3} \cdot E \cdot \Delta x, \ \omega = 0.397657$$

are shown in the Figures 2-a and 2-b. Results for

$$P_0 < 1 \cdot 10^{-4} \cdot E \cdot \Delta x, \ \omega = 0.397657$$

are shown in the Figures 2-c and 2-d.

From the Figure 2 follows that when

$$\omega = 0.397657$$

accurate solution is achieved when $\Delta x \le 0.05$. In Figure 3 results are shown for the following cases:

$$P_0 > 1 \cdot 10^{-3} \cdot E \cdot \Delta x,$$
$$P_0 \in \left(1 \cdot 10^{-3} \cdot E \cdot \Delta x; 1 \cdot 10^{-4} \cdot E \cdot \Delta x\right),$$
$$P_0 < 1 \cdot 10^{-4} \cdot E \cdot \Delta x.$$

When $\Delta x = 0.05$ there are

$$P_0 > 3.1154 \cdot 10^{-5},$$
$$P_0 \in \left(3.1154 \cdot 10^{-5}; 3.1154 \cdot 10^{-6}\right),$$
$$P_0 < 3.1154 \cdot 10^{-6}.$$

When $\Delta x = 0.01$ there are

$$P_0 > 6.23 \cdot 10^{-6},$$
$$P_0 \in \left(6.23 \cdot 10^{-6}; 6.23 \cdot 10^{-7}\right),$$
$$P_0 < 6.23 \cdot 10^{-7}.$$

Results for

$$P_0 > 1 \cdot 10^{-3} \cdot E \cdot \Delta x, \ \omega = 1.5016813$$

are shown in the Figure 3-a.

Calculation results with

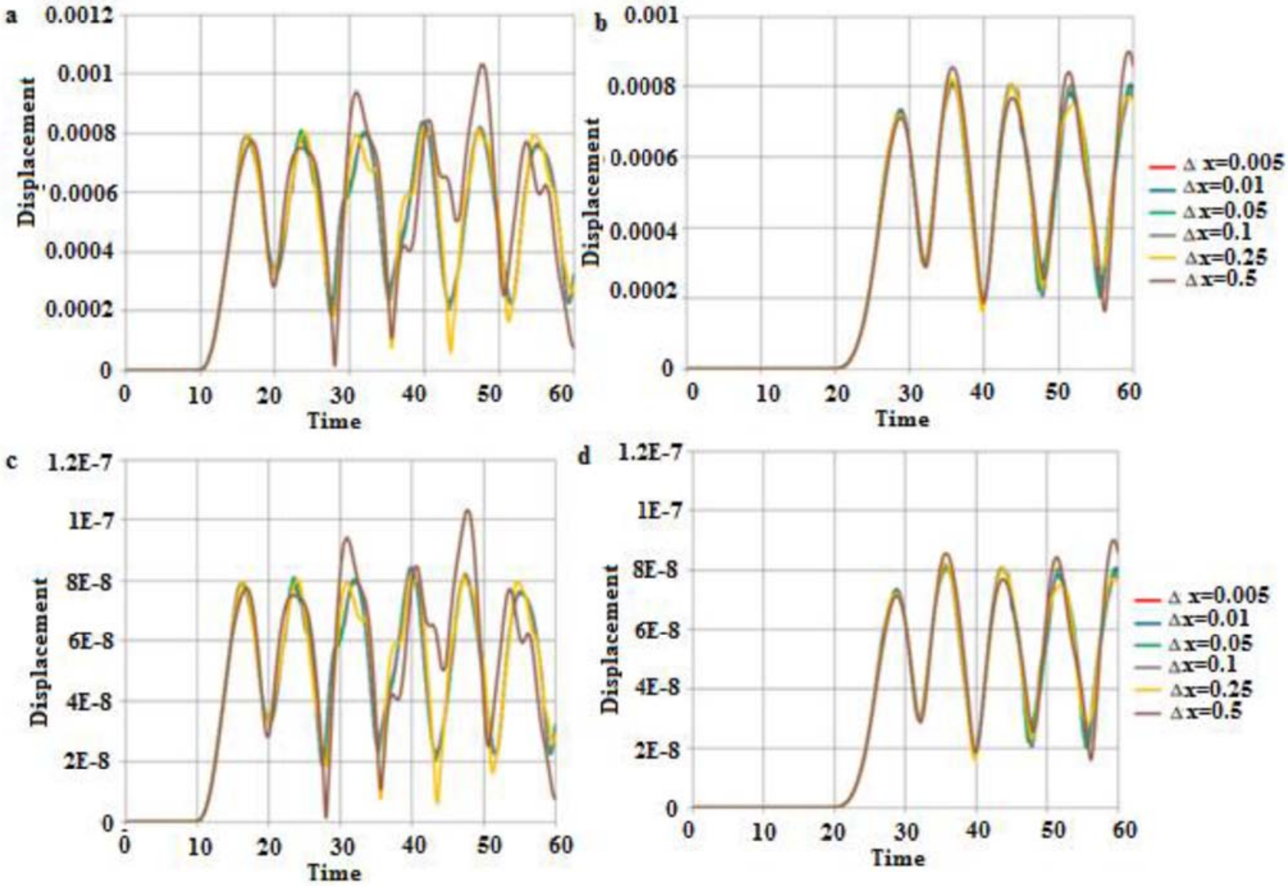

Figure 2. Displacement on the free surface when $\omega = 0.397657$ when the element dimension ranges from 0.005 to 0.5 *a) p.1 when* $P_0 > 1 \cdot 10^{-3} \cdot E \cdot \Delta x$, *b) p.2 when* $P_0 > 1 \cdot 10^{-3} \cdot E \cdot \Delta x$, *c) p.1 when* $P_0 < 1 \cdot 10^{-4} \cdot E \cdot \Delta x$, d) *p.2 when* $P_0 < 1 \cdot 10^{-4} \cdot E \cdot \Delta x$.

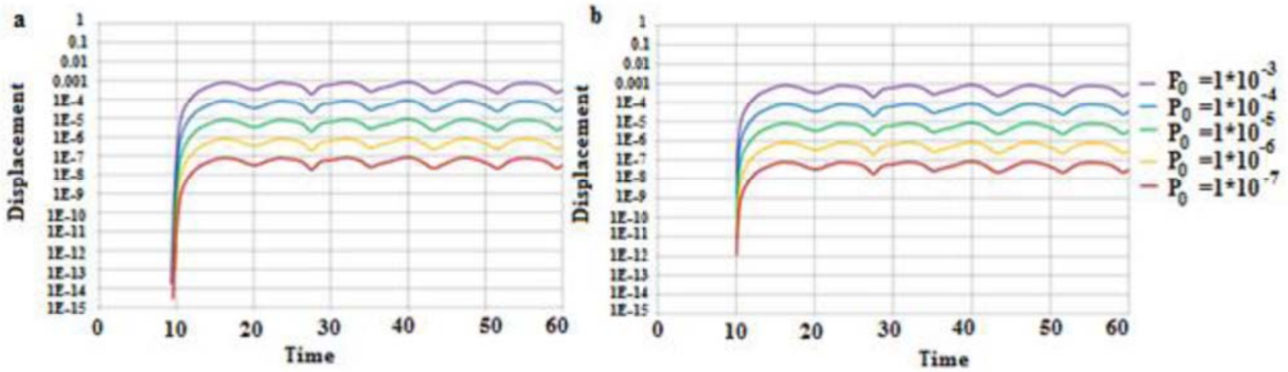

Figure 3. Displacement on the free surface for p.1 when a) $\Delta x = 0.05$, b) $\Delta x = 0.01$.

$$P_0 < 1 \cdot 10^{-4} \cdot E \cdot \Delta x, \omega = 1.5016813$$

are shown in the Figure 3-b.
From the Figure 4 follows that when $\omega = 1{,}5016813$ accurate solution is achieved when $\Delta x \leq 0.01$. In Figure 5 results are shown for the following cases:

$$P_0 > 1 \cdot 10^{-3} \cdot E \cdot \Delta x,$$
$$P_0 \in \left(1 \cdot 10^{-3} \cdot E \cdot \Delta x; 1 \cdot 10^{-4} \cdot E \cdot \Delta x\right),$$

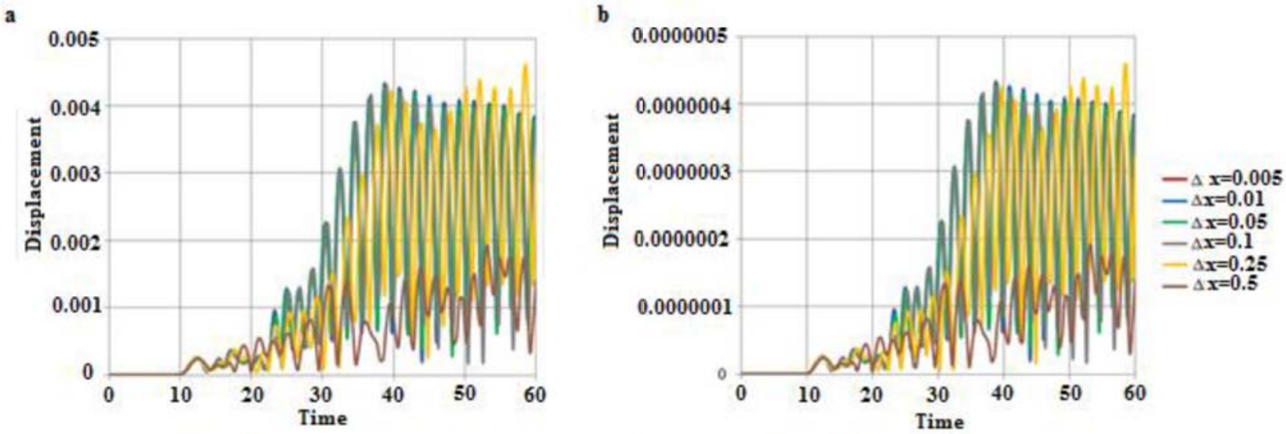

Figure 4. *Displacement on the free surface when* $\omega = 1.5016813$ *when the element dimension ranges from 0.005 to 0.5 a)* $P_0 > 1 \cdot 10^{-3} \cdot E \cdot \Delta x$, *b)* $P_0 < 1 \cdot 10^{-4} \cdot E \cdot \Delta x$.

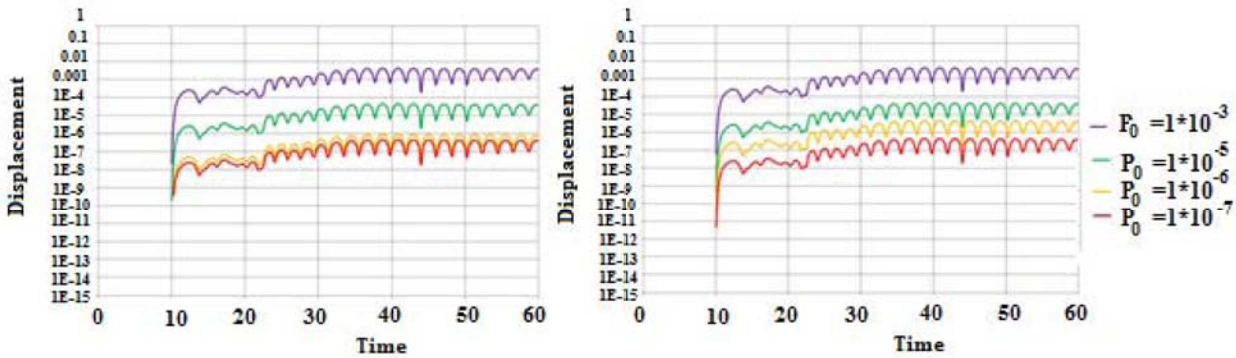

Figure 5. *Displacement on the free surface for p.1 when a)* $\Delta x = 0.05$, *b)* $\Delta x = 0.01$.

$$P_0 < 1 \cdot 10^{-4} \cdot E \cdot \Delta x.$$

When $\Delta x = 0.05$ there are

$$P_0 > 3.1154 \cdot 10^{-5},$$
$$P_0 \in \left(3.1154 \cdot 10^{-5}; 3.1154 \cdot 10^{-6}\right),$$
$$P_0 < 3.1154 \cdot 10^{-6}.$$

When $\Delta x = 0.01$ there are

$$P_0 > 6.23 \cdot 10^{-6},$$
$$P_0 \in \left(6.23 \cdot 10^{-6}; 6.23 \cdot 10^{-7}\right),$$
$$P_0 < 6.23 \cdot 10^{-7}.$$

It follows from the Figures 3 and 5 that amplitudes P_0 does not have a significant impact on the accuracy and stability of the solution.It means that in the investigated range of changes in external influences, the wave fields are described by linear equations, and the influence of geometrically nonlinear distortions can be neglected.

The dependence of the calculation time on element dimension is given in Table 4.1.

Table 4.1. *Dependence of the calculation time on element dimension.*

Element dimension Δx, cm	Calculation time, T, h.
0.005	4.4500
0.01	0.5200
0.05	0.0120
0.100	0.0100
0.250	0.0097
0.500	0.0070

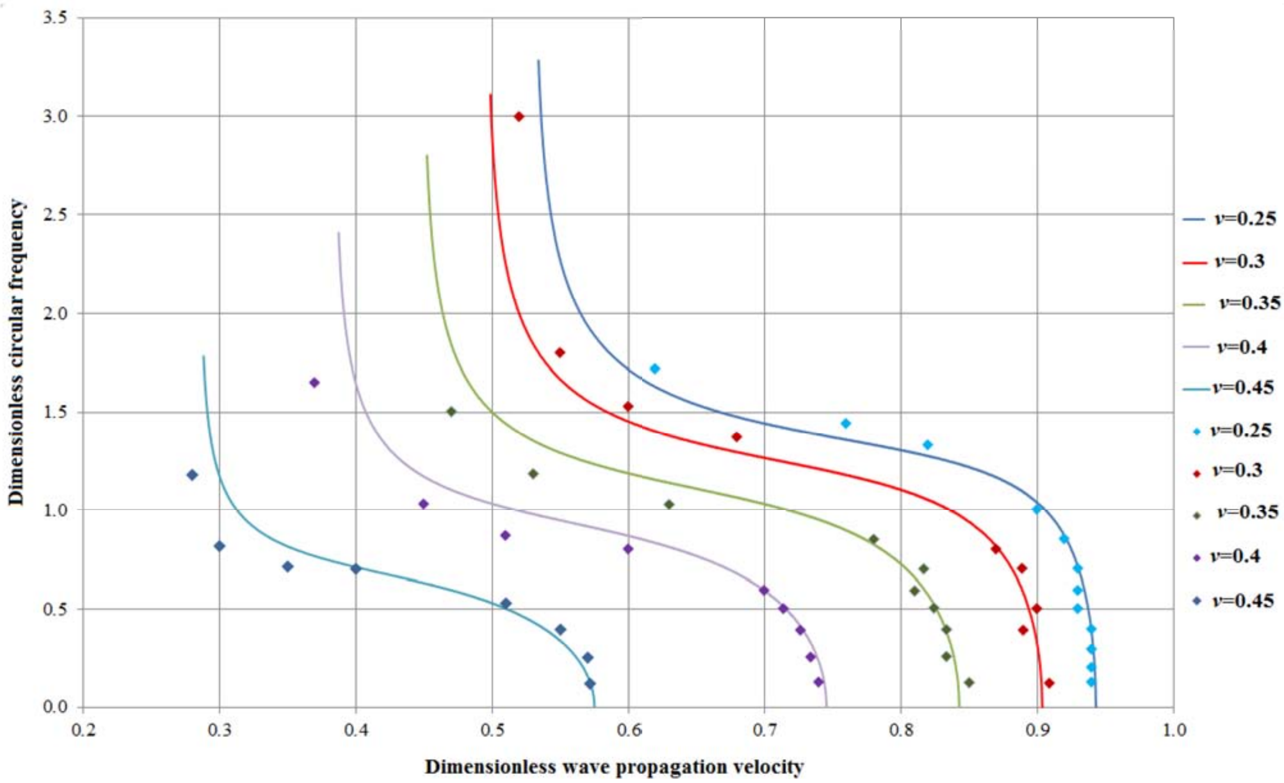

Figure 6. Comparative analysis of analytical (▬) and numerical (◆) solutions.

Thus, the optimal size of the element for this study is the size $\Delta x = 0.01$. It should be noted that for calculations with $\omega \to 0$ it is permissible to apply the size of the element $\Delta x = 0.05$. This will not affect the accuracy of the solution, but it will significantly reduce the calculation time.

When the Poisson ratio ranges from 0 to 0.5 comparative analysis of analytical and numerical solutions shown in Figure 6.

Also we studied the effect of element dimension on the Lamb waves polarization. The stability of the difference scheme was also achieved by fulfilling the Courant criterion. (3.3).

Study of the influence of the size of element dimension Δx on the accuracy of calculations and the stability of the difference scheme revealed that with the variation Δx in the interval $[0.005; 0.1]$ item size effect Δx on the accuracy of the solution is observed only at frequencies $\omega > 0.1$ and $\omega < 0.3$. When comparing results for $\Delta x = 0.005$, $\Delta x = 0.5$ and $\Delta x = 1$ item size effect Δx on the accuracy of the solution is observed only at frequencie $\omega > 0.1$ (Figure 7). For this case time, the speed of the solution decreases significantly.

5. CONCLUSIONS

It was first proposed the method of generating of surface waves using the FEM program complex. For the first time, a comparative analysis of the Lamb waves fundamental symmetric mode obtained by analytical methods and the finite element method. It was found stable finite element solution in the vicinity of the second limiting velocities.

Abaqus showed good results for solving problems of propagation of surface waves. We also to determine the range of problems in calculations by the FEM, the solution of which allowed us to minimize the errors caused by:

- the choice of the magnitude of the impact;
- selection of the frequency range;

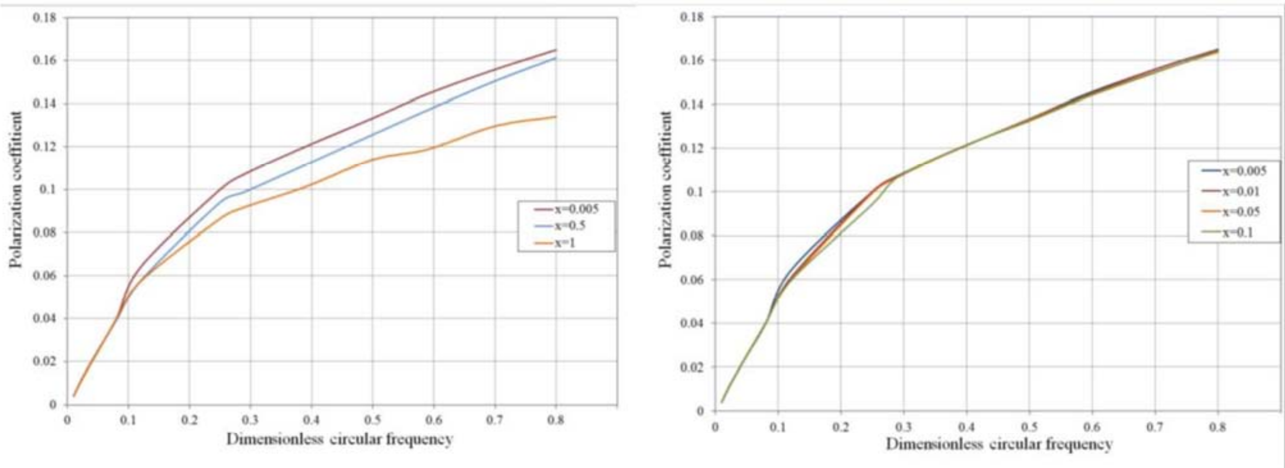

Figure 7. Investigation of mesh convergence forelement dimentions $\Delta x \in [0.005; 1]$.

- optimizing the time step using the finite element dimension.

The results of these studies are necessary for the application of acoustic non-destructive methods for diagnosing plate elements of building structures using Lamb waves.

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